

Third Edition

METAL FORMING

MECHANICS AND METALLURGY

William F. Hosford
Robert M. Caddell



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Mechanics and Metallurgy

THIRD EDITION

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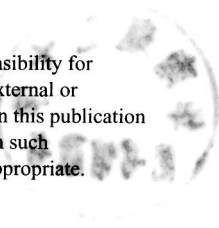
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METAL FORMING, THIRD EDITION

This book is designed to help the engineer understand the principles of metal forming and analyze forming problems – both the mechanics of forming processes and how the properties of metals affect the processes. The first third of the book is devoted to fundamentals of mechanics and materials; the middle to analyses of bulk forming processes such as drawing, extrusion, and rolling; and the last third covers sheet forming processes. In this new third edition, an entire chapter has been devoted to forming limit diagrams; another to various aspects of stamping, including the use of tailor-welded blanks; and another to other sheet forming operations, including hydroforming of tubes. Sheet testing is covered in a later chapter. Coverage of sheet metal properties has been expanded to include new materials and more on aluminum alloys. Interesting end-of-chapter notes and references have been added throughout. More than 200 end-of-chapter problems are also included.

William F. Hosford is a Professor Emeritus of Materials Science and Engineering at the University of Michigan. Professor Hosford is the author of more than 80 technical articles and a number of books, including the leading selling *Mechanics of Crystals and Textured Polycrystals*, *Physical Metallurgy*, *Mechanical Behavior of Materials*, and *Materials Science: An Intermediate Course*.

Robert M. Caddell was a professor of Mechanical Engineering at the University of Michigan, Ann Arbor.

Preface to Third Edition

My coauthor Robert Caddell died in 1990. I have greatly missed interacting with him.

The biggest changes from the second edition are an enlargement and reorganization of the last third of the book, which deals with sheet metal forming. Changes have been made to the chapters on bending, plastic anisotropy, and cup drawing. An entire chapter has been devoted to forming limit diagrams. There is one chapter on various aspects of stampings, including the use of tailor-welded blanks, and another on other sheet-forming operations, including hydroforming of tubes. Sheet testing is covered in a separate chapter. The chapter on sheet metal properties has been expanded to include newer materials and more depth on aluminum alloys.

In addition, some changes have been made to the chapter on strain-rate sensitivity. A treatment of friction and lubrication has been added. A short treatment of swaging has been added. End-of-chapter notes have been added for interest and additional end-of-chapter references have been added.

No attempt has been made in this book to introduce numerical methods such as finite element analyses. The book *Metal Forming Analysis* by R. H. Wagoner and J. L. Chenot (Cambridge University Press, 2001) covers the latest numerical techniques. We feel that one should have a thorough understanding of a process before attempting numerical techniques. It is vital to understand what constitutive relations are imbedded in a program before using it. For example, the use of Hill's 1948 anisotropic yield criterion can lead to significant errors.

Joining techniques such as laser welding and friction welding are not covered.

I wish to acknowledge the membership in the North American Deep Drawing Group from which I have learned much about sheet metal forming. Particular thanks are given to Alejandro Graf of ALCAN, Robert Wagoner of the Ohio State University, John Duncan of the University of Auckland, Thomas Stoughton and David Meuleman of General Motors, and Edmund Herman of Creative Concepts.

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1 Stress and Strain

An understanding of stress and strain is essential for analyzing metal forming operations. Often the words *stress* and *strain* are used synonymously by the nonscientific public. In engineering, however, stress is the intensity of force and strain is a measure of the amount of deformation.

1.1 STRESS

Stress is defined as the intensity of force, F , at a point.

$$\sigma = \partial F / \partial A \quad \text{as} \quad \partial A \rightarrow 0, \quad (1.1)$$

where A is the area on which the force acts.

