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DECENTRALIZED STOCHASTIC CONTROL OF LARGE-SCALE SYSTEMS VIA CONSTRAINED CONTROLLERS[1]

by

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Summary

This paper considers the decentralized control of large-scale stochastic systems. Specifically, each controller is allowed partial observations of the local subsystem state and complete observations of the interactions to the local subsystem; both observations taking place over noisy channels. Structural constraints are imposed on each controller and it is shown that the optimum parameters can be expressed in terms of the solution to a nonlinear two point boundary value problem.

I. Introduction

For a large-scale system the implementation of a controller derived by a straightforward application of existing stochastic control theory often requires a prohibitive amount of data handling and computational capability. The planning phase of implementation may require the solution of Riccati equations involving large matrices, while the on-line phase may involve the management of very large information flows. For example, the straightforward application of existing theory to the problem of controlling a string of high speed vehicles results in a control signal for each vehicle which depends on the state of each of the other vechicles in the string[1]. Thus, as the number of vehicles in the string increases the amount of on-line data handling increases significantly.

Much of the research to date in large scale system control has been oriented toward the resolution of problems that occur in the planning phase. For example the concept of ϵ-coupling[2,3] arose out of the desire to decouple the computations involved in solving the Riccati equations which result from the application of existing theory. Using the ϵ-coupling approach it is possible to approximate the solution to the Riccati equation by a sequence of solutions to decoupled equations of lower dimensionality. However, the problem of managing the on-line information flow remains. It is this aspect of large-scale systems control that is of interest in this paper.

The concept of decentralization[4] provides a technique for alleviating the on-line data handling requirements associated with the centralized controller. In the terminology introduced by Mesarovic[5], one form of decentralized controller structure can be visualized as consisting of a local controller or infimal unit M_i for each subsystem together with a supremal unit which coordinates the operations of the infimals. In this paper we are interested in the case where all of the coordination takes place in the planning phase. Chong and Athans[6] have considered a somewhat similar problem but have not related the results to the detailed structure of the system.

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In the next section the system model and problem formulation will be given. This is followed by a discussion of the controller structural constraints. It is then shown that the optimum parameters can be found in terms of the solution to a two-point boundary value problem.

II. System Model and Problem Formulation

Consider a system, S, composed of a collection $\{S_i : i=1,2,\ldots N\}$ of N interacting dynamical subsystems. We assume that the partitioning of S into the subsystems S_i is either given or that a "natural" partition exists. On the given time interval $[t_o,T]$ each subsystem is described by the following model

$$\dot{x}_i = A_i x_i + B_i m_i + L_{ii} u_i + w_i \tag{1-a}$$

$$y_i = H_i z_i + \eta_i \tag{1-b}$$

$$z_i' = [x_i' \; u_i']' \tag{1-c}$$

$$u_i = L_i x = \sum_{\substack{j=1 \\ j\neq i}}^{N} L_{ij} x_j \tag{1-d}$$

wherein for each $t\in[t_o,T]$ and each i

1. $x_i(t)$ is the state of S_i
2. $m_i(t)$ is the control input to S_i derived from the local control unit, M_i,
3. $u_i(t)$ is the interaction input to S_i derived from the other subsystems,
4. w_i is the local plant disturbance to S_i,
5. y_i is the output of S_i, and
6. η_i is the observation noise.

The initial state, $x_i(t_o)$ is assumed to be a gaussian random vector with mean $\bar{x}_i^o$ and covariance Σ_i^o and the noise processes w_i and η_i are assumed to be zero mean white gaussian processes with covariance W_i and N_i respectively. For i=1,2,..,N and $t\in[t_o,T]$,

$P_i \triangleq \{A_i, B_i, \{L_{ij} : i=1,2,\ldots,N\}\, H_i, \bar{x}_i^o, \Sigma_i^o, W_i, N_i\}$ is the set of parameters describing local subsystem S_i and

$Y_i(t) \triangleq \{(y_i(\tau),\tau) : \tau\in[t_o,T]\}$ is the on-line data available from S_i up to time t.

Letting $x' = [x_1'x_2'\ldots x_N']'$ and $m' = [m_1'm_2'\ldots m_N']'$ be, respectively, the composite state and control vectors, the stochastic control problem for the composite system (the overall problem) is to find a control law, M^*, such that $m^*(t) = M^*(Y(t))$ minimizes the cost functional

$$J = E\{x'(T)Fx(T) + \int_{t_o}^{T} x'(t)Qx(t) + m'(t)Rm(t)dt\} \tag{2}$$

where E denotes mathematical expectation. In (2) Y(t) denotes the information available to the controller up to time t, and F,Q,R are given matrices with F and Q positive semi-definite and R positive definite.

