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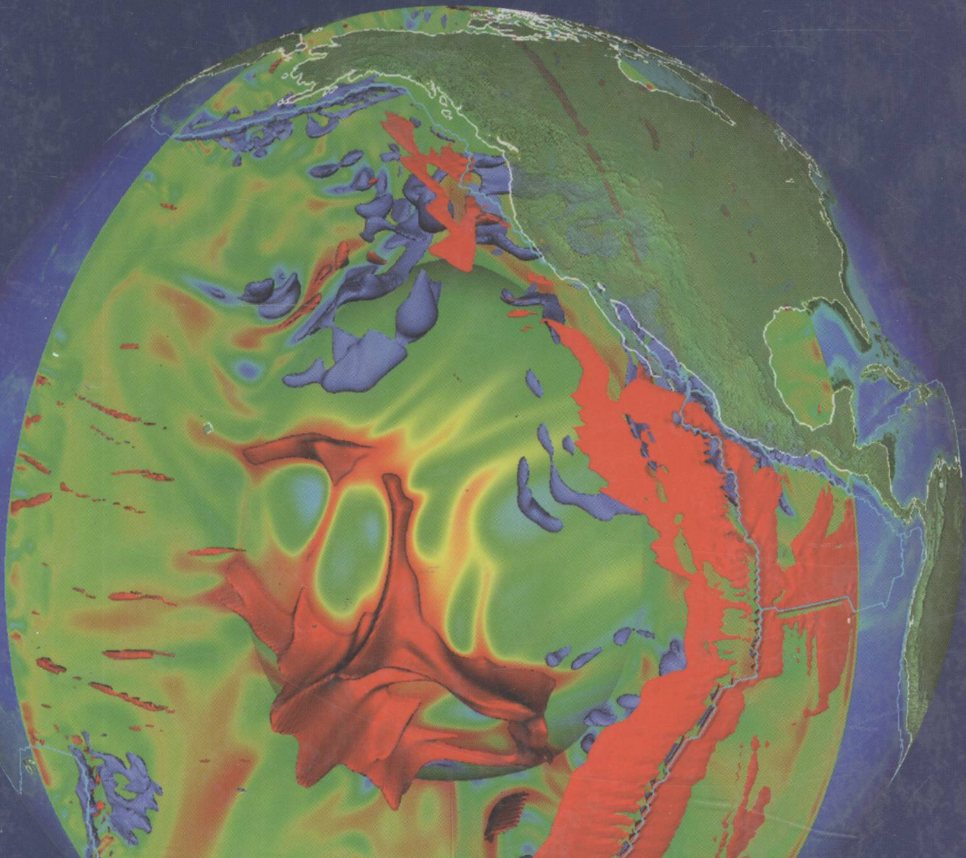
Brian L. N. Kennett and Hans-Peter Bunge



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Deformation in the Earth's Interior

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Geophysical Continua presents a systematic treatment of deformation in the Earth from seismic to geologic time scales, and demonstrates the linkages between different aspects of the Earth's interior that are often treated separately.

A unified treatment of solids and fluids is developed to include thermodynamics and electrodynamics, in order to cover the full range of tools needed to understand the interior of the globe. A close link is made between microscopic and macroscopic properties manifested through elastic, viscoelastic and fluid rheologies, and their influence on deformation. Following a treatment of geological deformation, a global perspective is taken on lithospheric and mantle properties, seismology, mantle convection, the core and Earth's dynamo. The emphasis throughout the book is on relating geophysical observations to interpretations of earth processes. Physical principles and mathematical descriptions are developed that can be applied to a broad spectrum of geodynamic problems.

Incorporating illustrative examples and an introduction to modern computational techniques, this textbook is designed for graduate-level courses in geophysics and geodynamics. It is also a useful reference for practising Earth Scientists. Supporting resources for this book, including exercises and full-colour versions of figures, are available at www.cambridge.org/9780521865531.

BRIAN KENNETT is Director and Distinguished Professor of Seismology at the Research School of Earth Sciences in The Australian National University. Professor Kennett's research interests are directed towards understanding the structure of the Earth through seismological observations. He is the recipient of the 2006 Murchison Medal of the Geological Society of London, and the 2007 Gutenberg Medal of the European Geosciences Union, and he is a Fellow of the Royal Society of London. Professor Kennett is the author of three other books for Cambridge University Press: *Seismic Wave Propagation in Stratified Media* (1983), *The Seismic Wavefield: Introduction and Theoretical Development* (2001), and *The Seismic Wavefield: Interpretation of Seismograms on Regional and Global Scales* (2002).

