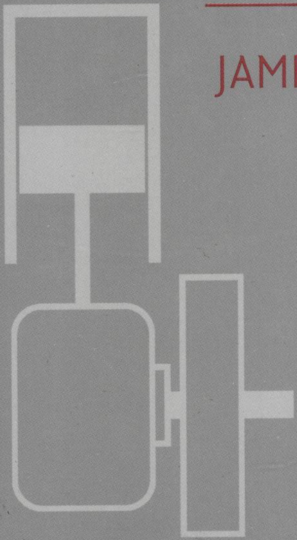
Mechanical Efficiency of Heat Engines

JAMES R. SENFT



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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo, Delhi

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9780521868808

First published 2007

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Senft, J. R. (James R.), 1942–
Mechanical efficiency of heat engines / James R. Senft.
p. cm.
Includes bibliographical references and index.
ISBN-13: 978-0-521-86880-8 (hardback)
ISBN-10: 0-521-86880-7 (hardback)
1. Heat-engines. 2. Mechanical efficiency. 3. Thermodynamics. I. Title.
TJ255.S36 2006
621.402'5 – dc22 2006022394

Mechanical Efficiency of Heat Engines

This book presents a newly developed general conceptual and basic quantitative analysis of the mechanical efficiency of heat engines. The book presents a theory of mechanical efficiency at a level of ideality and generality compatible with the treatment given to thermal efficiency in classical thermodynamics. This yields broad bearing results concerning the overall cyclic conversion of heat into usable mechanical energy. Most notably, the work reveals intrinsic limits on the overall performance of reciprocating heat engines. The ideal Stirling engine is shown to have the best overall performance potential of all possible engines operating under comparable conditions, which leads to mathematically explicit universal upper bounds on mechanical efficiency and cyclic work output. The theory describes the general effects of parameters such as compression ratio and external or buffer pressure on engine output. It also provides rational explanations of certain operational characteristics such as how engines generally behave when supercharged or pressurized. The results also identify optimum geometric configurations for engines operating in various regimes from isothermal to adiabatic. The basic mechanical efficiency results are extended to cover multi-workspace engines and heat pumps. Limited heat transfer and finite-time effects have also been incorporated into the work.

The main research interests of Prof. James Senft lie in the mathematical analysis of mechanisms and heat engines, with an emphasis on the Stirling engine. He has published more than 40 papers and several books in these areas. He holds the position of Professor Emeritus of Mathematics at the University of Wisconsin–River Falls. He has been a visiting research professor at the University of Rome, the University of Washington Joint Center for Graduate Study, the University of Zagreb, and the University of Calgary. Professor Senft has been a visiting fellow at the Australian National University Institute of Advanced Study and a visiting scientist at Argonne National Laboratory. He has received research grants from the Charles A. Lindbergh, Fulbright, and the National Science Foundations and has served as a consultant to the U.S. Department of Energy and NASA.

This book is dedicated
to my son
VICTOR
the engineer.

PREFACE

This book presents a general conceptual and basic quantitative analysis of the mechanical efficiency of heat engines. Typically, treatment of the mechanical efficiency of heat engines has been performed on a case-by-case basis. In ordinary practice, kinematic analysis and computer simulation of specific engine mechanisms coupled with calculated or measured pressure–volume cycles usually can indeed be effectively used for evaluating and locally optimizing engine designs. However, going beyond the specific and local requires broader insights that only a general theory can provide.

No general approach to mechanical efficiency of heat engines had been available until recently. This is in sharp contrast to the situation regarding the thermal efficiency of heat engines. Classical thermodynamics treats the subject of thermal efficiency in great generality. Its results, although obtained in a highly idealized setting, are of profound importance to engine theorists, designers, and practitioners. This book presents a theory of mechanical efficiency at a similar level of ideality and generality.

The first results in this area were published in 1985 and further developed in a series of papers up to the writing of this book. The work modeled the interaction between the mechanical section of an engine and its thermal section at a level compatible with that of classical thermodynamics. This yielded results of broad bearing concerning the overall cyclic conversion of heat into usable mechanical energy.

Most notably, the work uncovered intrinsic limits on the overall performance of reciprocating heat engines. The ideal Stirling engine was shown to have the potentially best overall performance of all possible engines operating under comparable conditions. The work provided mathematically explicit upper bounds on the mechanical efficiency and cyclic work output of engines having like characteristics. The theory described the effects of parameters such as compression ratio and external or buffer pressure on engine output. It also provided rational explanations of certain operational characteristics such as how engines generally behave when supercharged or pressurized. The results also identified optimum geometric configurations for engines operating in various regimes, from isothermal to adiabatic. Limited heat transfer effects have also been incorporated into the work, and results have been extended to multiworkspace engines and heat pumps as well. Most of this has been collected, organized, and presented in the pages that follow.