III. Controller Structure

In the absence of any constraints on the permissible information flow, the above problem has a well-known solution (see e.g.(7), p.414). That is, if $Y(t) = \bigcup_{i=1}^{N} (P_i U Y_i(t))$ then the optimal controller can be realized by

$$\begin{aligned} m^*(t) &= -P(t)\hat{x}(t) \\ \dot{\hat{x}}(t) &= A\hat{x}(t) + Bm^*(t) + K(t)[y(t)-H\hat{x}(t)] \\ \hat{x}(t_o) &= \bar{x}^o \end{aligned} \tag{3}$$

in which the optimal gain matrices, P and K, may be computed prior to t_o by solving the appropriate Riccati equations and A,B,H are formed from the corresponding subsystem matrices in an obvious manner.

In a large number of cases there are, of course, overriding technical and economical considerations which preclude the use of such a completely centralized controller. In order to accurately reflect the nature of control system design in a large scale system, one must incorporate constraints on the information flow structure. A basic and natural constraint that we place on the controller is that it should consist of a collection $\{M_i: i=1,2,\ldots,N\}$ of control units in which the information set available to each M_i is $I_i(t) = P_i U Y_i(t)$. That is, each control unit, M_i, is allowed the local on-line data from S_i as well as the parameters describing S_i+.

Motivated by the results obtained for the case of a linear system with gaussian disturbances and quadratic cost functional, we impose the further constraint that each M_i have the structure

$$\begin{aligned} \dot{\hat{x}}_i &= E_i\hat{x}_i + C_i m_i + G_i y_i \\ m_i &= D_i\hat{x}_i \end{aligned} \tag{4}$$

Here E_i is constrained to be an $n_i x n_i$ matrix and the signal $\hat{x}_i$ is regarded as an estimate of x_i. The rationale for choosing such a structure should be evident. In the absence of interactions and with local cost functionals defined appropriately the optimal unconstrained controller has the form given in (4).

Parameter Optimization

Under the above structural constraints the overall design problem becomes one of choosing $\{G_i, D_i, E_i, C_i, \hat{x}_i(t_o): i=1,2,..,N\}$ to minimize the cost given by (2) subject to the system constraints (1).

A natural requirement to impose on each controller is that for $i=1,2,\ldots,N$ $\hat{x}_i(t)$ be an unbiased estimate of $x_i(t)$ for every control and interaction input to S_i. Thus, we require that

$$E\{x_i(t)-\hat{x}_i(t) \,|\, Y_i(t)\} = 0 \quad \forall\, m_i, u_i \tag{4}$$

It is straightforward to show that a sufficient condition for obtaining unbiased estimates is to choose

$$\begin{aligned} E_i &= A_i - G_{i1}H_{i1} \\ C_i &= B_i \\ G_{i2} &= L_{ii} \end{aligned} \tag{5}$$

and $\hat{x}_i(t_o) = \bar{x}_i^o$, where $G_i = [G_{i1}\ G_{i2}]$.

Thus, subject to the constraint of unbiased estimates, the overall design problem reduces to the determination of $\{G_{i1}, D_i: i=1,2,\ldots,N\}$.

To determine necessary conditions for the optimal parameters, combine the system and controller dynamics to write

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} w \\ G\eta \end{bmatrix} \tag{6}$$

or

$$\dot{v} = \hat{A}\, v + \hat{w} \tag{6'}$$

where $G = \mathrm{diag}\{G_i: i=1,2,\ldots,N\}$

Defining $V(t) = E\{v(t)v'(t)\}$, and using the fact that the noise processes are white* one obtains**

$$\dot{V}(t) = \hat{A}V + V\hat{A}' + \hat{W} \tag{7}$$

where $\hat{W} = \left[\begin{array}{c|c} W & 0 \\ \hline 0 & G\bar{N}G' \end{array}\right]$, $\hat{A} = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right]$.

Using the fact that if Ψ is an nxn matrix and ν is an n-vector then $E\{\nu'\Psi\nu\} = \mathrm{trace}\{\Psi E(\nu\nu')\}$, one can write the cost functional (2) in the form

$$J = \mathrm{trace}\{\hat{F}V(T) + \int_{t_o}^{T} \hat{Q}(t)\, V(t)dt\} \tag{8}$$

where

$$\hat{F} = \left[\begin{array}{c|c} F & 0 \\ \hline 0 & 0 \end{array}\right] \quad \hat{Q}(t) = \left[\begin{array}{c|c} Q & 0 \\ \hline 0 & D'RD \end{array}\right]$$

and $D = \mathrm{diag}\{D_i: i=1,2,..,N\}$.

