COLLECTED WORKS
OF
THEODORE VON KÁRMÁN

1952-1963

COLLECTED WORKS OF THEODORE VON KARMAN

1952-1963

VON KARMAN INSTITUTE FOR FLUID DYNAMICS
INSTITUT VON KARMAN DE DYNAMIQUE DES FLUIDES
RHODE-ST-GENÈSE, BELGIUM
1975

von Kármán Institute for Fluid Dynamics
Institut von Kármán de Dynamique des Fluides
Chaussée de Waterloo, 72
1640 Rhode-St-Genèse, Belgium

Set and printed by Technical Editing and Reproduction Ltd
Harford House, 7–9 Charlotte St, London. W1P 1HD

VON KÁRMÁN COLLECTED WORKS COMMITTEE MEMBERS

Professor G. Gabrielli (Chairman)
Professor L. Crocco
Professor F. J. Malina
Professor F. E. Marble
Professor W. R. Sears
Dr F. L. Wattendorf
Professor J. F. Wendt
Mr. R. A. Willaume

The editors and publishers wish to thank the original copyright holders and the co-authors of certain papers for their generous permission to reprint the papers and articles contained in this volume. Individual acknowledgements precede each article.

VON KÁRMÁN COLLECTED WORKS COMMITTEE MEMBERS

The editors and publishers wish to thank the original copyright holders and the co-authors of certain papers for their generous permission to reprint the papers and articles contained in this volume. Individual acknowledgements precede each article.

CONTENTS

GENERAL INTRODUCTION

by Professor G. Gabrielli

In October 1972, on the occasion of the 20th Anniversary of AGARD, a solemn ceremony in memory of its founder, Theodore von Kármán, was held in Brussels. At the end of the commemorative speech I had the honor to deliver, I made a suggestion which has been in my mind for a long time and which, in my opinion, was the most valid and worthy manner in which to commemorate such a man and scientist as Theodore von Kármán.

He was the author of a collection of four volumes entitled "The Collected Works of Theodore von Kármán", issued by Butterworth Scientific Publications in 1956, covering the period 1902 to 1951. Often, when I consulted this collection, I though that it would be necessary to supplement it with another volume. And who, better than AGARD, could undertake its publication?

That is why, in Brussels, I terminated my commemorative speech with such a proposal, which met with the unanimous agreement of the National Delegates Board and the audience as well.

Soon after these celebrations, a Committee was established to select Theodore von Kármán's writings and to implement this initiative. After two years of exhaustive work and Committee meetings, its work was completed. This publication includes his writings over the period 1952 to 1963 – when von Kármán died – and covers a wide range of his intensive activity.

Theodore von Kármán liked to consider himself an engineer and, in this capacity, he considered the application of mathematics to engineering a means of properly interpreting physical phenomena. He also maintained that the development of mathematical analysis could not be separated from the development of physics, and particularly of mechanics. But he was also, and above all, an outstanding mathematician, with a particular gift for applied mathematics. This mental aptitude, together with his genius for perception, make his writings still up to date, not only in the depth and originality of the contents but also in the methodology which they represent.

In fact, the originality and depth of von Kármán's studies and research give not only an irreplaceable source of teaching and all-important scientific discoveries, but also an image of the man with all his ability of understanding and feeling for that which surrounds us. All this activity was based on a deep human warmth which he transmitted to those who had the good fortune to meet him. More than 10 years after his death, this work is aimed at being a meeting point to evoke the figure of a master of science and life, and to make available to scholars, pupils and friends of von Kármán a complete picture of the amazing scientific activity of this great man and an image of his unique personality.

I thank my fellow Committee Members for their dedicated and invaluable cooperation and, in particular, Professor W.R. Sears and Dr F.L. Wattendorf who accepted my invitation to write Prefaces for Parts I and II of the book. The other Committee Members (former students and friends of Dr von Kármán) were Dr F. Malina, Professor F. Marble, Professor L. Crocco and Mr R.A. Willaume and I thank them for their suggestions and advice. I would also express my thanks to AGARD and the von Kármán Institute for their support in bringing this project to fruition, and in particular to Mr O. Blichner, Director of AGARD, Mr R.A. Willaume, Director, Plans and Programmes of AGARD, Professor J. Smolderen, Director of the von Kármán Institute, Professor J. Wendt of the von Kármán Institute and Mr J. Trotman, Scientific Publications Executive of AGARD, for their assistance. The actual printing of this book has only been possible owing to the understanding and cooperation of Mr H. Stanton, a Director of Technical Editing and Reproduction Ltd. The extensive bibliography was produced with the help of the Members of the Technical Information Panel of AGARD, whose help I have greatly appreciated.

