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Many-Body Problems and Quantum Field Theory

An Introduction

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Translated by Steven Goldfarb

With 102 Figures, 7 Tables and 23 Exercises

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Foreword

This text is a revised and augmented version of a course given to graduate and Ph.D. students in the context of the doctoral school for physics in the French-speaking part of Switzerland. This doctoral school provides a common teaching program for the universities of Bern, Fribourg, Geneva, Neuchâtel and Lausanne, as well as for the Swiss Federal Institute of Technology in Lausanne. The scope of the course should be sufficiently general to interest both experimentalists and theoreticians wishing to engage in research in condensed matter or nuclear and particle physics. The prerequisites are an introductory course to quantum mechanics and elements of classical electromagnetism and statistical mechanics.

Our main concern was how to maintain a reasonably broad level of knowledge for students with different orientations, in a world of research where the price of survival is extreme specialization and competitiveness. Is it still possible in the available time to provide a cultural education in physics by relatively elementary means and in an optimized form? We believe that this is an essential pedagogical duty. Attempting to meet this challenge has determined the conception of this book: each individual part of it is standard and without novelty but should belong, in our opinion, to the basic culture of every physicist; only their common organization in a single house of decent size might possibly be put to our credit.

We have tried to keep a balance between formal developments and the physical applications: in fact they cannot be separated insofar as mathematical methods develop naturally under the necessity of resolving physical questions. Concerning the applications, we have always given a short description of the phenomenological context so that the main information about physical facts is available from the start without recourse to other sources. In the formal developments, we adopt the usual notation of physicists, while aiming at mathematical precision. The reader is warmly encouraged to improve his practice of the formalism by checking and reproducing for himself the algebra given in the text. Some more extended exercises are proposed at the end of each chapter in order to illustrate additional aspects not introduced in the main text.

For each of the systems discussed in this book, we have tried to exhibit how the main physical ideas can be captured in a formalized description by the appropriate tools. In this spirit several important branches of physics are represented: solid state physics (cohesion and dielectric properties of the electron gas, phonons and electron–phonon interactions), low temperature physics (superconductivity and superfluidity), nuclear physics (pairing of nucleons), matter and radiation (interaction of atoms with the quantum-electromagnetic field), particle physics (interaction by exchanged intermediate particles, mass generation by the Higgs mechanism).

These choices could be considered rather conservative, compared to topical new developing areas. However we think that they still serve as indispensable paradigms for the understanding of any more advanced subject. Also, in keeping with our aim of offering a broad formative view to our readers, they enable us to illustrate similarities and differences between concepts stemming from various domains in physics. In this respect, the first chapter presents a parallel exposition of classical electromagnetism and classical elasticity, with the purpose of introducing and comparing the notions of photon and phonon. Moreover, quantum fields (Chap. 8) cannot be understood without a good knowledge of their classical analogues. Chapter 2 is devoted to a simple description of collective effects due to Bose and Fermi statistics. Bose condensation is described and the role of Fermi statistics for the stability of matter and in astrophysical objects is discussed. In the third chapter we develop the so-called second quantized formalism in full generality without reference to any particular system, so that it will be available in any situation where the number of particles varies. Chapters 4, 5 and 6 are devoted to the use of the variational method. It is hoped that the reader will appreciate the wide range of applications of the idea of fermionic pairing formulated in the BCS theory for superconductivity (Chap. 5) as well as for nuclear matter (Chap. 6). The relationship between superfluidity (Chap. 7) and superconductivity on the one hand, and collective excitations of the nuclei on the other, is put into perspective. The quantum-electromagnetic field serves as a model for other quantized matter fields in Chap. 8. The concept of gauge theory is introduced and the close analogy between the Higgs mechanism and the Meissner effect displayed. The method of Feynman graphs is explained in Chaps. 9 and 10, stressing again the existence of a common language for condensed matter and field theory. We essentially give the physical interpretation of diagrams without performing the corresponding more technical quantitative calculations. The analysis is restricted to ground-state properties: non-zero temperature Green functions and the alternative functional integration viewpoint are not considered.

