FLOW AND TRANSPORT IN POROUS MEDIA

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Flow and Transport in Porous Media

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FLOW AND TRANSPORT IN POROUS MEDIA

Preface

The objective of Euromech Colloquia is to provide opportunities for European scientists, working in a specialized area of mechanics, to meet each other, and to discuss their current research activities. Meetings are rather small in size and informal in character.

In March 1980 the European Mechanics Committee decided to include a colloquium on Flow and Transport in Porous Media in the program for 1981, to be held at Delft, The Netherlands.

Because the response to the first announcement, which was sent to a selected number of prospective participants, was rather encouraging, with a variety of papers being offered, it was decided to print the papers submitted before the colloquium in a separate volume, published by A.A. Balkema, Rotterdam. The organizers are very happy that a large number of participants managed to complete their manuscript in the short time available.

It is our belief that the present book gives an up-to-date review of the state of the art in the field of Flow and Transport in Porous Media. This branch of science is rapidly growing, especially because of the importance of problems of pollution of groundwater, of heat transport and storage of solar energy in the ground, of hydraulics and mechanics in large granular structures, and of the simultaneous flow of different fluids in porous media. All these subjects are covered in this volume, together with a number of papers on the fundamental aspects of flow through porous media.

A. Verruijt
F.B.J. Barends

Contents

1. Dynamics of fluids in porous media

Proceedings of Euromech 143 / Delft / 2-4 September 1981

Shock-induced flow in a porous medium

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ABSTRACT

First experiments and evaluations were made of shock-induced flow in a solid matrix. The experiments were performed in a shock tube measuring pressure in front of and within the solid matrix as a function of time and location. Qualitative information on the flow pattern was obtained from shadowgraphs. These experimental data were compared with a numerical solution obtained by solving the transport equations for two-phase flow. Tentative adjustments of the drag and heat-transfer laws of Ergun (1952) and Yoshida et al. (1962) were made to fit the numerical results to the experimental data.

1 INTRODUCTION

Flow through porous media and packed beds occur in many branches of science and technology. A number of empirical correlations about the terms which describe momentum and energy exchange between the gas phase and the matrix of solid material are available. They have however been established for low Reynolds numbers and their extrapolation to high Reynolds numbers is not permissible, as was shown by Kuo and Nydegger (1978). They also fail at high Mach numbers and steep pressure gradients. Also time dependence may become important, e.g. when the flow is induced by a shock wave, as was found by Heilig and Reichenbach (1979,1980) who studied shock loading experimentally. They found that the drag coefficient for a single cylindrical body is strongly time dependent and its value can exceed the steady state value by a factor of up to 30, depending on Mach and Reynolds numbers. Schultz-Grunow (1972) studied the interaction of an incident shock wave with obstacles along walls. He observed increased pressure peaks depending on the details of the geometrical arrangement. Zloch (1974) investigated the decrease of shock strength in an array of axially arranged tublets experimentally and numerically. He used a shock tube to generate the incident shock. By making the area change rather large, he achieved steady-state boundary conditions at the entrance into the system of tublets. However, this is only a limiting case. For medium and large fractions of void volume the boundary conditions at the surface of the matrix change with time after shock impingement. This may be understood as an obstruction effect increasing with time.

In the present paper this more general case is studied experimentally and theoretically and first results are presented. The experiments are performed in a shock tube taking as matrix either a packed bed or an array of cylinders, with their axis perpendicular to the tube axis. Pressure profiles are measured in dependence upon time and location in front of and within the matrix. Also shadowgraphs are taken to obtain qualitative informations on the shock-induced flow pattern. These data are compared with numerical results obtained by solving the basic equations of two-phase flow using a method of characteristics. Particularly account was taken of coupling the two regions of flow in front of and within the matrix. For the drag and heat transfer the relations given by Ergun (1952) and Yoshida et al. (1962) were used. They had to be modified by a constant factor to obtain a first agreement between computational results and experimentally obtained pressure profiles. More detailed measurements of pressure and of heat transfer are in progress. It is expected that they will provide sufficient information to establish the drag and heat transfer laws valid under the conditions in more detail.