PART I

PREFACE
by Professor W.R.Sears

In his Preface to the *Collected Works of Theodore von Kármán, 1902–1951*, Dr Hugh L.Dryden expressed the hope that Professor von Kármán, having reached the age of seventy, would continue to contribute to science and technology and have many more fruitful years. This hope was realized; Theodore von Kármán lived almost to the age of eighty-two and continued to study, to write, to lecture, and to advise. The depth of his scientific insight and the breadth of his far-ranging technical interests are attested to by the present collection.

In this, the first half of the collection, the editors have gathered together what they believe are his most significant scientific and technical publications of this period. To make this selection, they have tried to distinguish between contributions obviously of lasting scientific value, which are collected in this part of the volume, and others whose purposes were more transient or whose lasting significance is more historical than scientific. Many of the latter are reproduced in the second part of this volume.

The scientific papers, like those of the preceding four volumes, represent a remarkably wide range of subjects. Clearly, many were inspired by practical engineering developments of the times. Typically, Professor von Kármán's studies in these years involve interdisciplinary areas; he was always intrigued and delighted by such combined fields as magnetofluid-mechanics and, especially, aerothermochemistry. At least five of the papers collected here are related to this subject, which concerns combustion, flame propagation, and heat release in flowing gases.

In his well-recognized role of Senior Scientist of the engineering world, Professor von Kármán, in his last years, contributed to meetings and publications in many countries and in several languages. His papers, therefore, have sometimes appeared in relatively obscure journals and proceedings. One of the virtues of this collection is that it will bring these writings to the attention of a wide audience. In the hope of performing another service, the editors have also prepared, with due humility, brief editorial comments as introductions to certain papers of this collection. These are intended to help the readers – in particular, younger readers – to relate these papers to the larger body of technical and scientific literature.

This paper is an abridged version of Professor von Kármán's contribution to the volume "General Theory of High-Speed Aerodynamics" (W.R.Sears, ed.), which constituted Volume VI of the series *High-Speed Aerodynamics and Jet Propulsion* published by Princeton University Press in 1954. This section (VI, A) is the opening section of the volume and serves as a general introduction and survey of the field of aerodynamic theory.

Proceedings of the First National Congress of Applied Mechanics, A.S.M.E., New York, 1952, pp.673–685.

1

ON THE FOUNDATIONS OF HIGH SPEED AERODYNAMICS*

1. INCOMPRESSIBLE FLOW

Looking back on fifty years of aerodynamics research during the first half of this century, it appears most remarkable that the crude approximation which considers the air as an incompressible nonviscous fluid proved itself so valuable in solving many problems of practical aircraft design.

In certain cases, for example in the performance calculations of airplanes and propellers, useful results could be obtained even by a further simplification, that is, by approximating the actual flow by infinitely small perturbations of a uniform and parallel airflow. Of course, it was recognized at the time that certain phenomena, especially those connected with drag and stall, require the consideration of viscosity. However, even then Prandtl's classical idea to restrict the influence of viscosity to the neighborhood of solid walls, i.e., the concept of the boundary layer, proved to be sufficiently exact for the description and, in some cases, for the prediction of the phenomena.

This state of affairs was fundamentally changed by the advent of high-speed aircraft. At first, it seemed that compressibility troubles could be met by compressibility corrections. However, very soon it became clear that the engineer needed a full grasp and knowledge of fluid mechanics over the entire speed range, extending from incompressible flow to flows with velocities large in comparison to the velocity of sound, and over a density range extending almost to a complete vacuum.

The concept of an incompressible fluid evidently ignores the fact that pressure variations are propagated in a fluid with a finite velocity. The assumption of the instantaneous propagation of pressure introduces essential mathematical simplifications which make possible the application of Laplace's equation and the methods of conformal transformation, i.e., the simplest and most popular means of mathematical physics. It also justifies

* Presented at the First U.S. National Congress of Applied Mechanics, Ann Arbor, Michigan, June 1951.

the concept of apparent mass in the study of nonsteady phenomena. It is one of the fundamental theorems of the mechanics of incompressible nonviscous fluid that an arbitrary continuous sequence of vortex-free flow patterns always represents a dynamically correct transient flow. It is evident that this means an enormous reduction of difficulties in dealing with nonsteady phenomena.