The research reported in these pages began during a stay as a visiting scientist at Argonne National Laboratory, was subsequently supported by three grants from the National Science Foundation, and was aided by visiting professorships at the University of Rome and the Australian National University. My own University of Wisconsin in River Falls sustained the work throughout and provided a sabbatical which made most of the compilation of this book possible. I am grateful for all of the opportunities given to me.

In addition to these institutional benefactors, a certain small group of people must be specially thanked for the completion of this book. These poor souls have been afflicted with my presence in their lives, and they each in their own way provided encouragement and motivation to me for continuing with the writing of this book especially at times when I did not want to.

First and foremost of this group is my wife, Gloria, who, having patiently suffered though all of my ups and downs for some 39 years, was particularly tried during the writing of this book. She never wavered in her belief that my duty was to finish the book I had been given to do.

Many colleagues at the university were also steadfastly supportive, especially David Yurchak and Keith Chavey, who served as chairs of the department through this ordeal, and Kevin McLaughlin of the Chemistry Department who helped by his constant cheerful interest.

I am grateful to Fr. John Beckfelt, pastor of St. Mary's Parish in Big River, who regularly supplied encouragement and wise advice. I thank all the Saints whom I asked for prayers on my behalf . . . and above all, I thank our good God who hears all prayer.

JRS

As God is above all created things, honors, and possessions,
so should our internal esteem of his Divine Majesty
surpass our esteem or idea of anything whatever.

Saint Aloysius Gonzaga

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1

ENERGY TRANSFERS IN CYCLIC HEAT ENGINES

Heat engines are made to provide mechanical energy from thermal energy. Efficiency is a convenient measure of how well this is done. The overall efficiency of an engine is usually thought of as the product of two more basic efficiencies: the thermal efficiency of the engine cycle and the cyclic mechanical efficiency of the complete device. The first is well treated in classical thermodynamics. The second, mechanical efficiency, is the subject of this work.

Analysis of the mechanical efficiency of heat engines can be only as general as the conceptual basis on which it is built. The model used here has a level of generality matching that used in classical thermodynamics to analyze the thermal efficiency of heat engines.

HEAT ENGINE DIAGRAMS

Figure 1.1 is a representation of a cyclic heat engine typically found in thermodynamics textbooks. G represents the body of the working substance, and T_H and T_C are the temperatures of the heat source and sink, respectively. The net or *indicated cyclic work* done by the engine working substance is the difference $Q_i - Q_o = W$ between the heat absorbed from the high temperature source and the heat rejected to the lower temperature reservoir during a complete cycle.

Although the diagram is adequate for discussing the *thermal efficiency* $\eta_t = W/Q_i$ of the cycle, it does not allow the analysis of all of

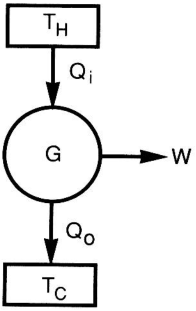

Figure 1.1 Cyclic heat engine diagram depicting heat transfers Q_i and Q_o to and from the working substance and indicated work output W.

the mechanical energy transfers that determine the mechanical efficiency of a complete engine. In fact, work must be done on the engine fluid to carry out half the cycle.

This is quite clear in looking at any pressure–volume (p–V) diagram of an engine cycle. An example is given in Figure 1.2 with characteristics that are typical of the cycles encountered in elementary thermodynamics and normal practice. What is termed as a *regular cycle* is described by a pair of functions p_c and p_e defined and continuous on a closed bounded interval $I = [V_m, V_M]$ representing the volume

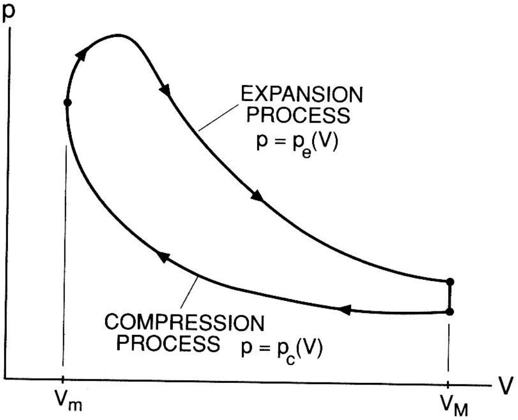

Figure 1.2 A regular cycle in the p–V plane.