Equations (7) and (8) can be viewed as a reformulation of the original optimization problem as a deterministic parameter optimization in which it is desired to minimize (8) subject to (7). Utilizing the matrix minimum principle[10] the necessary conditions for optimality can be written as

$$\dot{V} = \hat{A}V + V\hat{A}' + \hat{W} \tag{9-a}$$

$$\dot{P} = -\hat{Q} - \hat{A}'P - P\hat{A} \qquad P(T) = \hat{F} \tag{9-b}$$

$$\frac{\partial \Psi}{\partial D_i} = 0 \quad i=1,2,\ldots,N \tag{9-c}$$

$$\frac{\partial \Psi}{\partial G_{i1}} = 0 \quad i=1,2,\ldots,N \tag{9-d}$$

where

$$\Psi = \mathrm{tr}\{\hat{Q}V + (\hat{A}V + V\hat{A}' + \hat{W})P'\} \tag{9-e}$$

+ As pointed out by Aoki[8] it may be useful to allow on-line communication between the local controllers and thus increase the information set available to each M_i. However, in this paper we do not allow the exchange of information between local control units.

* We also assume that the individual noise processes are mutually independent.

** $\bar{N} = \mathrm{diag}\{N_i: i=1,2,\ldots,N\}$.

Equations (9-a) - (9-d) represent a two point boundary value problem which must be solved in order to determine the optimal parameters. The relations (9-c), (9-d) and (9-e) can be used to write the optimal D_i, G_{i1} i=1,2,...,N in terms of the solution to this boundary value problem. The interested reader is referred to [9] where a more detailed presentation of these results is given.

IV. Conclusions

A decentralized controller in which coordination takes place only in the planning phase of implementation has been considered. The particular form of the controller was chosen with a view toward satisfying certain natural constraints on the information flow structure which reflect the large-scale nature of the system to be controlled. The optimal parameter values were then shown to be expressable in terms of the solution to a nonlinear two point boundary value problem.

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COMPARISON OF TWO OPTIMAL TERMINAL CONTROL ALGORITHMS

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Summary

There are a wide variety of optimal control algorithms available today. They vary in their generality and computational efficiency. This paper presents a comparison of a "general" technique and a "special purpose" technique. The results show the computational advantages of using a method which is specially designed for the type of optimization problem being solved.

Comparison of the Algorithms

The purpose of this paper is to present an example of the huge computational advantages which can be realized by using special purpose control algorithms instead of general purpose control algorithms. The two techniques which will be compared are an optimal terminal control algorithm developed by Nahi and Wheeler[1] (hereafter called the special purpose algorithm) and a control algorithm based on one of Wolfe's quadratic programming techniques[2] (hereafter called the general purpose algorithm). Both algorithms involve transforming the original terminal control problem stated in terms of differential or difference equations into a quadratic programming problem. If the initial formulation is in terms of differential equations, the first step is to approximate the differential equations with difference equations. This approximation can be made arbitrarily good by using a very small sampling interval. Since the size of the resulting quadratic programming problem is inversely proportional to the sampling interval, the choice of a sampling interval which yields both a set of difference equations which are a "reasonable" approximation of the original differential equations and a "reasonably" sized quadratic programming problem is important. The relationship between the number of sampling intervals and computation time will be illustrated for both algorithms in the example. The solution to the quadratic programming problem will give the optimum values of the discrete control sequence.

Multiple amplitude-bounded controls and time-varying system coefficients can be handled by both techniques. The differences between the methods will be summarized below.

The special purpose algorithm is based on the control theory result that in a problem with n state variables, an optimal control sequence can always be found with no more than n of the magnitudes of the control sequence at less than the corresponding maximum allowable values. This result is used to enable the technique to avoid manipulating any matrices of dimension greater than n x n.

The general purpose algorithm must solve a quadratic program which includes a basis matrix whose dimensions are a function of the product of the number of state and control variables and the number of sampling intervals. For example in a problem with three state variables, two control variables, and ten sampling intervals, the linear constraint matrix would have fifty rows and therefore a 50 x 50 basis matrix. The special purpose algorithm would only involve a 3 x 3 matrix since there are three state variables; however, it is very important to note that the general purpose algorithm is much more powerful than the special purpose algorithm. For example, it will handle state variable constraints and summation (approximation to integral) type cost functions.[3]

Example

To establish the relative performance of the two algorithms the following example problem was run using both techniques. The operating time was fixed and a series of runs were made using different sampling intervals to illustrate the relationship between sampling interval length and computation time.

