



# **Materials and Acoustics Handbook**

**Edited by**  
**Michel Bruneau**  
**Catherine Potel**

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# Materials and Acoustics Handbook

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## **Materials and Acoustics Handbook**

## Foreword

It is with great pleasure that we are writing a few lines of introduction for the book *Materials and Acoustics Handbook*: after being persuaded that this book responds to an obvious need and that it contributes to the affirmation of a research field, which is currently experiencing much development, situated as it is at the crossroads of several disciplines.

It is interesting first of all to spend several moments reflecting on the title of this book. Those who have followed the recent developments of the fields to which this book makes reference will have noticed the structuring efforts carried out on the one hand by the Acoustics community and on the other hand the Material Mechanics community over the last 20 years. These respective structurings corresponded to the desire by researchers involved in these fields to be better identified and thus better recognized. However, they were also made necessary by the need for training specific to these fields, expressed by the world of work. A better identification of each of these fields is thus accompanied by the birth of new educational pathways in universities or engineering schools, of benefit, along with this book, as an extremely useful teaching tool. The two communities concerned, although certainly very different, have naturally interacted, and the creation of actions in the fields of “Materials for Acoustics” or “Acoustics for Materials” have been assisted, in which the role of each has been clearly identified, either as regards the final objective or as regards the means to achieve it. This book goes further, Materials and Acoustics are closely linked, and we are convinced this will play a driving role in bringing together these two communities, which is already underway.

The content of this book thus principally covers the propagation of acoustic waves in solid or porous, i.e. multi-phase (fluid-solid), materials. All of these terms can be discussed endlessly as they are evolving as research and applications advance. Thus, acoustics basically means what we assume on the one hand to be a propagation medium (fluid) and on the other hand a so-called audible range of

frequencies. Inevitable extensions are made to high or low frequencies (ultrasound or infrasound) and other media. In order to illustrate this, let us discuss the terms medium and frequency.

As far as the medium is concerned, propagation in a solid is more complicated than in a fluid (in any case, a fluid at rest and without dissipation), due to the presence of transverse waves. The (vectorial) “elastic waves” are more general than (scalar) “acoustic waves”. However, for the porous materials often used to reduce the “audible” noise and in this case called “acoustic materials”, we tend to talk about acoustic waves instead, even if all waves existing in solids are found there! Often, if the excitation is made via a fluid, the material in fact behaves in a similar way to a fluid, but this is not the case when the excitation is solid in nature. Many media other than acoustic materials are porous, and wave propagation within them is the object of countless works of research with essential applications such as geophysics or oil exploration.

As far as frequency is concerned, even if this work devotes a large part to ultrasound, particularly for applications such as non-destructive testing (NDT) and medical imaging, the methods developed are as general as possible. In certain problems, low frequency waves propagating in solids are in fact essential; for example, when it is a question of “seismic waves” in the earth. It is thus necessary not to immediately limit ourselves to a range of frequencies. Furthermore, it is becoming increasingly common to use temporal methods (rather than frequential methods) for both the generation of testing frequencies as well as for the analysis of the response of propagation media.

This book does not discuss vibroacoustics, another field where the mechanics of solids and acoustics are closely linked. Vibroacoustics are concerned with the response of structures (rather than propagation in a 3D element) and modeling tools corresponding to this are of a different type.

Having finished with these preliminaries, we will now move on to the essential matters. We will not analyze the book chapter by chapter. For the benefit of the reader however, it is useful to point out several notable characteristics in order to guide the reader through the various parts. Parts 1 to 5 contain the mathematical or physical foundations, useful theories for comprehension of the subject. Parts 6 to 8 are mainly dedicated to putting into practice the concepts and methods from Parts 1 to 5.

– Any reader concerned with becoming more familiar with the basics on linear waves will find a complete discussion on their propagation in different types of media, whether fluid or solid, isotropic or anisotropic, homogenous or stratified, in Part 1. This discussion can be used directly at a Master’s level, but can also be used

for a direct algorithmic implementation (we are thinking for example of transfer matrix formalism, which is well adapted to the systematic study of stratified media).

- This general discussion is completed by more technical results on research into Green functions in anisotropic media or into propagation in continuously stratified media, detailed in Chapters 4 and 19, which benefit from a second reading to deepen knowledge.

- Porous media, very important on the fundamental plane on that of its applications (acoustic comfort, imaging of biological tissues) and their modeling, are the subject of Part 2. The derivation of Biot's model (around which the international community is forming itself into regular annual world conferences) is discussed in several types of literature, ranging from everything macroscopic to the micro-macro passage through asymptotic developments. The presentation here favors average methods as well as the Lagrangian formalism approach. The first step clearly affects the essential role played by the microstructure of materials especially through the concepts of tortuosity or permeability as well as the frequency limit of different developed models. Part 2 furthers and completes the founding work by Johnson and the pioneering book by Jean-François Allard using recent developments. It also contains an opening on numerical methods (implementation of Biot's model in a finite element code).

- Still in Part 2 we will find, in addition to the highly expanded frequential analysis, a formulation in the temporal domain of the behavior laws of porous materials, particularly with the viscoelasticity models with fractional derivatives which are still relatively little-known concepts, at least sparingly covered by reference works.

- Nonlinear acoustics, which reveal for example wave propagation in granular media and more generally in heterogeneous nonlinear media or media with nonlinear interactions, are discussed in Chapter 17.

- In Part 3, under the rubric of experimental or numerical methods, we will find the foundations of the experimental approaches (impulse response of a sensor, problems of temporal reversal, an approach using inverse problems for the identification of characteristics of a propagation media).

Parts 6 to 8 echo the concepts and methods presented in the previous parts but on the level of applications and implementation:

- Any reader interested in non-destructive testing applications of linear propagation methods of the different types of waves described in Part 1 should look at Part 6. You will find there in particular a discussion on the application of Lamb waves for checking thin composite structures that are found throughout aeronautics (plates or shells).

– If we want to understand how to exploit the subtleties of nonlinear acoustics discussed in Chapter 17 within the framework of NDT we should move to Chapter 18, the defects often being accompanied by the apparition of nonlinearities (like frictional contact on the edges of a crack).

– If we would like to put the models of porous media from Part 2 into practice and measure, either in the frequential domain or the temporal domain, the different parameters existing in Biot's models, should go directly to Part 7.

– Finally, if we would like to have a complete view of the different imaging methods being developed in the biomedical domain, including ultrasonic tomography, we will be happy to discover it in Part 8.

The 50 or so authors contributing to this book offer us a panorama of the state of the art of the field of Materials and Acoustics in 2006 that is as complete as possible. The fact that this book was originally created in French shows in our eyes the driving force and creativity of the French School in this field. This is a reference work which will be of interest to both Master's and PhD students, as well as research engineers in the fields of materials, acoustic comfort, non-destructive testing and medical imaging – to mention only a few of the possible fields of application.

We must finally applaud the work of the coordinators of this book, Michel Bruneau and Catherine Potel. To obtain manuscripts from 50 authors, avoiding redundancies and interferences between the different contributions as far as possible, was not an easy task. They have managed it and for this they must be congratulated and thanked.

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## Preface

This collective work tries to fill a gap in scientific publishing in the fields grouped under the expression “Materials and Acoustics”. At least, that was the starting objective of the editorial committee who gave us the mission of coordinating everything that needed to be done.

From the preliminary outlines of the book, to the creation of the index, this project has enjoyed varying fortunes. However, the result is such that the basics of the fields covering the themes of “acoustics for materials” and “materials for acoustics”, from current results of basic research (Parts 1 to 5 for the essentials) to applications (especially Parts 6 to 8), are presented in this work.

The construction of this book has been governed by a certain logic, following which the essential, imperative elements generally precede presentations of applications. However, the contribution of each author remains understandable on its own.

Ignoring the fact that no-one can ever be completely happy, we would sincerely like to thank every author for the confidence they have put in us and for their support in this collective work.

*Michel BRUNEAU*  
*Catherine POTEL*

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