The book is not aimed at the specialist in any of the addressed topics. In fact, no chapter is intended to provide the up-to-date knowledge necessary for an immediate fight on the battlefront of research. We refer in particular to the present state of relativistic quantum field theory since, without mentioning electroweak theory and chromodynamics, no presentation of the Lorentz group or of the Dirac equation can be found here. From the viewpoint of condensed matter, high-T_c is only briefly touched, mesoscopic physics and highly correlated fermions are not discussed. To the prospective particle physicist, the book can merely give a complementary education on the use of similar techniques in condensed-matter physics. Conversely, physicists belonging to the latter discipline, although they may be aware of the importance of field theory for particle physics, should learn about fundamental ideas underlying both domains. We therefore hope that our readers will discover a certain unity of thinking among different domains of physics. In this case, this book will not have been written in vain.

This book was written at the instigation of the Troisième Cycle de la Physique en Suisse Romande, and in particular of J.-J. Loeffel. We are grateful to the many colleagues who provided useful suggestions or enlightenment on various points, namely to the late P. Huguenin and to B. Jancovici, D. Pavuna, J.-P. Perroud and G. Wanders. V. Savona helped with the elaboration of the exercises at the end of each chapter. We thank R. Fernandez for encouraging us to translate the book. S. Goldfarb translated and typed the whole text, including the numerous equations; D. Watson helped us to formulate additional material; L. Klinger and L. Trento drew the figures; our thanks go to all these people for their contributions. Finally we are indebted to the Institute of Condensed-Matter Physics and the Physics Section of the University of Lausanne, as well as to the Physics Department of the Swiss Federal Institute of Technology, Lausanne, for financial support.

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François Rothen

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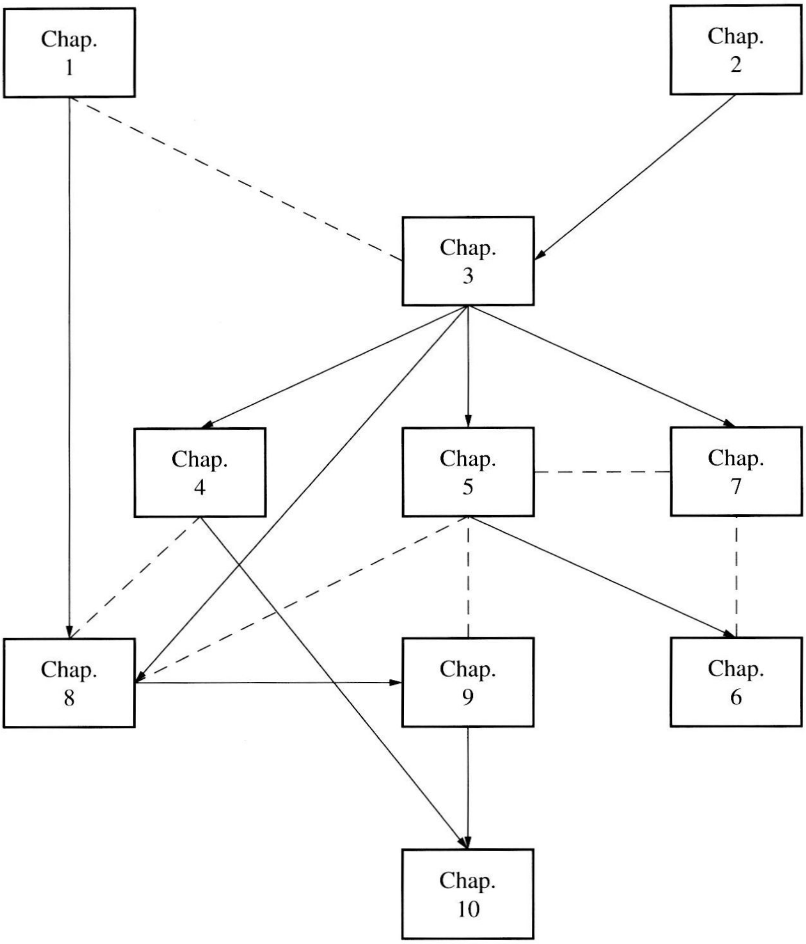
Reader's Guide



Logical organization of the material: the arrows indicate a recommended reading order

Chapter 1 offers an incomplete summary of quantum mechanics and of classical field theory. By consequence, it recalls notions which are useful, but which are generally part of an undergraduate repertoire. It is left to the reader to determine the necessity for a thorough study of its contents.