Problem Statement

$$\min_{u} \; (4-x_1(T))^2 + x_2(T)^2 + x_3(T)^2$$

subject to the constraints

$$\dot{x} = Ax + Bu$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -7 & 1 \\ 0 & 0 & -5 \end{bmatrix}$$

$$x(0) = [0 \quad 0 \quad 0]'$$

$$T = 6 \text{ seconds}$$

This example problem was solved using sampling intervals of 2, 1.2, .75, .6, and .4 seconds with each of the techniques discussed above. The algorithms were implemented in FORTRAN IV on a CDC 6600 computer. The required central processor time versus the number of sampling intervals for each algorithm is shown in figure one. The special purpose algorithm is seen to be more efficient than the general purpose algorithm as the number of sampling intervals increases.

The optimum value of the cost function for a terminal control problem will be zero if the desired terminal state lies in the reachable set[1] and greater than zero if it lies outside the reachable set. With all other factors held constant including the system operation time, the size of the reachable set is a monotone increasing function of the number of sampling intervals. For this example the desired terminal state is an element of the reachable set for all of the sampling intervals which were used; therefore, the optimum cost function value for each case is zero. The special purpose algorithm requires a finite number of steps to develop the optimum control sequence which yields the optimum value of the cost function so that the optimum value of the cost function is always achieved. The general purpose algorithm is a gradient technique, therefore in general it does not achieve the optimum value of the cost function in a finite number of steps. In the example calculations the general purpose algorithm was executed until the cost

function had become equal to .0001.

Conclusions

The special purpose algorithm was shown to be considerably more effective than the general purpose algorithm. The fact that the special purpose algorithm did not manipulate progressively larger matrices as the number of sampling intervals increased led to dramatic computational savings.

The principal conclusion which should be drawn from this result is that special purpose algorithms which exploit the properties of the type of problem being solved can be much more efficient than general purpose algorithms. The user must make a trade-off between the effort involved in developing a special purpose algorithm and the added computational cost of using a general purpose algorithm. Clearly if a problem is to be solved only a few times a general purpose algorithm should be used. On the other hand if the problem is to be solved in real-time a special purpose algorithm will probably be required.

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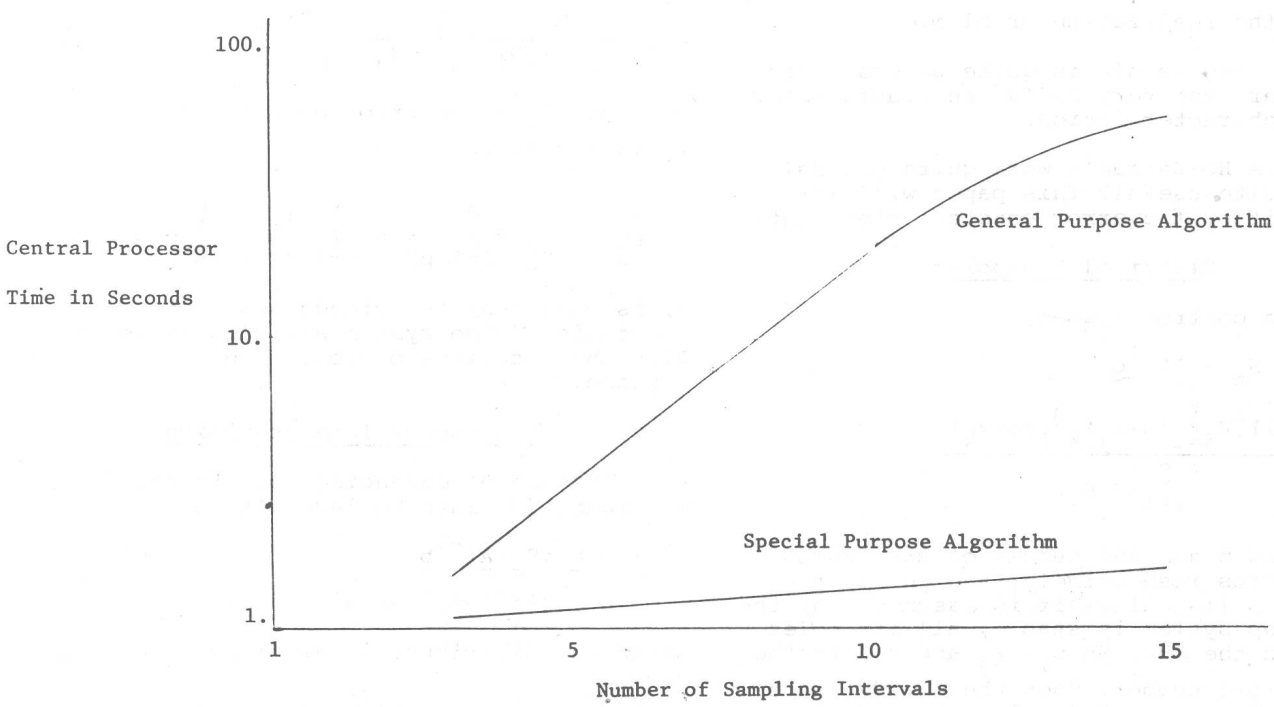