2. PROPAGATION OF PRESSURE

I believe the first calculation of the propagation of a pressure wave through air was made by Newton. Since he could not know the difference between isothermal and isentropic compression, his formula for the velocity of sound has a wrong numerical factor. He says, however, quite clearly in Theorem XXXVIII of his second book: "The velocities of pulses propagated in an elastic fluid are in a ratio compounded of the square root of the ratio of the elastic force directly, and the square root of the ratio of the density inversely; supposing the elastic force of the fluid to be proportional to its condensation."

He gives in the same book the following description of the mechanism of the resistance of a solid body moving in an elastic fluid: "Projectiles excite a motion in fluids as they pass through them, and this motion arises from the excess of the pressure of the fluid at the fore parts of the projectile above the pressure of the same at the hinder parts; and cannot be less in mediums infinitely fluid than it is in air, water, and quicksilver, in proportion to the density of matter in each. Now this excess of pressure does, in proportion to its quantity, not only excite a motion in the fluid, but also acts upon the projectile so as to retard its motion; and therefore the resistance in every fluid is as the motion excited by the projectile in the fluid; and cannot be less in the most subtle ether in proportion to the density of that ether, than it is in air, water, and quicksilver, in proportion to the densities of those fluids." We will see later that this concept of the drag was correct for the case of the supersonic motion of projectiles, whereas in the subsonic case — at least for rounded bodies — his description is at variance with d'Alembert's theorem.

3. SUPERSONIC VS. SUBSONIC FLOW

From Newton's concept of the propagation of motion in an elastic fluid, one arrives directly at the well-known picture of subsonic and supersonic flow. For the following considerations we restrict ourselves to flows produced by small disturbances, and neglect viscosity, i.e., absorption of energy in the air.

Since a slight pressure change is propagated at sound velocity, it is evident that the effect of pressure changes produced in the air by a body moving faster than sound cannot reach points ahead of the body. It may be said that the body is unable to send signals ahead. It is seen that there is a fundamental difference between subsonic and supersonic motion.

Consider the case of subsonic stationary motion, for example, the uniform level flight of an airplane. Then a pressure signal travels ahead at sound velocity minus flight velocity relative to the airplane, whereas a signal travels backward at a speed equal to the sum of the flight and sound velocity. So the distribution of the pressure effects is no longer symmetric; nevertheless, every point in space is reached by a signal, provided the flight started from an infinitely remote point. As can easily be seen, this is not the case in supersonic flight.

Consider the simplest case of a point source (Fig.1). Figure 1(a) shows the spherical surface that the pressure effect reaches in equal time intervals in the case of a point source at rest. Figure 1(b) shows the same surfaces relative to the point source moving with a speed less than that of the sound. Figure 1(c) represents the case of a point source moving with sonic velocity, and Figure 1(d) the case of a source moving faster than sound. It is seen that in the last case all action is restricted to the interior of a cone that includes all the spheres emitted by the source before the instant considered. The outside of this cone can be called the *zone of silence*. It is easily seen that the trigonometrical sine of the half vortex angle of the cone is equal to the reciprocal of the Mach number (ratio of source speed to sound speed). This angle is called the Mach angle. The cone that separates the zone of action from the zone of silence is called the Mach cone.