2 EXPERIMENTAL APPARATUS

The shock tube arrangement used for the experiments is represented schematically in Fig. 1. It consists of a tube of rectangular cross section 54 x 54 mm with the test section arranged at the end. The resulting

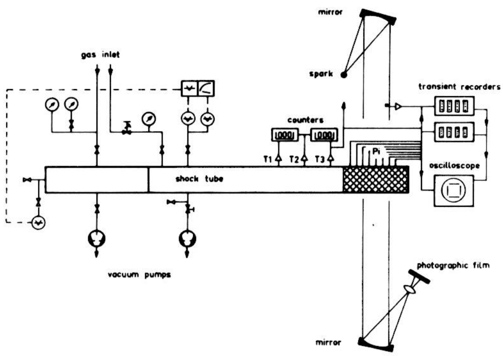

Fig. 1 Schematic diagram of the shock tube

wave system is represented schematically in Fig. 2. The Mach number of the incident shock wave was determined using film temperature gages. Pressure profiles were measured using seven gages of the type Kistler 603 B, 6031 and Piezotronics 113A21, 113A24. The signals were registered using oscilloscopes and transient recorders.

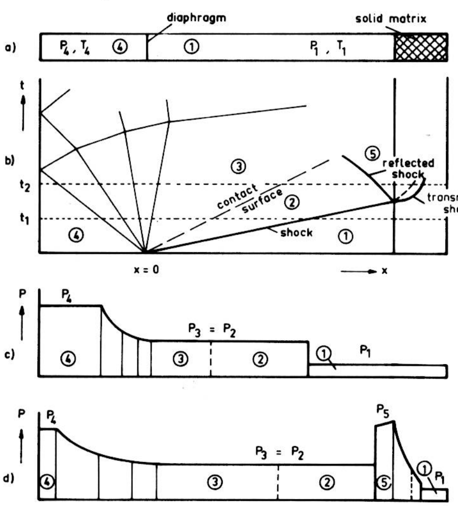

Fig. 2 Schematic representation of the wave system

Optical information on the flow pattern was obtained using a shadowgraph arrangement with single spark flashes initiated at suitable time intervals. A Cranz-Schardin multiple spark system is in preparation. As solid matrix a packed bed of spherical glas pellets of diameter 4, 5, 6 and 8 mm and also an array of cylinders 5 mm in diameter arranged at distances of 9 mm perpendicular to the tube axis was investigated.

3 EXPERIMENTAL RESULTS

Some of the experimental results are represented in Figs. 3 to 8. Figs. 3 to 6 refer to the fixed bed and Fig. 7 and 8 refer to the array of cylinders.

The shock wave incident upon the solid matrix is partially reflected and partially transmitted by the elements of the matrix. The complexity of the resulting wave and flow pattern may be gathered from the shadowgraphs of Fig. 7. In front of the array a first strong reflection is followed by various weaker shock waves which lead to an acceleration of the reflected shock and a continuous pressure increase in this region. Fig. 3 shows some typical pressure records obtained for a packed bed. The