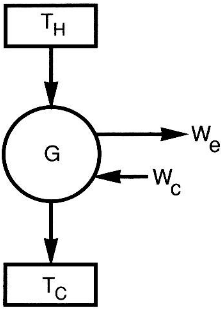

Figure 1.3 Heat engine diagram showing work transfers W_c and W_e with the working substance.

variation of the cycle. The functions represent the compression and expansion pressures of the cycle, with vertical segments supplied at volume extremes if necessary. For an engine, the cycle is oriented as shown in the figure, whereas compressors or heat pumps have the opposite orientation; the discussion will be limited to engines until a later chapter.

The area enclosed by the cycle in the p–V plane is the indicated work W of the engine. This net cyclic work is the difference between two distinct work processes. It is the difference between the work done by the engine fluid during expansion and the work done on it during compression. The *absolute expansion work* of the cycle is the area directly under the upper curve $p = p_e(V)$:

$$W_e = \int_{V_m}^{V_M} p_e(V)\,dV. \tag{1.1}$$

The *absolute compression work* of the cycle is the area directly below the cycle, which represents work that must be done on the engine substance to carry out the compression process described by the lower curve $p = p_c(V)$:

$$W_c = \int_{V_m}^{V_M} p_c(V)\,dV. \tag{1.2}$$

Both work quantities as defined here are positive, and $W = W_e - W_c$.

Figure 1.3 shows the individual expansion and compression work transfers. In a reciprocating or cyclic working engine, the expansion and compression processes do not take place simultaneously but rather sequentially. Thus to realize a self-acting reciprocating engine, means must be provided to divert and store some of the absolute expansion work and redirect it to the engine working fluid when it needs to carry out its absolute compression work.

THE BASIC CYCLIC HEAT ENGINE

Most practical engines have the features depicted conceptually in Figure 1.4. This is the type of engine to be dealt with here and will be referred to as a *reciprocating* or *cyclic kinematic* engine. The working substance, typically a gas, is contained in a capsule called the *workspace*, which is equipped with a means for varying the volume, usually a *piston*. Only a single body of working gas will be considered for the present. This does not represent a significant loss of applicability because most multi-cylinder engines can be considered as parallel connections of single-workspace engines. A later chapter will treat more complex arrangements of multi-piston engines.

The workspace is also equipped with means to interact thermally with heat reservoirs (not shown in Figure 1.4). The prime characteristic of the reciprocating engine is the *mechanism* linking the piston to the output *shaft*. This link is a kinematic one. The motion of the piston and

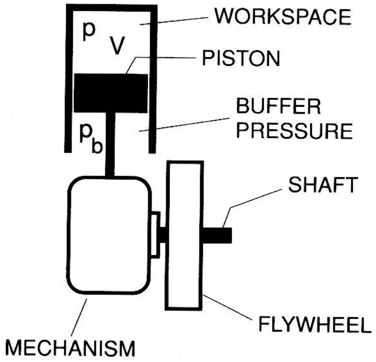

Figure 1.4 The elements of a reciprocating heat engine.

all other moving parts of the engine is completely constrained by the mechanism. The mechanism transmits force or torque as well as motion, so it is actually a *machine* in proper parlance, but the term *mechansim* will be used here to help avoid confusion with the engine as a whole.

The workspace usually contains other devices not shown in the figure such as valves or displacers or whatever may be necessary to carry the working fluid through the desired thermodynamic cycle. These devices, as well as auxiliary pumps, fans, etc., are kinematically linked to and are driven by the mechanism and are conceptually considered as part of the mechanism in the analysis here.

In the turbine type of heat engine, the expansion and compression processes take place simultaneously in different locations in the engine, and the processes are continuous. In cyclic kinematic engines, the processes are discrete and sequential. Because of this, a kinematic engine must be equipped with at least one work reservoir. For single-workspace engines with a rotating shaft output, this reservoir invariably takes the form of a *flywheel*, as Figure 1.4 depicts. Other devices can be used such as pendulums or springs at appropriate places. In multi-workspace engines, each workspace can use some of the others for this purpose as well.

Under steady state operation, the flywheel does not experience a net gain in energy over a cycle. During each cycle, it absorbs, stores, and returns energy to the engine that is necessary to sustain the cycle; the remainder is directed through the output shaft for use outside the engine.

BUFFER PRESSURE

The single-workspace engine needs nothing more in principle than the features described, but in practice it has a near constant external *buffer pressure* acting on the non-workspace side of the piston. The source of this pressure is usually due to the surrounding atmosphere, but sometimes a special enclosure or *buffer space* is constructed to permit the use of elevated pressure. As will be shown, buffer pressure