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ANALYSIS OF FAULTED POWER SYSTEMS

PAUL M. ANDERSON

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Analysis of Faulted Power Systems

PAUL M. ANDERSON
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PER UNIT CALCULATIONS

$$Z = \frac{S_{B\text{-}1\phi}}{V^2_{B\text{-}LN}} (Z \text{ ohm}) = \frac{S_{B\text{-}3\phi}}{V^2_{B\text{-}LL}} (Z \text{ ohm}) \text{ pu} = \frac{\text{Base MVA}_{3\phi}}{(\text{Base kV}_{LL})^2} (Z \text{ ohm}) \text{ pu}$$

$$Y = \frac{V^2_{B\text{-}LN}}{S_{B\text{-}1\phi}} (Y \text{ mho}) = \frac{V^2_{B\text{-}LL}}{S_{B\text{-}3\phi}} (Y \text{ mho}) \text{ pu} = \frac{(\text{Base kV}_{LL})^2 \; (Y\,\mu\text{mho})}{(\text{Base MVA}_{3\phi})\,(10^6)} = \frac{(\text{Base kV}_{LL})^2 \; (10^{-6})}{(\text{Base MVA}_{3\phi})\,(Z\text{ M}\Omega)}$$

$$Z_n = Z_o \left(\frac{V_{Bo}}{V_{Bn}}\right)^2 \left(\frac{S_{Bn}}{S_{Bo}}\right) \text{ pu}$$

PHASOR TRANSFORMATION

$$\mathcal{P}[a(t)] = \mathcal{P}[\sqrt{2}\,|A| \cos(\omega t + \alpha)] = |A| e^{j\alpha} = A$$

$$\mathcal{P}^{-1}[A] = \mathcal{R}e(\sqrt{2} A\, e^{j\omega t}) = a(t)$$

SYNTHESIS EQUATION

$$\mathbf{V}_{abc} = \mathbf{A}\,\mathbf{V}_{012}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{1}{h} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

ANALYSIS EQUATION

$$\mathbf{V}_{012} = \mathbf{A}^{-1}\,\mathbf{V}_{abc}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{h}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Analysis of Faulted Power Systems

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Preface to the IEEE Reissued Edition

Many textbooks have been written to describe a power system operating under balanced or normal conditions. A few texts also deal, at least in an elementary manner, with the unbalanced system conditions. The primary object of this book is to provide a text to be used by graduate students and to challenge the student to draw upon a background of knowledge from earlier studies. Since this is intended for advanced studies, the student reader should be able to use circuit concepts at the advanced undergraduate or beginning graduate level and should have a working knowledge of matrix notation. Particular stress here is placed upon a clear, concise notation, since it is the author's belief that this facilitates learning.

Although the thrust of the book is toward the solution of advanced problems, a thorough background is laid in the early chapters by solving elementary configurations of unbalanced systems. This serves to establish the algebraic style and notation of the book as well as to provide the reader with a growing knowledge and facility with symmetrical components. It also introduces the elementary concepts for those not familiar with this discipline. In either case, this background information of the early chapters should be studied since it establishes certain conventions used throughout the book.

The text contains sufficient material for an entire year of study on the subject. The first three chapters are introductory and may be covered rapidly in a graduate level class where the students have already been introduced to symmetrical components. This material should be reviewed, at least in a quick reading, as the matrix notation used throughout the book is introduced in these chapters.

The middle portion of the book, Chapters 4 through 7, treats the subject of *power system parameters.* Here, the sequence impedances of transmission lines, machines, and transformers are developed in detail. This is important material and is often omitted in the education of power engineers. Methods are used here that permit exact solutions of very general physical problems, such as finding the impedance of untransposed or partially transposed lines. Matrix methods are used to clarify computations and adapt them to computer usage.

The final portion of the book presents the application of symmetrical components to a variety of problems and provides an introduction to computer solution methods for large networks. Here, one learns to appreciate the use of matrix algebra in the solution of complex problems. This section also reinforces the engineer's appreciation of symmetrical components as a problem-solving technique.

In preparing the manuscript of a book of this type, one stands on the shoulders of giants. Several excellent books introduced these concepts in the first half of the twentieth century, in particular those by C. F. Wagner and R. D. Evans, G. O. Calabrese, and the books by Edith Clarke, are classics on symmetrical components and are still used by many of us. Their basic ideas are enlarged upon here, and the presentation simplified by the use of matrix methods, thereby aiding understanding and making computer solution much easier. The author's colleagues at Iowa State University played an important role in the development of the book. They used the book in its early stages, found many problems, and offered helpful suggestions for areas needing improvement. The electrical engineering department at Iowa State University was also helpful and understanding of the need for this effort and provided support in many ways.