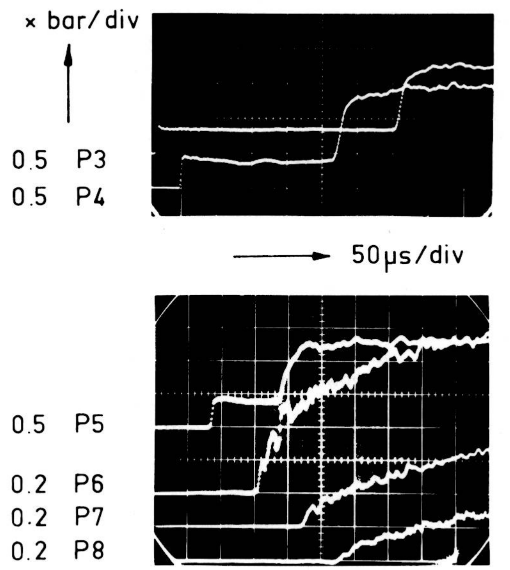

Fig. 3 Pressure records obtained for a packed bed

pressure gages No. 3 to 5 were placed in front of and No. 6 to 8 inside the bed. Probe No. 6 shows three distinctive pressure peaks which indicate shock waves, whereas probe No. 8 at a location farther inside the bed shows already an essentially continuous pressure increase, which however, as Fig. 7 shows, may still be caused by a multiplicity of weak shock waves.

Plotting the pressures measured by the

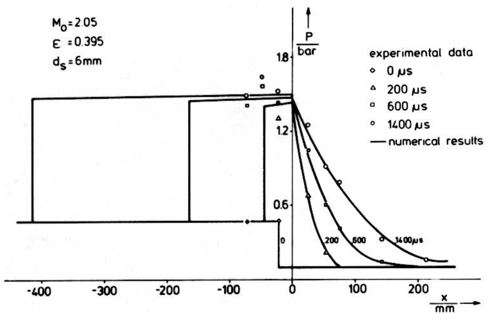

Fig. 4 Pressure p vs. location x for various times t. Points: measurements. Solid curves were obtained by numerical solution adjusting D and q.

different gages at fixed times vs. location the representations shown in Figs. 4 to 6 are obtained. In Fig. 4 values of pressure readings at various time intervals are shown. In Fig. 5 results are given for various pellet diameters and in Fig. 6 for various Mach numbers of the incident shock wave. The

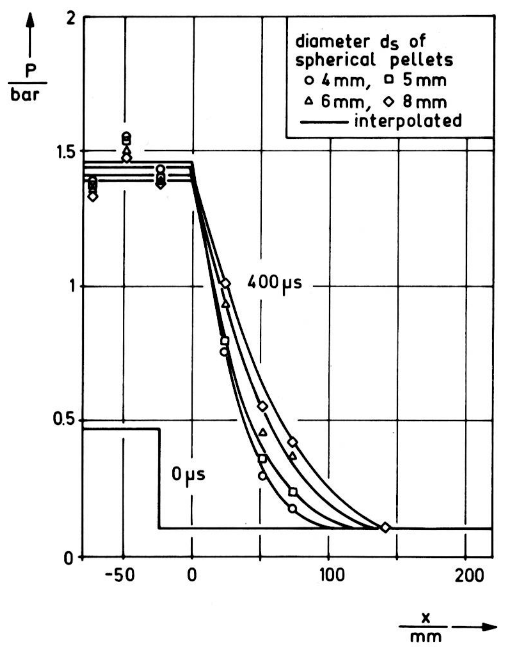

Fig. 5 Pressure p vs. location x for various diameters d_S of pellets. Points: measurements. Solid curves obtained by interpolating the measurements.

pressure level in front of the bed increases with decreasing diameter of the pellets and increasing Mach number approaching in the limit the value of a solid end wall.

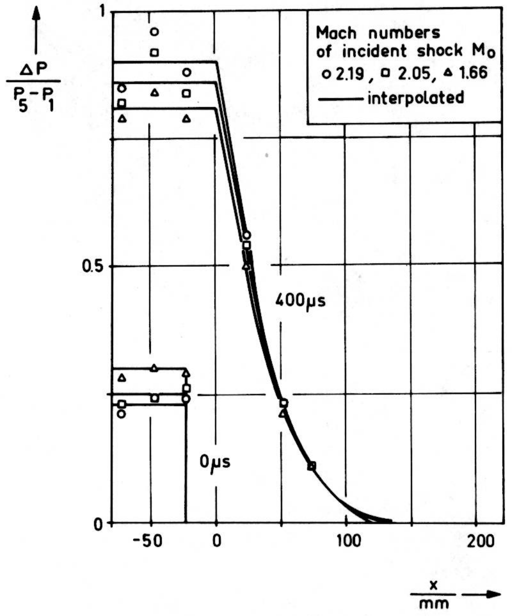

Fig. 6 Pressure p vs. location x for various Mach numbers M_o of incident shock. Points: measurements. Solid curves obtained by interpolating the measurements.