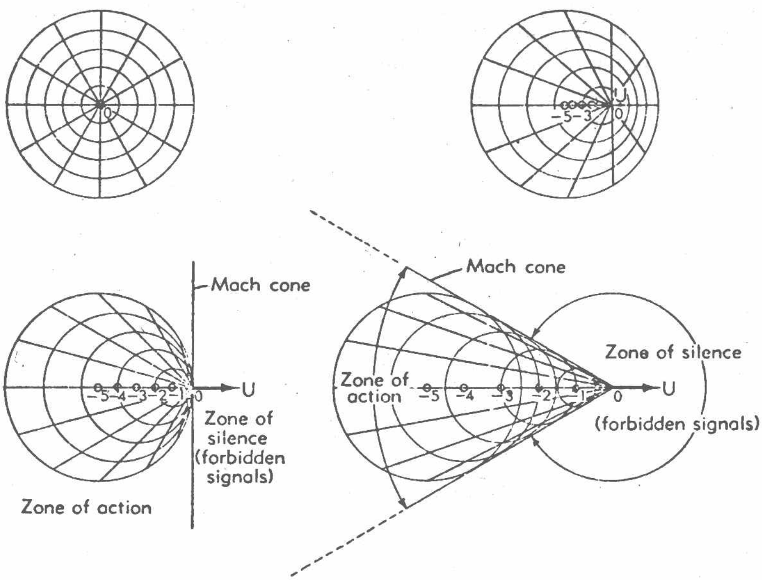

Figure 1

The points plotted in Figures 1(a)–(d) show the location of mass points that are supposed to be emitted from the source, and to move at sound velocity in all directions. They illustrate qualitatively the distribution of the density of action in the various cases. In the subsonic case one finds that the pressure effect not only decreases with increasing distance from the source, but is also dispersed in all directions. In the case of a body moving at supersonic velocity, the bulk of the effect is concentrated in the neighborhood of the Mach cone that forms the outer limit of the zone of action.

4. LINEARIZED THEORY OF SUPERSONIC FLOW

The so-called linearized theory of supersonic flow builds up the flow produced by the motion of a body by superposition of small disturbances such as considered in the last paragraph. One can develop in this way relatively simple methods for the computation of velocity and pressure distributions in the field and also for the computation of the forces, lift and drag, acting on moving bodies. In the last decade very extended analytical work has been done using the linearized theory of supersonic flow. This grew out from such modest beginnings as the computation of the drag of slender bodies, published by N.Moore and the writer in 1932, the article on "Problem of resistance in compressible fluids", presented by the writer at the Volta Congress for high speed in 1935 in Rome, Ackeret's work on the lift of a two-dimensional thin airfoil published in 1928, and Busemann's work of 1935. In the case of vortex-free flow, the equations of motion can be reduced to equations analogous to the wave equation. The coordinate parallel to the direction of the main flow, or to the motion of the missile, wing or body, plays the role of the time coordinate. Hence, the well-established methods of finding solutions of the wave equation can be used in a greater number of problems of practical importance, similarly to the capabilities of the methods for the solution of Laplace's equation in the case of incompressible flow.

The concept of the linearized supersonic flow theory also reveals the existence of a novel kind of drag which we do not encounter in subsonic motion, and which we designate as "wave drag".

As before, we neglect viscosity and assume that the motion of the body produces disturbances that can be considered small. At a certain distance from the moving body this second assumption will, in general, be satisfied.

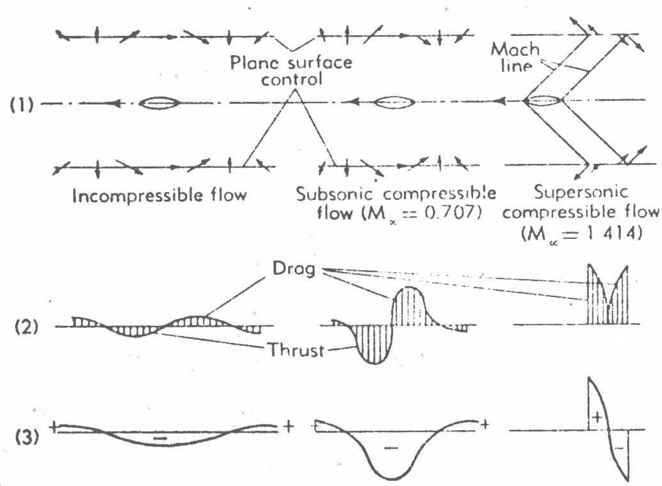

Figure 2

Now consider the body and the surrounding air inside of a cylindrical control surface as one mechanical system. Then one finds that, because of the concentrated action that characterizes the propagation of pressure from a source moving at supersonic velocity, the total flux of momentum of the air masses entering and leaving the cylindrical boundary remains finite even when the boundary is removed to an arbitrarily large distance.