This new printing of the book by IEEE Press provides software that was especially developed at Iowa State University for the solution of exactly the kind of problems introduced in this book. The program PWRMAT was developed by the author, fellow faculty members, and graduate students for the purpose of providing the engineer with a convenient method of solving the many problems associated with matrices of complex numbers. The program has been improved over the years by Iowa State University and has been used and enjoyed by many students for at least twenty years. Iowa State University has graciously agreed that the program could be distributed with this printing of the book to make problem solving easier. This is important, since the drudgery of solving these complex problems by calculator distracts the engineering student from the objective of learning the concepts. The software and text file versions of the user's manual are attached to the book on diskettes. The software is easy to master and to use. It permits the user to write small programs in a simple language that can read a user-created data file to solve the problem at hand and print the computed results in an orderly matrix notation. Operations such as matrix inversion or reduction are accomplished with ease. It is hoped that the addition of this software will help many new readers in their desire to become more proficient with the important subjects covered in this book.

P. M. Anderson
San Diego, California

Preface

Many textbooks have been written dealing with the power system that operates under balanced or normal conditions. A few, particularly the excellent recent text by W. D. Stevenson [9], deal in an elementary way with both normal and faulted systems. Most of the books treating unbalanced and faulted systems, however, have been in print for years and are inadequate for several reasons. Nevertheless, in spite of their date of copyright, the serious student should become familiar with the famous works of C. F. Wagner and R. D. Evans [10], the outstanding volumes by Edith Clarke [11] and the more recent work of Calabrese [24].

The goal here is to produce a text to be used by graduate students, one that can draw upon a background of knowledge from previous courses. Since this is an advanced text, the student should be able to employ circuit concepts not usually taught to undergraduates and should recognize the beauty and simplicity of matrix notation. Particular stress is placed upon a clear, concise notation since it is the author's belief that this facilitates learning.

Although the thrust of the book is toward the solution of advanced problems, a thorough background is laid by solving elementary configurations of unbalanced systems. This serves to establish the algebraic style and notation of the book. It also serves to introduce the elementary concepts to the uninitiated. Thus for some it will be an organized review and for others an introduction to the solution of faulted networks. In either case this background should be studied since it establishes certain conventions used later.

The text contains sufficient material for a two-semester or three-quarter treatment of the subject. The first three chapters are introductory and may be covered rapidly in a graduate class where the students have already been introduced to symmetrical components. This material should be reviewed, at least in a quick reading, as the matrix notation used throughout the book is introduced in these chapters.

The middle portion of the book, Chapters 4 through 7, treat the subject of *power system parameters*. Here the sequence impedances of lines, machines, and transformers are developed in detail. This is important material and is often omitted in the education of power engineers. Methods are used here which permit exact solutions of very general physical problems such as finding the impedance of untransposed or partially transposed lines. Matrix methods are used to clarify these computations and adapt them to computer usage.

The final portion of the book presents the application of symmetrical components to a variety of problems and provides an introduction to computer solution of large networks. Here one learns to appreciate the use of matrix algebra in the solution of complex problems. This section also reinforces the engineer's appreciation of symmetrical components as a problem-solving technique.

At Iowa State University we have found it convenient to cover most of the first 10 chapters in a two-quarter sequence, leaving computer applications as a separate course. This means that some of the sections in Chapters 1-10 must be omitted, but the student is encouraged to pursue these on his own. In this two-quarter presentation Chapters 1-3 are skimmed quickly since the course carries an undergraduate prerequisite which introduces symmetrical components. Then the balance of the first quarter is spent on power system parameters, leaving the applications for the second quarter.

This book would not have been possible without the unique contribution of many individuals to whom the author is greatly indebted. Several Iowa State University colleagues, particularly W. B. Boast, J. W. Nilsson, and J. E. Lagerstrom (now of the University of Nebraska), are largely responsible for the author's interest in the subject. These three were also responsible for the organization and teaching of a short course in symmetrical components, taught in connection with the Iowa State University A-C Network Analyzer for 10 years or so. The author's interest in this course, first as a student and later as a teacher, helped him gain competence in the subject. Indeed, many ideas expressed here are taken directly or indirectly from the short course notes. The influence of the late W. L. Cassell must also be mentioned, for his insistence on a clear notation has contributed to the education and understanding of many students, the author included. The author is particularly indebted to David D. Robb who used much of the book in a graduate class and made countless valuable suggestions for improvements. Portions of the computer solutions presented are the work of J. R. Pavlat and G. N. Johnson, and these contributions are gratefully acknowledged.

Finally, I wish to express my thanks to the Electrical Engineering Department of Iowa State University and to W. B. Boast, head of the department, for giving me the opportunity to prepare this material. Special thanks are due to my wife, Ginny, who provided both moral support and expert proofreading, and to my editor, Nancy Bohlen, who is a marvel with both mathematical notation and eccentric authors.

List of Symbols

1. CAPITALS

A	ampere, unit symbol abbreviation for current
$\mathbf{A}$	complex transformation matrix; transmission parameter matrix; node incidence matrix
A	magnetic vector potential
$\mathcal{A}$	inverse transmission parameter matrix
B	$= \mathcal{I}m\, Y$, susceptance
$\mathbf{B}$	complex transformation matrix; shunt susceptance matrix
C	capacitance
C	coulomb, unit symbol abbreviation for charge
$\mathbf{C}$	complex transformation matrix; Maxwell's coefficients; capacitance coefficients
D	distance or separation
E	source emf; voltage
$\mathbf{E}$	primitive source voltage vector
F	farad, unit symbol abbreviation for capacitance
F, F'	fault point designation
F-D-Q	rotor circuits of a synchronous machine
G	$= \mathcal{R}e\, Y$, conductance
$\mathbf{G}$	inverse hybrid parameter matrix
GMD, GMR	mutual geometric mean distance, geometric mean radius
H	henry, unit symbol abbreviation for inductance
Hz	hertz, unit symbol abbreviation for frequency
$\mathbf{H}$	hybrid parameter matrix
I	rms phasor current
$\mathbf{I}_{abc}$	$= [I_a\ I_b\ I_c]^t$, line current vector
$\mathbf{I}_{012}$	$= [I_{a0}\ I_{a1}\ I_{a2}]^t$, sequence current vector
I_B	base line current, A
J	joule, unit symbol abbreviation for energy
$\mathbf{J}$	primitive current source vector
K	dielectric constant
$\mathbf{K}$	Kron's transformation or connection matrix
L	inductance
LL	line-to-line
LN	line-to-neutral
$\mathbf{L}$	inductance matrix
M	$= 10^6$, mega, a prefix

M . . . . mutual inductance
M_{ij} . . . . minor of a matrix
M . . . . two-port network vector
N . . . . newton, unit symbol abbreviation for force
N . . . . zero potential bus designation
N . . . . two-port network vector
$\mathcal{P}$ . . . . phasor operator
P . . . . average power; transformer circuit designation
P . . . . Vandermonde matrix; potential coefficient matrix; Park's transformation matrix
Q . . . . average reactive power; transformer circuit designation; total charge; phasor charge density
R . . . . $= \mathcal{R}e\, Z$, resistance; transformer circuit designation
R . . . . resistance matrix
S . . . . $= P + jQ$, complex apparent power
S_B . . . . base apparent power, VA
SLG . . . . single-line-to-ground
T . . . . time; time constant; torque; equivalent circuit configuration
T_B . . . . base time, s
$\mathbf{T}_\phi$ . . . . twist matrix
U . . . . unit matrix
V . . . . rms phasor voltage
V . . . . volt, unit symbol abbreviation for voltage
VA . . . . voltampere, unit symbol abbreviation for apparent power
$\mathbf{V}_{abc}$ . . . . $= [V_a \ V_b \ V_c]^t$, phase voltage vector
$\mathbf{V}_{012}$ . . . . $= [V_{a0} \ V_{a1} \ V_{a2}]^t$, sequence voltage vector
V_B . . . . base voltage, V
W . . . . watt, unit symbol abbreviation for power
Wb . . . . weber, unit symbol abbreviation for magnetic flux
X . . . . $= \mathcal{I}m\, Z$, reactance
$\mathcal{Y}$ . . . . primitive admittance matrix
Y . . . . $= G + jB$, complex admittance
Y_B . . . . base admittance, mho
Y . . . . admittance matrix
$\mathcal{Z}$ . . . . primitive impedance matrix
Z . . . . $= R + jX$, complex impedance
Z_B . . . . base impedance, Ω
Z . . . . impedance matrix

2. LOWERCASE

a . . . . $= e^{j2\pi/3}$, 120° operator
ac . . . . alternating current
a-b-c . . . . stator circuits of a synchronous machine; phase designation
adj . . . . adjoint (of a matrix)
b . . . . $= \omega c$, line susceptance per unit length
ber, bei . . . . real, imaginary Bessel functions
c . . . . capacitance per unit length
dc . . . . direct current

