Mathematics and Statistics for Economists

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MATHEMATICS AND STATISTICS FOR ECONOMISTS

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HOLT, RINEHART AND WINSTON

New York · Chicago · San Francisco · Toronto

January , 1965

Printed in the United States of America
Library of Congress Catalog Card Number: **53–6883**
28463–0113

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PREFACE

The present book grew out of a definite need felt during many years of teaching at Iowa State College. There seems to be no text adapted to the American student of economics (undergraduate or graduate) who has had no thorough training in college mathematics, but who is willing to acquire some of the mathematical equipment necessary nowadays, for a serious study of economics.

This book has been written in an attempt to meet such a specific need. It includes some applications of elementary mathematics to economics, as well as topics in calculus, probability, and elementary statistics. The examples are taken from economics. The student is not burdened by the necessity of familiarizing himself with mechanics and other branches of physics, which traditionally supply much of the illustrative materials in elementary calculus texts. It is felt that the economics student who wants to learn mathematics and statistics has not the time to study those topics which are somewhat remote from his field of interest.

This book is addressed specifically to the future econometrician—a student of economics who is willing to use the tools of mathematics and statistics in his economic investigations. Little mathematical preparation is required of the student. It is believed that some knowledge of algebra and elementary trigonometry will be sufficient, although familiarity with elementary economics is required. In this connection, two somewhat advanced books on economic theory are recommended: Kenneth E. Boulding, *Economic Analysis* (New York: Harper & Brothers, 1941) and George J. Stigler, *The Theory of Price* (New York: The Macmillan Company, 1946). The student will gain a greater insight into the theoretical economic problems used as illustrations if he reads the relevant chapters of one or both texts. It is to be hoped that, *after* mastering this text, the student or reader will possess sufficient knowledge in mathematics and statistics to understand most of the articles published in such journals as *Econometrica*, the *Review of Economic Studies*, and the *Journal of the American Statistical Association*.

It is evident that a book which is planned, not for the future professional mathematician, but for the future econometrician, cannot be entirely rigorous in the proofs of the mathematical theorems involved. Intuitive proofs and demonstrations are frequently substituted for mathematical rigor, where the more adequate proof is beyond the scope of the book and also beyond the powers of most readers or students. Rigorous treatment is already available in many books on advanced calculus, algebra, and statistics. Some of these books are indicated in the postscript.

Many empirical examples are included in the exercises of this book. They represent the efforts of econometricians to utilize statistical methods to obtain theoretically meaningful economic relationships. The statistical methods used by these econometricians are not always the most modern ones. In spite of this fact, it seemed worth while to include, as illustrations, some of the older results, found with the help of somewhat antiquated statistical methods. It should be emphasized that the empirical relationships given in the examples are to be interpreted with some caution. They represent merely efforts to estimate some kind of average relationship between the variables indicated. It is to be hoped that these examples, indicative of the great theoretical interest and practical potentialities of econometrics, will make the study of mathematics and statistics more interesting to the economist, and will inspire him to future studies in the field.

Some of the examples in the text require the use of mathematical tables. The following set of tables can be recommended: H. D. Larsen, *Rinehart Mathematical Tables, Formulas, and Curves*, Enlarged Edition (New York: Rinehart & Company, Inc., 1953).

I should like to express my gratitude to a number of my colleagues at Iowa State College who have taken a kindly interest in this text and given me assistance in various ways. I am particularly obliged to Professor Edward S. Allen (Department of Mathematics), Professor Dio L. Holl (Head, Department of Mathematics), Professor William G. Murray (Head, Department of Economics), and Professor E. R. Smith (Department of Mathematics). I am also indebted to Professor C. V. Newsom for improvements in the manuscript. I have to thank Mr. F. Jarred (Melbourne, Australia) for helping me with the answers to the problems.

Problems marked by * contain important ideas and theorems which will be required later. Problems and sections marked ** are somewhat more difficult and may be omitted.

