# Constructive Nonlinear Control

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With 41 Figures



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# Preface

In this book several streams of nonlinear control theory are merged and directed towards a constructive solution of the feedback stabilization problem. Analytic, geometric and asymptotic concepts are assembled as design tools for a wide variety of nonlinear phenomena and structures. Differential-geometric concepts reveal important structural properties of nonlinear systems, but allow no margin for modeling errors. To overcome this deficiency, we combine them with analytic concepts of passivity, optimality and Lyapunov stability. In this way geometry serves as a guide for construction of design procedures, while analysis provides robustness tools which geometry lacks.

Our main tool is passivity. As a common thread, it connects all the chapters of the book. Passivity properties are induced by *feedback passivation* designs. Until recently, these designs were restricted to weakly minimum phase systems with relative degree one. Our recursive designs remove these restrictions. They are applicable to wider classes of nonlinear systems characterized by feedback, feedforward, and interlaced structures.

After the introductory chapter, the presentation is organized in two major parts. The basic nonlinear system concepts - passivity, optimality, and stability margins - are presented in Chapters 2 and 3 in a novel way as design tools. Most of the new results appear in Chapters 4, 5, and 6. For cascade systems, and then, recursively, for larger classes of nonlinear systems, we construct design procedures which result in feedback systems with optimality properties and stability margins.

The book differs from other books on nonlinear control. It is more design-oriented than the differential-geometric texts by Isidori [43] and Nijmeijer and Van der Schaft [84]. It complements the books by Krstić, Kanellakopoulos and Kokotović [61] and Freeman and Kokotović [26], by broadening the class of systems and design tools. The book is written for an audience of graduate students, control engineers, and applied mathematicians interested in control theory. It is self-contained and accessible with a basic knowledge of control theory as in Anderson and Moore [1], and nonlinear systems as in Khalil [56].

For clarity, most of the concepts are introduced through and explained by examples. Design applications are illustrated on several physical models of practical interest.

The book can be used for a first level graduate course on nonlinear control, or as a collateral reading for a broader control theory course. Chapters 2, 3, and 4 are suitable for a first course on nonlinear control, while Chapters 5 and 6 can be incorporated in a more advanced course on nonlinear feedback design.

* * *

The book is a result of the postdoctoral research by the first two authors with the third author at the Center for Control Engineering and Computation, University of California, Santa Barbara. In the cooperative atmosphere of the Center, we have been inspired by, and received help from, many of our colleagues. The strongest influence on the content of the book came from Randy Freeman and his ideas on inverse optimality. We are also thankful to Dirk Aeyels, Mohammed Dahleh, Miroslav Krstić, Zigang Pan, Laurent Praly and Andrew Teel who helped us with criticism and advice on specific sections of the book. Gang Tao generously helped us with the final preparation of the manuscript. Equally generous were our graduate students Dan Fontaine with expert execution of figures, Srinivasa Salapaka and Michael Larsen with simulations, and Kenan Ezal with proofreading.

Our families contributed to this project by their support and endurance. Ivana, Edith, Simon and Filip often saw their fathers absent or absent-minded. Our wives, Natalie, Seka, and Anna unwaveringly carried the heaviest burden. We thank them for their infinite stability margins.

* * *

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Mrdjan Janković
Petar Kokotović

*Santa Barbara, California, August 1996*

# Contents

# Chapter 1

# Introduction

Control theory has been extremely successful in dealing with linear time-invariant models of dynamic systems. A blend of state space and frequency domain methods has reached a level at which feedback control design is systematic, not only with disturbance-free models, but also in the presence of disturbances and modeling errors. There is an abundance of design methodologies for linear models: root locus, Bode plot[illegible]-optimal control, eigenstructure assignment, H-infinity, $\mu$-synth[illegible]trix inequalities, etc. Each of these methods can be [illegible]ation, tracking, disturbance attenuation and similar [illegible]jectives.

The situati[illegible]ferent for nonlinear models. Although several nonlinear [illegible] are beginning to emerge, none of them taken alone is suffic[illegible]sfactory feedback design. A question can be raised whether a [illegible] methodology can encompass all nonlinear models of practical [illegible], and whether the goal of developing such a methodology should even [illegible]e pursued. The large diversity of nonlinear phenomena suggests that, with a single design approach most of the results would end up being unnecessarily conservative. To deal with diverse nonlinear phenomena we need a comparable diversity of design *tools and procedures.* Their construction is the main topic of this book.