Figure 2 refers to the case of a two-dimensional symmetric airfoil, with sharp leading edge, moving through the air initially at rest. Let us consider the flow through a plane parallel to the plane of symmetry at a certain distance from the body. The diagram first shows the distribution of induced velocities (1) and the horizontal component of momentum transfer (2) along this plane for three cases. It is evident that the reaction of outgoing flow having a horizontal component opposite to the flight direction and incoming flow with a horizontal component in the flight direction is equivalent to a propulsive thrust acting on the body. Conversely, outgoing flow with a component in the flight direction and incoming flow directed opposite to the flight direction give rise to a drag. In the two subsonic cases ($M = 0$ and $M = 0.707$), thrust and drag contributions are balanced, and the total horizontal momentum transfer is equal to zero. This is in accordance with d'Alembert's theorem.

The influence of increasing Mach number is essentially to increase the magnitude of the induced velocities, and to increase the concentration of the disturbance in the region extending laterally outward from the body. The increase in the concentration of action is also illustrated by the pressure distribution (3) on the control surface. In the supersonic case ($M = 1.414$), the disturbance is restricted to two strips bounded by two Mach lines. These lines are the intersections of planes that are envelopes of the Mach cones starting from points of the leading and trailing edges of the airfoil. The horizontal component of the outward flow is in the flight direction; that of the inward flow is opposite to the flight direction. Hence, both represent drag on the body. This type of drag is called "wave drag".

The linearized theory, however, has serious limitations. First, it gives only a first approximation, since all deviations from the uniform parallel flow are considered infinitely small and therefore additive. This is justified only for very thin or slender bodies. If the disturbance caused by the presence of the body cannot be considered small, the linearized theory does not apply, or at least needs importance corrections. One of the necessary corrections is to take into account the existence of discontinuities (shockwaves) in supersonic flow.

Second, there are speed ranges in which the linearization of the equation of motion even for small disturbances is not justified. The conditions for the validity of the linearized theory are twofold:

(a) The perturbation velocities must be small in comparison with both the mean stream velocity and the velocity of sound.

(b) The perturbation velocities must be small in comparison with the difference of the mean stream velocity and the sound velocity.

Condition *(a)* excludes the case of very high velocities; evidently if the mean stream velocity is several times larger than the sound velocity, disturbances which are small relative to the mean stream velocity may be of the same order of magnitude as the sound velocity. This speed range is called the "hypersonic range".

On the other hand, condition *(b)* excludes the range near $M = 1$ from the velocity of the linearized theory. We call this range the "transonic range".

5. FINITE DISTURBANCES IN COMPRESSIBLE FLOW SHOCKWAVES

The solution of the equations of motion in the case of finite disturbances of a uniform parallel stream of an ideal compressible fluid is, in general, a cumbersome mathematical problem, which has been treated by a great number of authors with relatively little success. In the case of a supersonic stream, at least in the domains where the flow remains supersonic, the method of characteristics can be applied with good results. Also the transfer of the computation from the physical plane to the so-called hodograph plane, i.e., the replacement of space coordinates by velocity coordinates as independent variables, is helpful in many problems, since the equations in the hodograph plane become linear by means of the Legendre transformation. In addition to computation difficulties a new phenomenon has to be taken into account: from the mathematical point of view – the possibility of discontinuous solutions; from the physical point of view – the existence of shockwaves.

In subsonic flow the only possible discontinuous change of the velocity is tangential to a surface consisting of streamlines. Such a discontinuity does not violate any mechanical law because of the nonexistence of shear stresses in the fluid. In the supersonic flow, however, a discontinuous change of velocity is also possible along streamlines. Such discontinuous change is called a *shock*. In fact, the laws of mechanics are satisfied provided the fluid element conserves its mass, momentum and energy. The physical reason why a discontinuous change is only possible in supersonic flow can be easily seen. The theorem of conservation of mass calls for the equality of the so-called Fano number (product of density and normal velocity) on both sides of the surface. The question arises whether this product can have the same value for different individual values of velocity and density while at the same time the momentum and energy of the fluid element remain unchanged. Now, if we consider the expansion or compression process of a gas, we find that the Fano number has a maximum when the velocity of the gas is equal to the velocity of sound. Consequently, in the case of a shock, the velocity normal to the discontinuity surface has to be subsonic on one side and supersonic on the other side. Therefore no discontinuous change can occur in a purely subsonic flow. The rule, however, does not exclude a discontinuous change in a purely supersonic flow, since it refers to the components of the velocity normal to the discontinuity surface. Thus an "oblique shock" is possible also in a purely supersonic flow.

One gains a certain insight into the physical relationships by the consideration of the following problem: