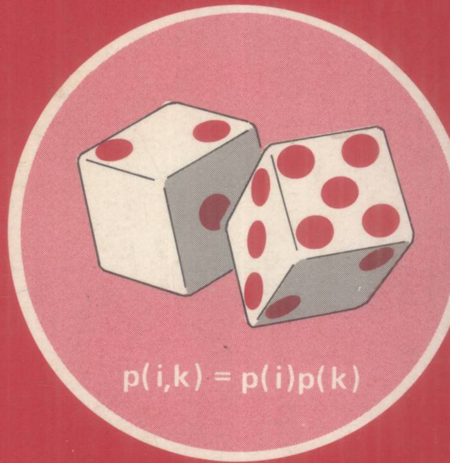
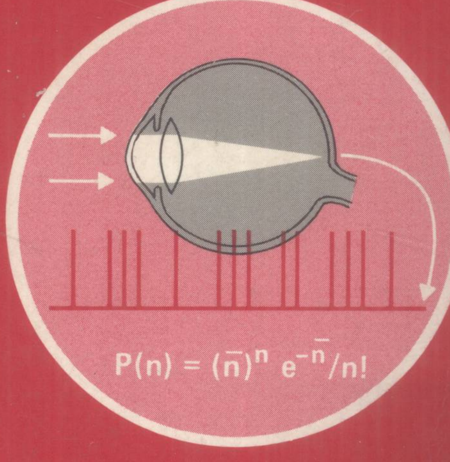
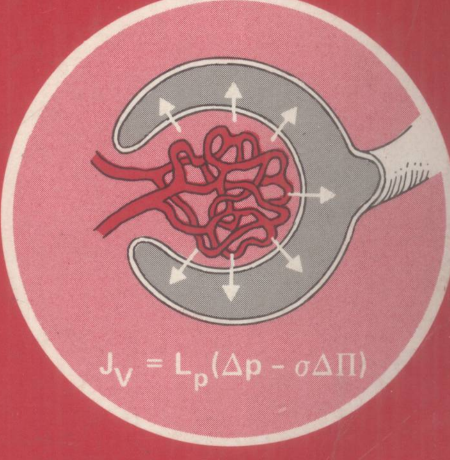
PHYSICS

WITH ILLUSTRATIVE EXAMPLES FROM MEDICINE AND BIOLOGY

Volume 2: STATISTICAL PHYSICS







Felix M. H. Villars
George B. Benedek

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Volume 2
STATISTICAL PHYSICS

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PREFACE

The structure of Classical Physics rests on three cornerstones: (1) Mechanics, (2) Electricity and Magnetism, and finally (3) Statistical Mechanics and Thermodynamics.

Newtonian Mechanics, the primary subject of Volume I of this series, provides a deterministic description of physical phenomena. If the forces acting on a body are known, and if the initial coordinates and momenta are known, then Newton's Laws of motion permit us to predict with great precision the subsequent motion of the body. Mechanics is a manifestly causal science. Newton's laws of motion provide the link between the causes (the forces) and the effects (the motion).

Electricity and Magnetism, the subject of Volume III of this series also provides a deterministic description of physical phenomena. In this case Maxwell's equations provide a deterministic framework which links the electric charges and their motion with the electric and magnetic fields.

The dominant role of these two disciplines and their customary location at the beginning of the physics curriculum naturally creates in the student's mind the view that physics is a rigidly deterministic discipline.

In the present volume we develop and present the ideas of statistical physics, of which statistical mechanics and thermodynamics are but one part. We seek to demonstrate to students, early in their career, the power, the broad range, and the astonishing usefulness of a probabilistic, non-deterministic view of the origin of a wide range of physical phenomena. By applying this approach analytically and quantitatively to problems such as: the size of random coil polymers; the diffusive flow of solutes across permeable membranes; the survival of bacteria after viral attack; the attachment of oxygen to the binding sites on the hemoglobin molecule; and the effect of solutes on the boiling point and vapor pressure of volatile solvents; we demonstrate that the probabilistic analysis of statistical physics provides a satisfying understanding of important phenomena in fields as diverse as physics, biology, medicine, physiology, and physical chemistry.

The development of the ideas of statistical physics does not follow a traditional and well established pattern as in the case of mechanics or electromagnetism. It is quite impossible to summarize the essence of statistical physics with a few compact expressions like Maxwell's Equations or Newton's Equations. We have nevertheless endeavored to develop our views of statistical physics into a logically satisfying structure starting with the elements of the theory of probability and concluding, in Chapter 4, with the deep insights of statistical mechanics and thermodynamics.

In Chapter 1 we present, along with some historical vignettes, a development of the fundamental ideas of probability. By considering the coin-toss process, which in fact is isomorphic to a wide variety of biological and physical processes, we develop the very basic binomial probability distribution. This distribution is then applied to three problems: the sex distribution of children in families of various size; the mean square radius of random coil polymers; and the mean and mean square electric charge and solubility of the protein hemoglobin, in solution.