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Preface

Geophysical Continua is designed to present a systematic treatment of deformation in the Earth from seismic to geologic time scales. In this way we demonstrate the linkages between different aspects of the Earth's interior that are commonly treated separately. We provide a coherent presentation of non-linear continuum mechanics with a uniform notation, and then specialise to the needs of particular topics such as elastic, viscoelastic and fluid behaviour. We include the concepts of continuum thermodynamics and link to the properties of material under pressure in the deep interior of the Earth, and also provide the continuum electrodynamics needed for conducting fluids such as the Earth's core.

Following an introduction to continuum methods and the structure of the Earth, Part I of the book takes the development of continuum techniques to the level where they can be applied to the diverse aspects of Earth structure and dynamics in Part II. At many levels there is a close relation between microscopic properties and macroscopic consequences such as effective rheology, and so Part II opens with a discussion of the relation of phenomena at the atomic scale to continuum properties. We follow this with a treatment of geological deformation at the grain and outcrop scale. In the subsequent chapters we emphasise the physical principles that allow understanding of Earth processes, taking a global perspective towards lithospheric and mantle properties, seismology, mantle convection, the core and Earth's dynamo. We make links to experimental results and seismological observations to provide insight into geodynamic interpretations.

The material in the book has evolved over a considerable time period and has benefited from interactions with many students in Cambridge, Canberra, Princeton and Munich. Particular thanks go to the participants in the Geodynamics Seminar in Munich in 2005, which helped to refine Part I and the discussion of the lithosphere in Part II.

In a work of this complexity covering many topics with their own specific notation it is difficult to avoid reusing symbols. Nevertheless we have have tried to sustain a unified notation throughout the whole book and to minimise multiple use.

We have had stimulating discussions with Jason Morgan, John Suppe and Geoff Davies over a wide range of topics. Gerd Steinle-Neumann provided very helpful

input on mineral properties and *ab initio* calculations, and Stephen Cox provided valuable insight into the relation of continuum mechanics and structural geology.

Special thanks go to the Alexander von Humboldt Foundation for the Research Award to Brian Kennett that led to the collaboration on this volume.

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Contents

1

Introduction

The development of quantitative methods for the study of the Earth rests firmly on the application of physical techniques to the properties of materials without recourse to the details of atomic level structure. This has formed the basis of seismological methods for investigating the internal structure of the Earth, and for modelling of mantle convection through fluid flow. The deformation behaviour of materials is inextricably tied to microscopic properties such as the elasticity of individual crystals and processes such as the movement of dislocations. In the continuum representation such microscopic behaviour is encapsulated in the description of the rheology of the material through the connection between stress and strain (or strain rate).

Different classes of behaviour are needed to describe the diverse aspects of the Earth both in depth and as a function of time. For example, in the context of the rapid passage of a seismic wave the lithosphere may behave elastically, but under the sustained load of a major ice sheet will deform and interact with the deeper parts of the Earth. When the ice sheet melts at the end of an ice age, the lithosphere recovers and the pattern of post-glacial uplift can be followed through raised beaches, as in Scandinavia.

The Earth's core is a fluid and its motions create the internal magnetic field of the Earth through a complex dynamo interaction between fluid flow and electromagnetic interactions. The changes in the magnetic field at the surface on time scales of a few tens of years are an indirect manifestation of the activity in the core. By contrast, the time scales for large-scale flow in the silicate mantle are literally geological, and have helped to frame the configuration of the planet as we know it.

We can link together the many different facets of Earth behaviour through the development of a common base of continuum mechanics before branching into the features needed to provide a detailed description of specific classes of behaviour. We start therefore by setting the scene for the continuum representation. We then review the structure of the Earth and the different types of mechanical behaviour that occur in different regions, and examine some of the ways in which information at the microscopic level is exploited to infer the properties of the Earth through both experimental and computational studies.