If the stress is the same everywhere,

$$\sigma = F / A. \quad (1.2)$$

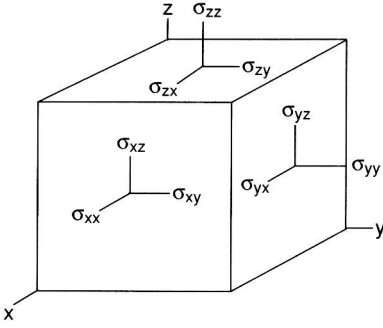
There are nine components of stress as shown in Figure 1.1. A normal stress component is one in which the force is acting normal to the plane. It may be tensile or compressive. A shear stress component is one in which the force acts parallel to the plane.

Stress components are defined with two subscripts. The first denotes the normal to the plane on which the force acts and the second is the direction of the force.* For example, σ_{xx} is a tensile stress in the x -direction. A shear stress acting on the x -plane in the y -direction is denoted σ_{xy} .

Repeated subscripts (e.g., σ_{xx} , σ_{yy} , σ_{zz}) indicate normal stresses. They are tensile if both subscripts are positive or both are negative. If one is positive and the other is negative, they are compressive. Mixed subscripts (e.g., σ_{zx} , σ_{xy} , σ_{yz}) denote shear stresses. A state of stress in tensor notation is expressed as

$$\sigma_{ij} = \begin{vmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{vmatrix}, \quad (1.3)$$

* The use of the opposite convention should cause no problem because $\sigma_{ij} = \sigma_{ji}$.



1.1. Nine components of stress acting on an infinitesimal element.

where i and j are iterated over x , y , and z . Except where tensor notation is required, it is simpler to use a single subscript for a normal stress and denote a shear stress by τ . For example, $\sigma_x \equiv \sigma_{xx}$ and $\tau_{xy} \equiv \sigma_{xy}$.

1.2 STRESS TRANSFORMATION

Stress components expressed along one set of axes may be expressed along any other set of axes. Consider resolving the stress component $\sigma_y = F_y/A_y$ onto the x' and y' axes as shown in Figure 1.2.

The force F'_y in the y' direction is $F'_y = F_y \cos \theta$ and the area normal to y' is $A_{y'} = A_y / \cos \theta$, so

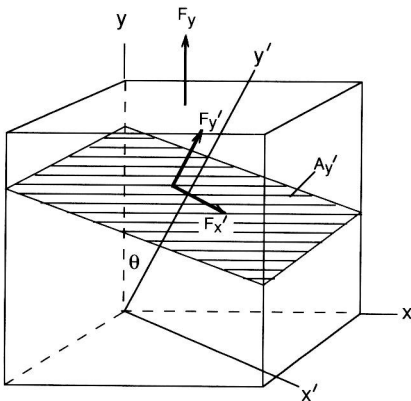
$$\sigma_{y'} = F'_{y'} / A_{y'} = F_y \cos \theta / (A_y / \cos \theta) = \sigma_y \cos^2 \theta. \quad (1.4a)$$

Similarly

$$\tau_{y'x'} = F_{x'} / A_{y'} = F_y \sin \theta / (A_y / \cos \theta) = \sigma_y \cos \theta \sin \theta. \quad (1.4b)$$

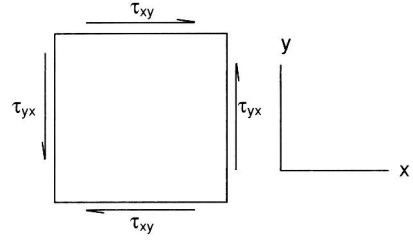
Note that transformation of stresses requires two sine and/or cosine terms.