1. Classical Fields and Their Associated Particles

1.1 Introduction

Introductory quantum physics texts often begin with a historical review. A typical starting point is the analysis of the black-body radiation spectrum, conducted in 1900 by Max Planck and traditionally considered the birth of quantum physics. This is a natural choice, as it is on this occasion that Planck first introduced *his* constant h.

Such an historical approach, however, may lead to serious conceptual difficulties and does not necessarily develop a clear pedagogical logic. The black-body radiation problem immediately introduces a large number of particles, photons, and its analysis requires a good understanding of statistical physics, as well as quantum-field theory. Moreover, the photon, introduced by Einstein in 1905, obeys non-trivial Bose statistics. A more intuitive approach would be to first examine the problem of a single quantum particle before moving on to more complex systems. History, however, chose the opposite and the model of the hydrogen atom created by N. Bohr, the first single-particle problem explicitly involving the Planck constant, did not appear until 1913.

This fact reveals a profound reality: *there exists no quantum system that is strictly a single-particle problem*. In other words, all problems involving a single particle (or a fixed number of particles) result from an approximation which is in general valid only if the energy of the system is weak.

Nevertheless, the formalism of quantum physics is equally well suited for single body problems as for analyses involving a large number of particles. Its methods apply to the problem of a charged particle in a Coulomb potential, for example, just as well as to the analysis of a collection of photons in a reflecting cavity, or the dynamics of nucleons in a nucleus or of electrons in a solid. However, although the passage from classical physics to quantum physics is well known for the case of a massive particle such as an electron, its generalization to electromagnetic radiation or to the vibrations of a crystal lattice is not so evident. Difficulties arise with the introduction of the wave–particle duality as it is expressed for the electron, the photon or the phonon.

The discovery of the electron is commonly attributed to J.-J. Thompson (1897). He recognized that, under the influence of magnetic fields, "cathode rays" behave like jets of particles, with their charge-to-mass ratios remaining constant. Millikan later determined the charge of these particles. So, classically, one could then think of the electron as a particle characterized by its mass and kinematical properties. In 1923, however, an hypothesis of de Broglie attributed a wavelength to the electron and in 1926, Schrödinger gave his name to the famous equation which describes its wave-like character. This new quantum description, illustrated by the wave–particle duality, motivated the association of a wavefunction to the electron obeying Schrödinger's equation of motion. Thus the particle aspect of the electron was imposed before its wave aspect.

In the case of radiation, history followed the reverse path for reasons which were not accidental. Planck and Einstein associated the electromagnetic field with a quantum particle. The famous Planck relation $\Delta E = h\nu = \hbar\omega$ was introduced in 1900 to account for the black body spectrum. In Planck's mind, ΔE represented a minimal exchange of energy between radiation and matter. But in 1905 Einstein interpreted $h\nu$ as the energy of a constituent particle of radiation, the photon. Hence electromagnetic radiation, whose wave properties had been recognized since the work of Young, Fresnel, Fizeau, Kirchhoff and Maxwell, simultaneously acquired the properties of a particle. It should be noted that, while Newton did indeed consider radiation as a flux of particles, he was acting on an hypothesis (one would call it a model today) which was not based on any experimental results. So, in this case, the wave aspect of radiation appears to have preceded its particle aspect.

Another fundamental difference between the electron and the photon is that the latter has zero mass. For non-relativistic massive particles, the passage to quantum theory is achieved by applying the correspondence principle to the Hamiltonian mechanics. Consider the case of an atom in a situation where the kinetic energy is small compared to the rest energy of the particles and small compared to the binding nuclear and ionization energies: the nucleus remains stable and the number of electrons does not change. Even if it is necessary to introduce spin and the Pauli exclusion principle, the particle composition of the atom (nucleons and electrons) remains unchanged; the passage to quantum mechanics via the correspondence principle affects the dynamics without changing either the number or the nature of the constituent particles of the atom.

For a relativistic particle of zero mass, such as the photon, the transition to quantum mechanics is less direct. Under the form of radiation confined to the interior of a cavity, an electromagnetic field is continuously absorbed or reflected by the walls of the vessel. The number of photons is not in general a constant of motion. In other words, the interaction of a photon with other particles cannot be represented by a potential energy. In relativistic physics, interactions are written in terms of exchanges of particles and collisions. The photon thus does not have a classical counterpart and the passage to the quantum description cannot be made by following the same path as in the case of an electron or an atom.