The results obtained with the array of cylinders are illustrated in Figs. 7 and 8.

Fig. 7 shows shadowgraphs of the resulting wave and flow pattern. Its various features may be explained by comparison with the flow and wave pattern observed for a single cylinder (Heilig 1980). Remarkable is the strong effect of the boundary layer along the front of the cylinders upon the light refraction and also the formation of dual vortices behind the obstacles. Between the cylinders a supersonic jet is formed which leads to a bow shock wave in front of the next cylinder. This jet and the recirculation region is separated by a shear layer well visible e.g. in the second photograph of Fig. 7. The pressure readings obtained at the locations 4 to 7 as indicated in Fig. 7 are shown in Fig. 8. The pressure curve of gage No. 4 shows quite clearly the incident shock wave and the pattern of the reflected shock waves. Also at the probes located inside the arry various waves may be clearly distinguished. The pressure values belonging to the various regions of the flow field are obtained from these measurements.

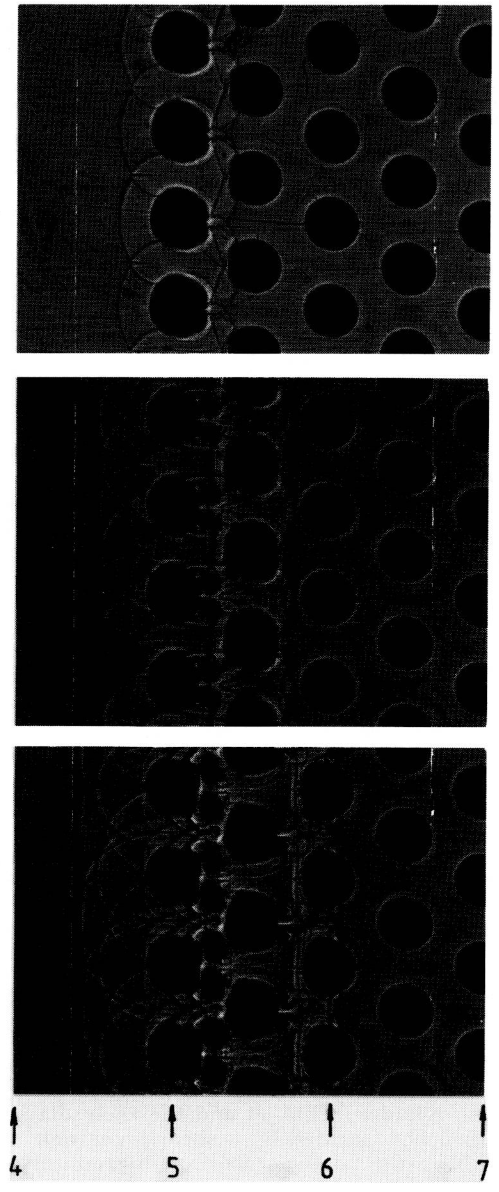

Fig. 7 Shadowgraphs of the shock induced flow through a cylindrical array taken at three distinct time intervals after shock impingement. Arrows indicate the location of pressure probes.