Chapter 2 starts with an analysis of the one dimensional random walk problem in terms of the Bernoulli probability distribution. The time space evolution of this distribution leads to an understanding of the fundamental phenomena of diffusion and transport of solute particles in solution and across biological and synthetic membranes. This subject in fact is carried up to the most current level of understanding of the process of passive transport. Applications are made to artificial kidney dialysis, to the physiological function of the kidney and to the passage of solute and solvent across capillary walls.

Chapter 3, entitled "Poisson Statistics", begins with a careful analysis of the general conditions under which the Poisson probability distribution is applicable. In fact this distribution appears in the analysis of a wide variety of subjects ranging from economics to epidemiology to sports and war. The Poisson distribution plays a central role in the understanding of the detection of light at both very low and high light illumination levels as the student will see from our discussion of the famous Hecht, Shlaer, Pirenne experiment and the threshold intensity discrimination experiments of H. Barlow. We also analyze, in this chapter, the Luria-Delbrück experiment which proved, with the aid of purely statistical reasoning, that bacterial immunity to viral attack is the result of mutation and is not the result of the transmission of an acquired characteristic.

This volume closes with Chapter 4 which deals with the Boltzmann probability distribution and the thermal equilibrium of many body systems. In this final chapter we have devoted particular attention to the clear development and physical understanding of the central ideas of statistical mechanics e.g. the Boltzmann factor and the entropy. We recognize the entropy as the quantity whose maximization establishes the equilibrium state (Second Law of Thermodynamics), and use this to develop thermodynamics as a description of the relationship between equilibrium states of simple and composite systems. We believe that Chapter 4 contains many unique applications. For example we present the theory for the temperature dependence of the vapor pressure of a solid and show that this classical macroscopic phenomenon permits a quite accurate determination of Planck's constant. We also present a combined microscopic-thermodynamic theory which describes, with good accuracy, the shape of the important oxygen dissociation curves of myoglobin and hemoglobin. We believe that Chapter 4 will prove useful not only to students of physics and the life sciences, but will be particularly valuable for students of physical chemistry and chemical thermodynamics.

At the end of each chapter the reader will find many problems which clarify the physical, biological, or mathematical ideas presented in the text. The problems are graded in order of difficulty and are classified according to the textual topic where they apply.

The essential mathematical skills needed in the theory of probability are the ability to count possible patterns, and the ability to sum series, particularly the geometric series and the binomial series. These skills employ a knowledge of high school algebra and a student versed in this subject should encounter little difficulty with the mathematics of probability theory. Elementary differential and integral calculus are employed here on about the same level as in Volume I. In the present volume we frequently use the partial derivative of a function $f(x,y,z)$ of several variables. The student should understand that the partial derivative of f with respect to some variable, say z, $(\partial f/\partial z)$ is the same as the usual first derivative except that in calculating it one holds constant all the other independent variables $(x,y = \text{const.})$.

The experienced teacher will recognize that this volume contains more material than can normally be presented in a one semester course. The text has been written in such a way as to permit selective omissions without loss of information necessary for subsequent sections. For example, one need not present all the applications included in Chapter 1. Also, in Chapter 2, one might elect not to include section 2.5 or 2.6. Chapter 3 is largely self contained and is not a prerequisite for Chapter 4. In Chapter 4 the Instructor may wish, for example, to present the statistical mechanics portion and omit the thermodynamics which follow. In general we encourage the Instructor to make use of the flexibility offered by the structure of this text, so as to employ that material most suited to his particular objectives.

We have presented the subject of Statistical Physics (Vol. 2) before that of Electricity and Magnetism (Vol. 3) because of our deeply felt belief in the overriding importance of probabilistic ideas in physics. These ideas are central in the description of phenomena of crucial importance for students of physics, chemistry, and the life sciences. We recognize, of course, Electricity and Magnetism is taught conventionally immediately after Mechanics. Volume 3, <u>Electricity and Magnetism with Illustrative Examples from Medicine and Biology</u>, will be written in such a way as to permit the Instructor to maintain that conventional pattern, if he desires.

ACKNOWLEGDEMENTS

We acknowledge, with gratitude, the continued encouragement and farsighted support of Professor Irving London, M.D., and the Harvard-M.I.T. Program in Health Sciences and Technology.

We are deeply indebted to our exceptional secretary Miss Mary Patton Fitzgerald. Despite the heavy press of many other duties she typed this book with remarkable accuracy and speed. She is responsible for the visual balance and clarity of this text.

Felix M.H. Villars

George B. Benedek

Cambridge, Mass.

August 1, 1974

For the convenience of readers interested in Volumes 1 and 3
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PHYSICS: WITH ILLUSTRATIVE EXAMPLES FROM MEDICINE AND BIOLOGY

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