1.1 Continuum properties

A familiar example of the concept of a continuum comes from the behaviour of fluids, but we can use the same approach to describe solids, glasses and other more general substances that have short-term elastic and long-term fluid responses. The behaviour of such continua can then be established by using the conservation laws for linear and angular momentum and energy, coupled to explicit descriptions of the relationship between the stress, describing the force system within the material, and the strain, which summarises the deformation.

We adopt the viewpoint of continuum mechanics and thus ignore all the fine detail of atomic level structure and assume that, for sufficiently large samples,:

- the highly discontinuous structure of real materials can be replaced by a smoothed hypothetical continuum; and
- every portion of this continuum, however small, exhibits the macroscopic physical properties of the bulk material.

In any branch of continuum mechanics, the field variables (such as density, displacement, velocity) are conceptual constructs. They are taken to be defined at all points of the imagined continuum and their values are calculated via axiomatic rules of procedure.

The continuum model breaks down over distances comparable to interatomic spacing (in solids about 10^{-10} m). Nonetheless the *average* of a field variable over a small but *finite* region is meaningful. Such an average can, in principle, be compared directly to its nominal counterpart found by experiment – which will itself represent an average of a kind taken over a region containing many atoms, because of the physical size of any measuring probe.

For solids the continuum model is valid in this sense down to a scale of order 10^{-8} m, which is the side of a cube containing a million or so atoms.

Further, when field variables change slowly with position at a microscopic level $\sim 10^{-6}$ m, their averages over such volumes (10^{-20} m^3 say) differ insignificantly from their centroidal values. In this case pointwise values can be compared directly to observations.

Within the continuum we take the behaviour to be determined by
a) conservation of mass;
b) linear momentum balance: the rate of change of total linear momentum is equal to the sum of the external forces; and
c) angular momentum balance.
The continuum hypothesis enables us to apply these laws on a local as well as a global scale.

1.1.1 Deformation and strain

If we take a solid cube and subject it to some deformation, the most obvious change in external characteristics will be a modification of its shape. The specification of this deformation is thus a geometrical problem, that may be carried out from two different viewpoints:
a) with respect to the undeformed state (Lagrangian), or
b) with respect to the deformed state (Eulerian).
Locally, the mapping from the deformed to the undeformed state can be assumed to be linear and described by a differential relation, which is a combination of pure stretch (a rescaling of each coordinate) and a pure rotation.

The mechanical effects of the deformation are confined to the stretch and it is convenient to characterise this by a *strain* measure. For example, for a wire under load the strain ϵ would be the relative extension, i.e.,

$$\epsilon = \frac{\text{change in length}}{\text{initial length}}, \qquad (1.1.1)$$

The generalisation of this idea requires us to introduce a *strain tensor* at each point of the continuum to allow for the three-dimensional nature of deformation.

1.1.2 The stress field

Within a deformed continuum a force system acts. If we were able to cut the continuum in the neighbourhood of a point, we would find a force acting on a cut surface which would depend on the inclination of the surface and is not necessarily perpendicular to the surface (Figure 1.1).

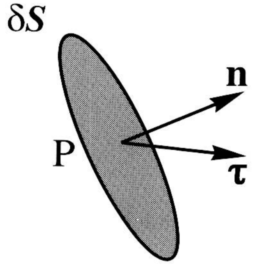

Figure 1.1. The force vector $\boldsymbol{\tau}$ acting on an internal surface specified by the vector normal $\mathbf{n}$ will normally not align with $\mathbf{n}$.

This force system can be described by introducing a *stress tensor* $\boldsymbol{\sigma}$ at each point, whose components describe the loading characteristics, and from which the force vector $\boldsymbol{\tau}$ can be found for a surface with arbitary normal $\mathbf{n}$.

For a loaded wire, the stress σ would just be the force per unit area.

1.1.3 Constitutive relations

The specification of the stress and strain states of a body is insufficient to describe its full behaviour, we need in addition to link these two fields. This is achieved by introducing a *constitutive relation*, which prescribes the response of the continuum to arbitrary loading and thus defines the connection between the stress and strain tensors for the particular material.

At best, a mathematical expression provides an approximation to the actual behaviour of the material. But, as we shall see, we can simulate the behaviour of a wide class of media by using different mathematical forms.

We shall assume that the forces acting at a point depend on the *local* geometry of deformation and its history, and possibly also on the history of the local temperature. This concept is termed the *principle of local action*, and is designed to exclude 'action at a distance' for stress and strain.