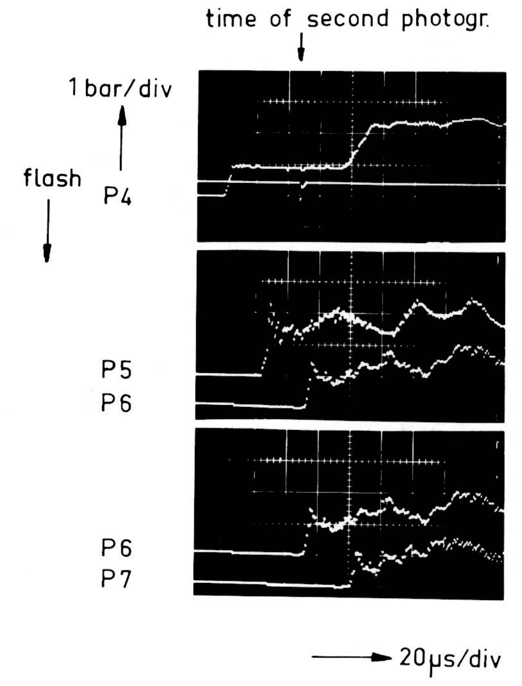

Fig. 8 Pressure readings corresponding to Fig. 7

4 THEORY: BASIC EQUATIONS

In the one-dimensional conservation equations for unsteady two-phase flow (Wallis 1969) the following terms are neglected:

a) heat loss to the tube wall
b) friction force exerted on the gas by the tube wall
c) work done due to viscous stress generated from velocity gradient in the gas phase
d) homogeneous heat dissipation
e) heat conduction in the gas phase
f) temperature dependence of constant pressure specific heat.

After rearranging various terms and inserting the perfect gas law the following set of partial differential equations is obtained (Wallis 1969, Zucrow and Hoffman 1976)

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} = -\rho \frac{\partial u}{\partial x} \qquad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x} + \frac{s}{\varepsilon} D\right) \qquad (2)$$

$$\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} = -\gamma p \frac{\partial u}{\partial x} - (\gamma - 1)\frac{s}{\varepsilon}(q - Du) \qquad (3)$$

The heat flux q and the drag force D are defined by (Bird et al. 1960, Krier and

Rajan 1975)

$$q = h(T - T_s) \tag{4}$$

$$D = \frac{\varepsilon^3}{3(1-\varepsilon)} \rho u^2 f \tag{5}$$

T_s denotes the surface temperature of the matrix material and f is the nondimensional friction factor. For spherical pellets the specific surface s is (Bird et al. 1960)

$$s = \frac{6(1-\varepsilon)}{d_s} \tag{6}$$

Under assumption of an uniform pellet temperature an energy balance leads to the following ordinary differential equation for T_s

$$\frac{\partial T_s}{\partial t} = \frac{s}{1-\varepsilon} \frac{q}{\rho_s c_s} \tag{7}$$

These equations are used to describe the flow inside the solid matrix and also - setting D=q=0 - the flow in the upstream region in front of the matrix. These two regions are coupled by

the energy equation for quasi-steady flow of perfect gas

$$\frac{\gamma}{\gamma-1} \frac{p}{\rho} + u^2/2 = \text{const.} \tag{8}$$

the isentropic law

$$p/\rho^\gamma = \text{const.} \tag{9}$$

and the continuity equation for quasi-steady flow

$$\rho u A = \text{const.} \tag{10}$$

The applicability of these equations as coupling conditions rests upon the fact that the extension of the transition zone between solid matrix and free flow is small compared to the characteristic length of the problem. The conditions have e.g. been discussed by Laporte (1954). They establish a dependence between porosity ε, the states in front of the incident shock wave and the Mach numbers of the incident, reflected and transmitted shock waves.

For the friction factor f the following relation (Ergun 1952, Bird et al. 1960) was used.

$$f = \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{75}{Re_s} + 0.875 \frac{(1-\varepsilon)}{\varepsilon^3} \tag{11}$$

Here the Reynolds number Re_s is defined by

$$Re_s = \varepsilon \frac{\rho u d_s}{\mu} \tag{12}$$

The heat transfer law as given by Yoshida et al., 1962 (see also Bird et al. 1960) was used

$$j_H = 0.91\, Re^{-0.51} \quad (Re < 50)$$
$$j_H = 0.61\, Re^{-0.41} \quad (Re > 50) \tag{13}$$

where the so-called Colburn factor j_H and the Reynolds number Re are defined by

$$j_H = \frac{h}{\rho u c \varepsilon} Pr^{2/3} \tag{14}$$

$$Re = \frac{\rho u \varepsilon}{s \mu \psi} \tag{15}$$

For spherical pellets the shape factor ψ is equal to one.