Pairs of shear stresses with the same subscripts in reverse order are always equal (e.g., $\tau_{ij} = \tau_{ji}$). This is illustrated in Figure 1.3 by a simple moment balance on an



1.2. The stresses acting on a plane, A' , under a normal stress, σ_y .

1.3. Unless $\tau_{xy} = \tau_{yx}$, there would not be a moment balance.



infinitesimal element. Unless $\tau_{ij} = \tau_{ji}$, there would be an infinite rotational acceleration. Therefore

$$\tau_{ij} = \tau_{ji}. \quad (1.5)$$

The general equation for transforming the stresses from one set of axes (e.g., n, m, p) to another set of axes (e.g., i, j, k) is

$$\sigma_{ij} = \sum_{n=1}^3 \sum_{m=1}^3 \ell_{im} \ell_{jn} \sigma_{mn}. \quad (1.6)$$

Here, the term ℓ_{im} is the cosine of the angle between the i and m axes and the term ℓ_{jn} is the cosine of the angle between the j and n axes. This is often written as

$$\sigma_{ij} = \ell_{im} \ell_{jn} \sigma_{mn}, \quad (1.7)$$

with the summation implied. Consider transforming stresses from the x, y, z axis system to the x', y', z' system shown in Figure 1.4.

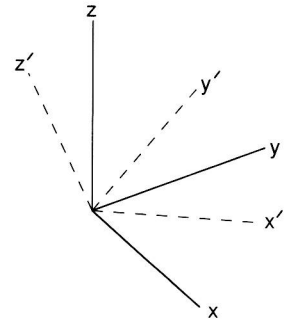
Using equation 1.6,

$$\begin{aligned} \sigma_{x'x'} = & \ell_{x'x} \ell_{x'x} \sigma_{xx} + \ell_{x'x} \ell_{x'y} \sigma_{xy} + \ell_{x'x} \ell_{x'z} \sigma_{xz} \\ & + \ell_{x'y} \ell_{x'x} \sigma_{yx} + \ell_{x'y} \ell_{x'y} \sigma_{yy} + \ell_{x'y} \ell_{x'z} \sigma_{yz} \\ & + \ell_{x'z} \ell_{x'x} \sigma_{zx} + \ell_{x'z} \ell_{x'y} \sigma_{yz} + \ell_{x'z} \ell_{x'z} \sigma_{zz} \end{aligned} \quad (1.8a)$$

and

$$\begin{aligned} \sigma_{x'y'} = & \ell_{x'x} \ell_{y'x} \sigma_{xx} + \ell_{x'x} \ell_{y'y} \sigma_{xy} + \ell_{x'x} \ell_{y'z} \sigma_{xz} \\ & + \ell_{x'y} \ell_{y'x} \sigma_{yx} + \ell_{x'y} \ell_{y'y} \sigma_{yy} + \ell_{x'y} \ell_{y'z} \sigma_{yz} \\ & + \ell_{x'z} \ell_{y'x} \sigma_{zx} + \ell_{x'z} \ell_{y'y} \sigma_{yz} + \ell_{x'z} \ell_{y'z} \sigma_{zz} \end{aligned} \quad (1.8b)$$

1.4. Two orthogonal coordinate systems.



These can be simplified to

$$\sigma_{x'} = \ell_{x'x}^2 \sigma_x + \ell_{x'y}^2 \sigma_y + \ell_{x'z}^2 \sigma_z + 2\ell_{x'y}\ell_{x'z}\tau_{yz} + 2\ell_{x'z}\ell_{x'x}\tau_{zx} + 2\ell_{x'x}\ell_{x'y}\tau_{xy} \quad (1.9a)$$

and

$$\begin{aligned} \tau_{x'y'} = & \ell_{x'x}\ell_{y'x}\sigma_x + \ell_{x'y}\ell_{y'y}\sigma_y + \ell_{x'z}\ell_{y'z}\sigma_z + (\ell_{x'y}\ell_{y'z} + \ell_{x'z}\ell_{y'y})\tau_{yz} \\ & + (\ell_{x'z}\ell_{y'x} + \ell_{x'x}\ell_{y'z})\tau_{zx} + (\ell_{x'x}\ell_{y'y} + \ell_{x'y}\ell_{y'x})\tau_{xy}. \end{aligned} \quad (1.9b)$$