Solids

Solids are a familiar part of the Earth through the behaviour of the outer layers, which exhibit a range of behaviours depending on time scale and loading.

We can illustrate the range of behaviour with the simple case of extension of a wire under loading. The tensile stress σ and tensile strain ϵ are then typically related as shown in Figure 1.2.

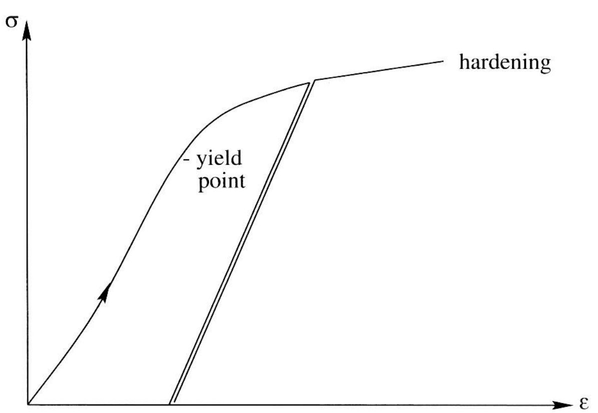

Figure 1.2. Behaviour of a wire under load

Elasticity

If the wire returns to its original configuration when the load is removed, the behaviour is said to be elastic:
(i) linear elasticity $\sigma = E\epsilon$ – usually valid for small strains;
(ii) non-linear elasticity $\sigma = f(\epsilon)$ – important for rubber-like materials, but not significant for the Earth.

Plasticity

Once the yield point is exceeded, permanent deformation occurs and there is no unique stress–strain curve, but a unique $d\sigma - d\epsilon$ relation. As a result of microscopic processes the yield stress rises with increasing strain, a phenomenon known as work hardening. Plastic flow is important for the movement of ice, e.g., in glacier flow.

Viscoelasticity (rate-dependent behaviour)

Materials may creep and show slow long-term deformation, e.g., plastics and metals at elevated temperatures. Such behaviour also seems to be appropriate to the Earth, e.g., the slow uplift of Fennoscandia in response to the removal of the loading of the glacial ice sheets.

Elementary models of viscoelastic behaviour can be built up from two basic building blocks: the *elastic spring* for which

$$\sigma = m\,\epsilon, \tag{1.1.2}$$

and the *viscous dashpot* for which

$$\sigma = \eta\,\dot{\epsilon}. \tag{1.1.3}$$

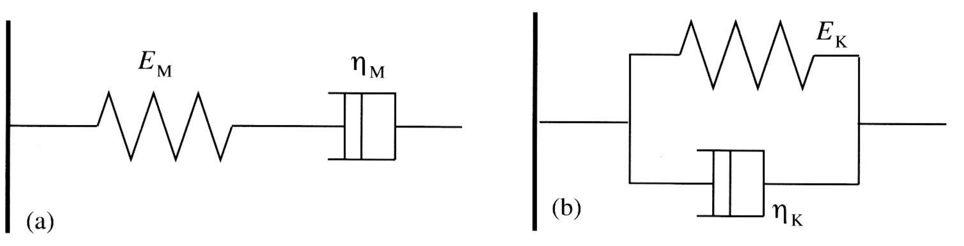

Figure 1.3. Mechanical models for linear viscoelastic behaviour combining a spring and viscous dashpot: (a) Maxwell model, (b) Kelvin–Voigt model.

The stress-strain relations depend on how these elements are combined.

(i) Maxwell model
The spring and dashpot are placed in series (Figure 1.3a) so that

$$\dot{\sigma} = E_M(\dot{\epsilon} + \epsilon/\tau_M); \tag{1.1.4}$$

this allows for instantaneous elasticity and represents a crude description of a fluid. The constitutive relation can be integrated using, e.g., Laplace transform methods and we find

$$\sigma(t) = E_M\left(\epsilon(t) + \int^t dt'\,\epsilon(t')\exp[-(t-t')/\tau_M]\right), \tag{1.1.5}$$

so the stress state depends on the history of strain.

(ii) Kelvin–Voigt model
The spring and dashpot are placed in parallel (Figure 1.3b) and so

$$\sigma = E_K(\dot{\epsilon} + \epsilon/\tau_K), \tag{1.1.6}$$