Once the "tools and procedures" attitude is adopted, an immediate task is to determine the areas of applicability of the available tools, and critically evaluate their advantages and limitations. With an arsenal of tools one is encouraged to construct design procedures which exploit structural properties to avoid conservativeness. Geometric and analytic concepts reveal these properties and are the key ingredients of every design procedure in this book.

Analysis is suitable for the study of stability and robustness, but it often disregards structure. On the other hand, geometric methods are helpful in determining structural properties, such as relative degree and zero dynamics, but, taken alone, do not guarantee *stability margins*, which are among the prerequisites for *robustness*. In the procedures developed in this book, the geometry makes the analysis constructive, while the analysis makes the geometry more robust.

Chapters 2 and 3 present the main geometric and analytic tools needed for the design procedures in Chapters 4, 5, and 6. Design procedures in Chapter 4 are constructed for several types of cascades, and also serve as building blocks in the construction of recursive procedures in Chapters 5 and 6.

The main recursive procedures are *backstepping* and *forwarding*. While backstepping is known from [61], forwarding is a procedure recently developed by the authors [46, 95]. This is its first appearance in a book. An important feature of this procedure is that it endows the systems with certain optimality properties and desirable stability margins.

In this chapter we give a brief preview of the main topics discussed in this book.

## 1.1 Passivity, Optimality, and Stability

### 1.1.1 From absolute stability to passivity

Modern theory of feedback systems was formed some 50-60 years ago from two separate traditions. The Nyquist-Bode *frequency domain methods*, developed for the needs of feedback amplifiers, became a tool for servomechanism design during the Second World War. In this tradition, feedback control was an outgrowth of linear network theory and was readily applicable only to linear time-invariant models.

The second tradition is more classical and goes back to Poincaré and Lyapunov. This tradition, subsequently named the *state-space approach*, employs the tools of nonlinear mechanics, and addresses both linear and nonlinear models. The main design task is to achieve stability in the sense of Lyapunov of feedback loops which contain significant nonlinearities, especially in the actuators. A seminal development in this direction was the *absolute stability* problem of Lurie [70].

In its simplest form, the absolute stability problem deals with a feedback loop consisting of a linear block in the forward path and a nonlinearity in the feedback path, Figure 1.1. The nonlinearity is specified only to the extent that it belongs to a "sector", or, in the multivariable case, to a "cone". In other words, the admissible nonlinearities are linearly bounded. One of the absolute

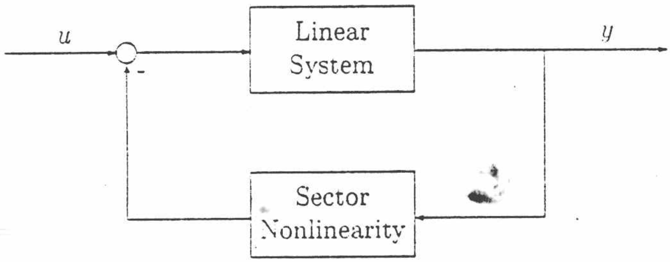


**Figure 1.1:** The absolute stability problem.

stability results is a Lyapunov function construction for this class of systems. The stability property is "absolute" in the sense that it is preserved for any nonlinearity in the sector. Hence, a "sector stability margin" is guaranteed.

During a period of several years, the frequency domain methods and the absolute stability analysis coexisted as two separate disciplines. Breakthroughs by Popov in the late 1950's and early 1960's dramatically changed the landscape of control theory. While Popov's stability criterion [87] was of major importance, even more important was his introduction of the concept of *passivity* as one of the fundamental feedback properties [88].

Until the work of Popov, passivity was a network theory concept dealing with rational transfer functions which can be realized with passive resistances, capacitances and inductances. Such transfer functions are restricted to have *relative degree* (excess of the number of poles over the number of zeros) not larger than one. They are called *positive real* because their real parts are positive for all frequencies, that is, their phase lags are always less than 90 degrees. A key feedback stability result from the 1960's, which linked passivity with the existence of a quadratic Lyapunov function for a linear system, is the celebrated Kalman-Yakubovich-Popov (KYP) lemma also called *Positive Real Lemma.* It has spawned many significant extensions to nonlinear systems and adaptive control.