5 RESULTS AND DISCUSSION

In Fig. 9 some profiles obtained numerically for pressure p, temperature T and flow velocity u are plotted vs. coordinate at t = 300 μs after the shock impingement upon the solid matrix. In front of the matrix the reflected shock wave propagates to the left. A continuous increase of p and T and a decrease of u is observed between this shock wave and the surface. This may be understood as an obstruction effect of the matrix increasing with time. At the front of the matrix (location at x=0) the gas observes the balance equations formulated in Eqs. (8) to (10) resulting in an acceleration of the flow in a zone assumed here to be infinitely thin. This leads to the discontinuous changes of variables of state at x=0, causing the observed drops of pressure and temperature. The transmitted shock wave located at x ≈ 130 mm has already been attenuated considerably caused by the drag force D. At the contact surface between the gas originally seperating the matrix and the gas outside, a temperature jump and a pronounced peak are observed.

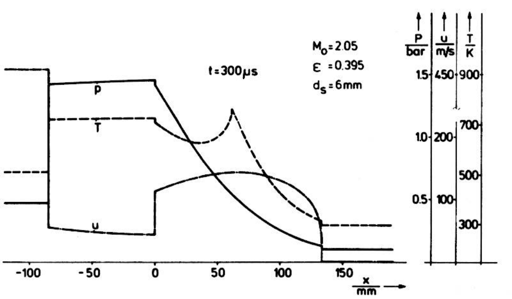

Fig. 9 Profiles of T, u and p 300 μs after shock impingement

In Fig. 10 the various influences contributing to this effect can be studied. Here the temperature profiles are plotted vs. x for a fixed time interval introducing various values for drag D and heat transfer q. A comparison between curves 1 and 2 shows that the heat flux q gives only a small contribution to the reduction of the shock strength. The comparison between curves 1 and 2 on the one side and 3 to 5 on the other side shows that this reduction is essentially due to the drag force term in the balance of momentum (Eq. (2)). Curves 3 and 5 differ in the presence of the drag force term in the energy balance (Eq. (3)), which represents the heat dissipation caused by the flow resistance of the solid matrix. It can be gathered that this term causes the strong temperature increase in the environment of the contact surface, which can be observed in comparing these two curves.

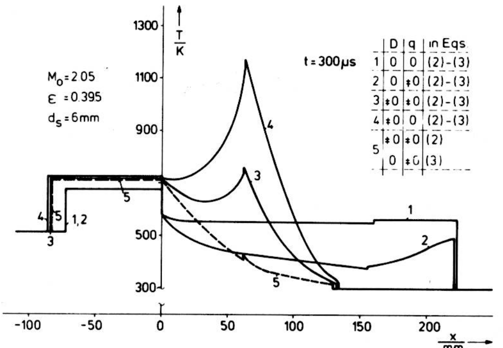

Fig. 10 Numerical solution of T vs. x for various values of D and q in Eqs. (2)-(3)

Fig. 11 shows the pressure profiles corresponding to the temperature profiles of Fig. 10 exhibiting again the strong difference in shock attenuation caused by the drag force in the momentum equation.

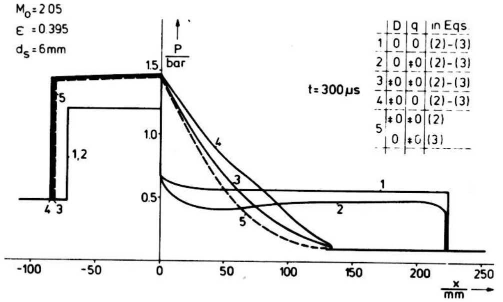

Fig. 11 Numerical solution of p vs. x for various values of D and q in Eqs. (2)-(3)

Results obtained by the numerical computation have also been plotted in Fig. 4 (solid curves). It is found that the dependences of drag force D and heat transfer q as formulated by Eqs. (11) and (13) lead to pronounced deviation between experiment and theory. Acceptable agreement for the pressure distribution inside the packed bed is achieved by multiplying the drag force term D by a factor of 1.5 and the heat-transfer term q by a factor of 4. However, this procedure does not lead to a good agreement in front of the matrix. The fact, that the original terms have to be altered may be essentially due to the unsteady behavior of the flow and to the high Reynolds and Mach numbers when the flow initially is induced by the transmitted shock wave.