Sources of numerical examples

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I

SOME APPLICATIONS OF ELEMENTARY MATHEMATICS TO ECONOMICS

1

FUNCTIONS AND GRAPHS

1. Functions

The function concept is one of the most important in both pure and applied mathematics. The variable y is said to be a function of the variable x when y depends upon x in such a way that the fixing of x determines one or more corresponding values of y. The variable x, whose value may be arbitrarily assigned (except for the nonpermissible values) is called the *independent variable*. The variable y, whose numerical value is determined after a permissible value has been given to x, is the *dependent variable*. If we want to indicate that y is a function of x, without fixing the specific form of the function, we write $y = f(x)$. Instead of f, we may use also, as functional symbols, such letters as g, h, F, G, and so on. Also, we write $y = y(x)$.

EXAMPLE 1

Let y depend on x through the medium of the formula $y = 1/x$. Thus the formula denotes a functional relationship. If $x = 3$, we have $y = 1/3$. If $x = -7$, we have $y = -1/7$. If $x = 8$, we have $y = 1/8$. If $x = 1/3$, we have $y = 1/(1/3) = 3$, and so forth. We see that for all permissible values of x (that is, all values except $x = 0$) we can find the corresponding y. Specifically, we say then that *y is a function of x*.

It is easy to illustrate, from economics, the concept of functional relationship. Consider, for instance, a demand function. Here the quantity demanded (dependent variable y) may be considered as a function of the price (independent variable x) of a certain commodity. In production theory,

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the quantity of the product (dependent variable y) may be considered as a function of the amount of labor involved in production (independent variable x), and so on.

EXAMPLE 2

If $f(x)$ is the function of Example 1, we may write, for instance,

$$y = f(x) = y(x) = \frac{1}{x}.$$

When we substitute specific values for x, we may write

$f(4) = 1/4$, which means that x has been replaced by 4,

$f(-3) = 1/(-3) = -1/3$, which means that -3 has been substituted for x,

$f(5/3) = 1/(5/3) = 3/5$, and so on.

Instead of a specific number we may substitute into $f(x)$ an algebraic expression for x; thus,

$$f\left(\frac{1}{a}\right) = \frac{1}{\frac{1}{a}} = a,$$

$$f\left(\frac{x}{y}\right) = \frac{y}{x}.$$

EXERCISES 1

1. Let $y = f(x) = 3 - 2x + x^2$. Find $f(0), f(-2), f(5), f(-1)$.

2. Let $y = f(x) = 2x/(x^2 - 1)$. (a) Find $f(0)$; $f(-6)$; $f(5)$; $f(2)$; $f(-3/4)$. (b) Are the values $x = 1$ and $x = -1$ permissible for the given function?

3. Let $y = f(x) = (2x^2 - 4x + 6)/2x$. (a) Find $f(1)$; $f(0)$; $f(-1)$; $f(1/5)$; $f(-1/3)$. (b) Is the value $x = -2$ permissible? (c) Is the value $x = 0$ permissible?

4. Let $y = f(x) = 2^x$. Find $f(1)$; $f(2)$; $f(4)$.

5. Let $y = f(x) = (2x - 1)^2$. Find $f(0)$; $f(-1)$; $f(3)$; $f(-1/5)$.

6. Let $y = f(x) = [2(x - 1)^2 + 5]/(x + 3)^2$. (a) Find $f(0)$; $f(-1)$; $f(5)$; $f(-1/2)$. (b) Is $x = 3$ a permissible value? (c) Is $x = -3$ a permissible value of x?

7. Let $y = f(x) = x^3$. Find $f(0)$; $f(-2)$; $f(4)$; $f(10)$; $f(-10)$.

8. Let $f(x) = 2x^2 - 1$. Find (a) $f(1)$, $-f(0)$; (b) $2f(1)$, $-3f(-1)$; (c) $[f(3)]/[f(-2)]$.

9. Let $f(x) = a + bx$, where a and b are arbitrary constants. Find (a) $f(0)$; (b) $f(-2)$; (c) $f(1)$; (d) $f(a)$; (e) $f(-a)$; (f) $f(a/b)$; (g) $f(-a/b)$.

10. Let $f(x) = ax^2 + bx + c$, where a, b and c are constants. Find (a) $f(0)$; (b) $f(-1)$; (c) $f(1)$; (d) $f(a - b)$; (e) $f(b - a)$.

****11.** Let $f(x) = 5x$. Find $f(a)$; $f(b)$; $f(a + b)$; $f(a - b)$. Show that $f(a + b) = f(a) + f(b)$. Show that $f(a - b) = f(a) - f(b)$.