d_0, d_2 . . . . zero, negative sequence electrostatic unbalance factors
d, q . . . . stator circuits, referred to the rotor
det . . . . determinant (of a matrix)
e . . . . base for natural logarithms
f . . . . frequency
f_k . . . . kth fraction of total line length
g . . . . ground terminal
h . . . . two-port hybrid parameter designation
h . . . . a constant (1 or $\sqrt{3}$)
i . . . . instantaneous current
i . . . . instantaneous current vector
j . . . . $=\sqrt{-1}$, 90° operator
k, k′ . . . . constants used in computing L, C
k . . . . $= 10^3$, kilo, a prefix
k . . . . $=\sqrt{3/2}$, a constant used in synchronous machine theory
ℓ . . . . inductance per unit length; leakage inductance
ln, log . . . . natural (base e), base 10 logarithms
m . . . . $= 10^{-3}$, milli, a prefix; a constant used in computing skin effect
m . . . . mutual inductance per unit length
m_0, m_2 . . . . zero, negative sequence electromagnetic unbalance factors
m . . . . complex transformation ratio
n . . . . number of phases; number of nodes; $= 10^{-9}$, nano, a prefix
n . . . . neutral terminal; neutral voltage; turns ratio; number of turns
pu . . . . per unit
p . . . . instantaneous power
q . . . . linear charge density of a wire
q . . . . vector of linear charges on a group of wires
r . . . . radius; internal (source) resistance; resistance per unit length
s . . . . line length, length of section k, slip of an induction motor
s . . . . speed voltage vector
t . . . . time
u . . . . unit step function
v . . . . instantaneous voltage
v . . . . instantaneous voltage vector
x . . . . line reactance per unit length; internal (source) reactance
y . . . . two-port admittance parameter designation
z . . . . two-port impedance parameter designation; internal (source) impedance; impedance per unit length
$\bar{z}$ . . . . transmission line primitive impedance

3. UPPERCASE GREEK

Δ . . . . delta connection; determinant (of a matrix)
Σ . . . . summation symbol
Ω . . . . ohm, unit symbol abbreviation for impedance

4. LOWERCASE GREEK

α . . . . phase angle
α_R, α_L . . . . ac/dc skin effect ratios
δ . . . . torque angle of a synchronous machine
δ_{ij} . . . . Kronecker delta
ϵ . . . . $= \epsilon_0 \kappa$, permittivity
θ . . . . phase angle; rotor angle of a synchronous machine
κ . . . . dielectric constant
λ . . . . element of Vandermonde matrix; flux linkage
μ . . . . $= \mu_0 \mu_r$, permeability (μ_0, free space; μ_r, relative)
μ . . . . $= 10^{-6}$, micro, a prefix
π . . . . pi, 3.14159265. . .
ρ . . . . resistivity
τ . . . . time constant
ϕ . . . . magnetic flux; phase angle
ω . . . . radian frequency; synchronous machine speed

5. SUBSCRIPTS

a . . . . phase *a*; armature
A . . . . phase *a*
b . . . . phase *b*
B . . . . phase *b*
B . . . . base quantity
c . . . . phase *c*; core loss quantity
C . . . . phase *c*; transformer circuit designation
d . . . . direct axis; direct axis circuit quantity
D . . . . direct axis damper winding quantity
e . . . . excitation quantity, of a transformer
eq . . . . equivalent circuit quantity; equivalent spacing
env . . . . envelope of an ac wave
F . . . . referring to the fault point; field winding
f . . . . referring to the fault point
g . . . . referring to the fault point
H . . . . transformer winding designation
LN . . . . line-to-neutral
LL . . . . line-to-line
m . . . . magnetizing quantity (in a transformer); motor quantity; mutual (coupling or GMD)
max . . . . maximum
min . . . . minimum
M . . . . mutual (frequently $M0, M1, M2$)
n . . . . neutral
q . . . . quadrature axis; quadrature axis circuit quantity
Q . . . . quadrature axis damper winding quantity
r . . . . rotor quantity
R . . . . rotor quantity
s . . . . source quantity; stator quantity; self (GMD)

S .... stator quantity; transformer circuit designation; self (frequently $S0, S1, S2$)
sym .... symmetrical
T .... transformer circuit designation
u .... per unit
X .... transformer winding designation
Y .... transformer winding designation
$1\phi, 3\phi$ .... single-phase; three-phase
0, 1, 2 .... zero, positive, negative sequence quantity
$0, \Sigma, \Delta$ .... zero, sum, difference sequence quantity
$0, \Delta$ .... initial condition; change condition

6. SUPERSCRIPTS

$(\ )^t$ .... transpose (of a matrix)
$(\ )^{-1}$ .... inverse (of a matrix)
$(\tilde{\ })$ .... (tilde), distinguishing mark for various quantities
$(\hat{\ })$ .... (circumflex), distinguishing mark for various quantities
$(\dot{\ })$ .... $= d/dt$, derivative with respect to time
$(\ )^*$ .... conjugate, of a phasor or a matrix
$(\ )'$ .... (prime), transient
$(\ )''$ .... (double prime), subtransient

7. NUMERAL SYMBOLS

1ϕ .... single-phase
2LG .... double-line-to-ground
3ϕ .... three-phase
1LO .... one line open
2LO .... two lines open

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