1.3 PRINCIPAL STRESSES

It is always possible to find a set of axes along which the shear stress terms vanish. In this case σ_1 , σ_2 , and σ_3 are called the principal stresses. The magnitudes of the principal stresses, σ_p , are the roots of

$$\sigma_p^3 - I_1\sigma_p^2 - I_2\sigma_p - I_3 = 0, \quad (1.10)$$

where I_1 , I_2 , and I_3 are called the *invariants* of the stress tensor. They are

$$\begin{aligned} I_1 &= \sigma_{xx} + \sigma_{yy} + \sigma_{zz}, \\ I_2 &= \sigma_{yz}^2 + \sigma_{zx}^2 + \sigma_{xy}^2 - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} - \sigma_{xx}\sigma_{yy}, \quad \text{and} \\ I_3 &= \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\sigma_{yz}\sigma_{zx}\sigma_{xy} - \sigma_{xx}\sigma_{yz}^2 - \sigma_{yy}\sigma_{zx}^2 - \sigma_{zz}\sigma_{xy}^2. \end{aligned} \quad (1.11)$$

The first invariant $I_1 = -p/3$ where p is the pressure. I_1 , I_2 , and I_3 are independent of the orientation of the axes. Expressed in terms of the principal stresses, they are

$$\begin{aligned} I_1 &= \sigma_1 + \sigma_2 + \sigma_3, \\ I_2 &= -\sigma_2\sigma_3 - \sigma_3\sigma_1 - \sigma_1\sigma_2, \quad \text{and} \\ I_3 &= \sigma_1\sigma_2\sigma_3. \end{aligned} \quad (1.12)$$

EXAMPLE 1.1: Consider a stress state with $\sigma_x = 70$ MPa, $\sigma_y = 35$ MPa, $\tau_{xy} = 20$, $\sigma_z = \tau_{zx} = \tau_{yz} = 0$. Find the principal stresses using equations 1.10 and 1.11.

SOLUTION: Using equations 1.11, $I_1 = 105$ MPa, $I_2 = -2050$ MPa, $I_3 = 0$. From equation 1.10, $\sigma_p^3 - 105\sigma_p^2 + 2050\sigma_p + 0 = 0$, so

$$\sigma_p^2 - 105\sigma_p + 2,050 = 0.$$

The principal stresses are the roots $\sigma_1 = 79.1$ MPa, $\sigma_2 = 25.9$ MPa, and $\sigma_3 = \sigma_z = 0$.

EXAMPLE 1.2: Repeat Example 1.1 with $I_3 = 170,700$.

SOLUTION: The principal stresses are the roots of $\sigma_p^3 - 105\sigma_p^2 + 2050\sigma_p + 170,700 = 0$. Since one of the roots is $\sigma_z = \sigma_3 = -40$, $\sigma_p + 40 = 0$ can be factored out. This gives $\sigma_p^2 - 105\sigma_p + 2050 = 0$, so the other two principal stresses are $\sigma_1 = 79.1$ MPa, $\sigma_2 = 25.9$ MPa. This shows that when σ_z is one of the principal stresses, the other two principal stresses are independent of σ_z .

1.4 MOHR'S CIRCLE EQUATIONS

In the special cases where two of the three shear stress terms vanish (e.g., $\tau_{yx} = \tau_{zx} = 0$), the stress σ_z normal to the xy plane is a principal stress and the other two principal stresses lie in the xy plane. This is illustrated in Figure 1.5.