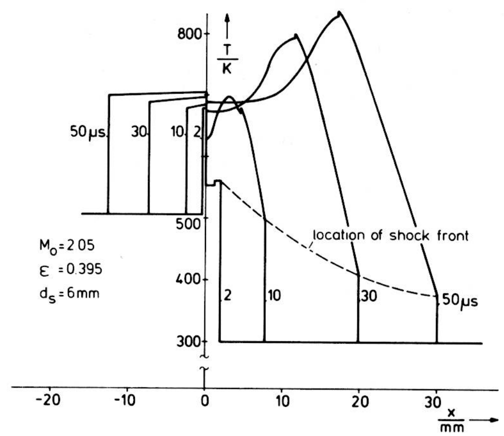

Fig. 12 Development of temperature profiles and decrease of shock strength with time obtained by numerical solution

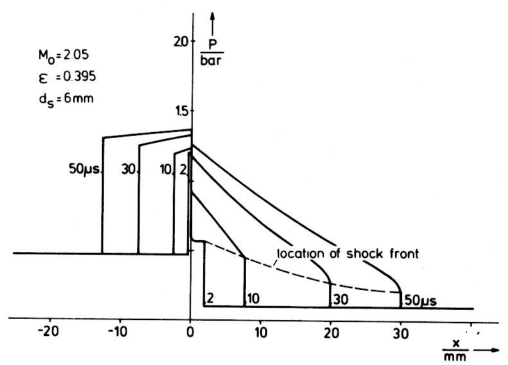

Fig. 13 Development of pressure profiles and decrease of shock strength with time obtained by numerical solution

Figures 12 and 13 show the development of temperature and pressure profiles within the solid matrix for short time intervals.

It can be seen, that the attenuation of the transmitted shock is very rapid. The reflected shock, however, is reenforced due to the obstruction of the matrix increasing with time.

6 CONCLUSIONS

Shock loading of a solid matrix consisting of a packed bed of spherical pellets and an array of cylinders perpendicular to the flow has been investigated experimentally and theoretically. The experiments were performed in a shock tube measuring the pressure distribution in dependence upon time with the Mach number of the incident shock and with pellet diameter as parameters. Also shadowgraphs were taken giving qualitativ information about the flow and wave pattern. All the essential results obtained experimentally so far (cf. Sec. 3) were confirmed by comparison with a numerical solution obtained by solving the basic balance equations of two phase flow (cf. Sec. 5). The relations used commonly for the drag force and heat transfer between the gas and the solid matrix had to be adjusted. It was found that in the average for the investigated shock induced flow they posses a higher value than under steady state conditions. For the tentative correction chosen the pressure development inside the bed could be represented sufficiently well, however not the distribution in front of the bed. Further measurements are in progress to obtain additional information about the interaction terms.

Nomenclature

- A cross-sectional area of the tube
- c specific heat
- D drag force acting on gas per unit wetted area of porous medium or of particles
- d_s diameter of pellets
- f friction factor
- h heat transfer coefficient
- j_H Colburn-factor
- M_o Mach number of incident shock
- p pressure
- Pr Prandtl number
- q heat flux per unit wetted area of porous medium or of particles
- Re Reynolds number
- s specific wetted surface area of porous medium or of packed bed
- t time
- T absolute temperature
- u velocity
- x coordinate

Greek Symbols

- γ ratio of specific heats
- ε porosity, volume fraction of void
- μ dynamic viscosity
- ρ density

Subscript

- s solid

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