### 1.1.2 Passivity as a phase characteristic

The most important passivity result, and also one of the fundamental laws of feedback, states that *a negative feedback loop consisting of two passive systems is passive.* This is illustrated in Figure 1.2. Under an additional detectability condition *this feedback loop is also stable.*

To appreciate the content of this brief statement, assume first that the two

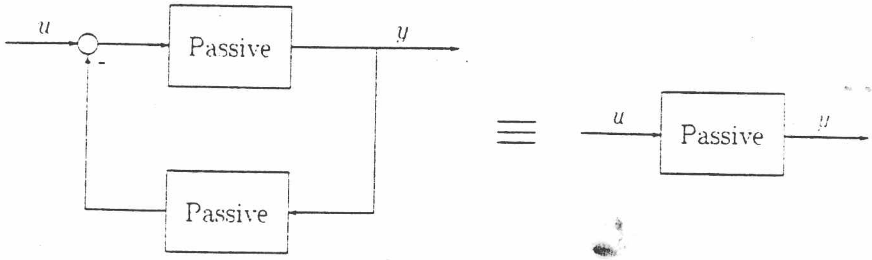


**Figure 1.2:** The fundamental passivity result.

passive blocks in the feedback connection of Figure 1.2 are linear. Then their transfer functions are positive real, that is, with the phase lag not larger than 90 degrees. Hence, the phase lag over the entire feedback loop is not larger than 180 degrees. By the Nyquist-Bode criterion, such a linear feedback loop is stable for all feedback gains, that is, it possesses an "infinite gain margin".

When the two blocks in the feedback loop are nonlinear, the concept of passivity can be seen to extend the Nyquist-Bode 180 degree phase lag criterion to nonlinear systems. For nonlinear systems, passivity can be therefore interpreted as a "phase" property, a complement of the gain property characterized by various small gain theorems such as those presented in [18].

In the early 1970's, Willems [120] systematized passivity (and dissipativity) concepts by introducing the notions of *storage function* $S(x)$ and *supply rate* $w(u, y)$, where $x$ is the system state, $u$ is the input, and $y$ is the output. A system is passive if it has a positive semidefinite storage function $S(x)$ and a bilinear supply rate $w(u, y) = u^T y$, satisfying the inequality

$$S(x(T)) - S(x(0)) \leq \int_0^T w(u(t), y(t))\, dt \tag{1.1.1}$$

for all $u$ and $T \geq 0$. Passivity, therefore, is the property that the increase in storage $S$ is not larger than the integral amount supplied. Restated in the derivative form

$$\dot{S}(x) \leq w(u, y) \tag{1.1.2}$$

passivity is the property that the rate of increase of storage is not higher than the supply rate. In other words, any storage increase in a passive system is due solely to external sources. The relationship between passivity and Lyapunov stability can be established by employing the storage $S(x)$ as a Lyapunov function. We will make a constructive use of this relationship.

### 1.1.3 Optimal control and stability margins

Another major development in the 1950's and 1960's was the birth of optimal control twins: Dynamic Programming and Maximum Principle. An optimality result crucial for feedback control was the solution of the optimal linear-quadratic regulator (LQR) problem by Kalman [50] for linear systems $\dot{x} = Ax + Bu$. The well known optimal control law has the form $u = -B^T P x$, where $x$ is the state, $u$ is the control and $P$ is the symmetric positive definite solution of a matrix algebraic Riccati equation. The matrix $P$ determines the optimal value $x^T P x$ of the cost functional, which, at the same time, is a Lyapunov function establishing the asymptotic stability of the optimal feedback system.

A remarkable connection between optimality and passivity, established by Kalman [52], is that a linear system can be optimal only if it has a passivity property with respect to the output $y = B^T P x$. Furthermore, optimal linear systems have infinite gain margin and phase margin of 60 degrees.

These *optimality, passivity, and stability margin* properties have been extended to nonlinear systems which are affine in control:

$$\dot{x} = f(x) + g(x)u \tag{1.1.3}$$

A feedback control law $u = k(x)$ which minimizes the cost functional

$$J = \int_0^\infty (l(x) + u^2)dt \tag{1.1.4}$$

where $l(x)$ is positive semidefinite and $u$ is a scalar, is obtained by minimizing the Hamiltonian function

$$\mathcal{H}(x, u) = l(x) + u^2 + \frac{\partial V}{\partial x}(f(x) + g(x)u) \tag{1.1.5}$$

If a differentiable optimal value function $V(x)$ exists, then the optimal control law is in the "$L_g V$-form":

$$u = k(x) = -\frac{1}{2} L_g V(x) = -\frac{1}{2}\frac{\partial V}{\partial x} g(x) \tag{1.1.6}$$

The optimal value function $V(x)$ also serves as a Lyapunov function which, along with a detectability property, guarantees the asymptotic stability of the optimal feedback system. The connection with passivity was established by Moylan [80] by showing that, as in the linear case, the optimal system has an infinite gain margin thanks to its passivity property with respect to the output $y = L_g V$.