For these conditions $\ell_{x'z} = \ell_{y'z} = 0$, $\tau_{yz} = \tau_{zx} = 0$, $\ell_{x'x} = \ell_{y'y} = \cos \phi$, and $\ell_{x'y} = -\ell_{y'x} = \sin \phi$. Substituting these relations into equations 1.9 results in

$$\begin{aligned}\tau_{x'y'} &= \cos \phi \sin \phi (-\sigma_x + \sigma_y) + (\cos^2 \phi - \sin^2 \phi) \tau_{xy}, \\ \sigma_{x'} &= (\cos^2 \phi) \sigma_x + (\sin^2 \phi) \sigma_y + 2(\cos \phi \sin \phi) \tau_{xy}, \quad \text{and} \\ \sigma_{y'} &= (\sin^2 \phi) \sigma_x + (\cos^2 \phi) \sigma_y + 2(\cos \phi \sin \phi) \tau_{xy}.\end{aligned}\quad (1.13)$$

These can be simplified with the trigonometric relations

$$\sin 2\phi = 2 \sin \phi \cos \phi \quad \text{and} \quad \cos 2\phi = \cos^2 \phi - \sin^2 \phi \quad \text{to obtain}$$

$$\tau_{x'y'} = -\sin 2\phi (\sigma_x - \sigma_y)/2 + (\cos 2\phi) \tau_{xy}, \quad (1.14a)$$

$$\sigma_{x'} = (\sigma_x + \sigma_y)/2 + \cos 2\phi (\sigma_x - \sigma_y)/2 + \tau_{xy} \sin 2\phi, \quad \text{and} \quad (1.14b)$$

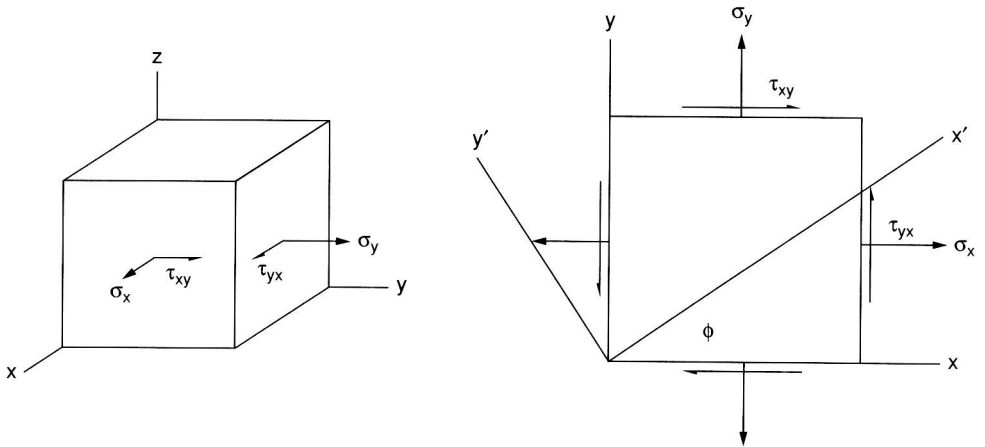
$$\sigma_{y'} = (\sigma_x + \sigma_y)/2 - \cos 2\phi (\sigma_x - \sigma_y)/2 + \tau_{xy} \sin 2\phi. \quad (1.14c)$$

If $\tau_{x'y'}$ is set to zero in equation 1.14a, ϕ becomes the angle θ between the principal axes and the x and y axes. Then

$$\tan 2\theta = \tau_{xy}/[(\sigma_x - \sigma_y)/2]. \quad (1.15)$$

The principal stresses, σ_1 and σ_2 , are then the values of $\sigma_{x'}$ and $\sigma_{y'}$

$$\begin{aligned}\sigma_{1,2} &= (\sigma_x + \sigma_y)/2 \pm (1/2)[(\sigma_x - \sigma_y) \cos 2\theta] + \tau_{xy} \sin 2\theta \quad \text{or} \\ \sigma_{1,2} &= (\sigma_x + \sigma_y)/2 \pm (1/2)[(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2]^{1/2}.\end{aligned}\quad (1.16)$$



1.5. Stress state for which the Mohr's circle equations apply.