Figure 1

STATE SPACE INTERPRETATION OF ERROR CONSTANTS

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Introduction

For a given linear system

$$\dot{\underset{\sim}{x}} = \underset{\approx}{A}\underset{\sim}{x} + \underset{\sim}{b}u \tag{1}$$

$$y = \underset{\sim}{C}^T\underset{\sim}{x} \tag{2}$$

the input-output transfer function is

$$T(s) = \underset{\sim}{C}^T(S\underset{\approx}{I} - \underset{\approx}{A})^{-1}\underset{\sim}{b} \tag{3}$$

Ho and Kalman[1] performed the Laurent expansion of T(s)

$$T(s) = \underset{\sim}{C}^T(S\underset{\approx}{I}-\underset{\approx}{A})^{-1}\underset{\sim}{b} = \sum_{i=1}^{\infty}\frac{\underset{\approx}{J}_i}{s^i} \tag{4}$$

where

$$\underset{\approx}{J}_i = \underset{\sim}{C}^T\underset{\approx}{A}^{i-1}\underset{\sim}{b} \tag{5}$$

are known as Markov parameters. Then they organized Hankel's matrix $\underset{\approx}{H}$ be defined by

$$\underset{\approx}{H} = \begin{bmatrix} J_1 & J_2 & J_3 & \cdots & J_q \\ J_2 & J_3 & \cdots & & \\ J_3 & \cdots & & & \\ \vdots & & & & \\ J_p & & & & J_{p+q-1} \end{bmatrix} \tag{6}$$

to solve the realization problem.

While the result is quite general, it is, however, not very useful in practice for transfer-characterization.

Why is Ho-Kalman's work quite general but not quite useful? This paper will interpret it by the error constant viewpoint.

Classical Viewpoint

For a control system,

$$T(s) = \underset{\sim}{C}^T(S\underset{\approx}{I} - \underset{\approx}{A})^{-1}\underset{\sim}{b}$$

$$= \frac{(-1)^{\ell}K\prod_{j=1}^{\ell}(s-z_j)\prod_{k=1}^{h}(s+z_k)}{\prod_{i=1}^{n}(s+p_i)} \tag{7}$$

where ℓ and h are the number of RHP zeros and LHP zeros respectively, and n is the number of poles. Since it is assumed that the closed loop system is stable, all its poles must be in the LHP. So p_i, z_j and z_k are the positive real number. Then the steady state error constants are defined in terms of the successive coefficients in a Maclaurin expansion of T(s).

$$T(s) = \frac{K_p}{1+K_p} - \frac{1}{K_v}s - \frac{1}{K_a}s^2 - \cdots \tag{8}$$

and these error constants can be calculated as follows.

(a) position error constant K_p

$$K_p = \frac{(-1)^{\ell}K\prod_{i=1}^{\ell}(-z_j)\prod_{k=1}^{h}(z_k)}{\prod(p_i)-(-1)^{\ell}K\prod_{j=1}^{\ell}(-z_j)\prod_{k=1}^{h}(z_k)} \tag{9}$$

so, if

$$K = \frac{\prod_{i=1}^{n}(p_i)}{\prod_{j=1}^{\ell}(z_j)\prod_{k=1}^{h}(z_k)} \tag{10}$$

in (9), then $K_p = \infty$ and $T(0) = 1$, which implies that steady position error is zero.

(b) velocity error constant K_v

If $K = \infty$, i.e., $T(0) = 1$, then

$$\frac{1}{K_v} = \sum_{i=1}^{n}\frac{1}{p_i} + \sum_{j=1}^{\ell}\frac{1}{z_j} - \sum_{k=1}^{n}\frac{1}{z_k} \tag{11}$$

(c) acceleration error constant K_a

If $T(0) = 1$, then

$$-\frac{2}{K_a} = \frac{1}{K_v^2} + \sum_{i=1}^{n}\frac{1}{p_i^2} - \sum_{j=1}^{\ell}\frac{1}{z_j^2} - \sum_{k=1}^{h}\frac{1}{z_k^2} \tag{12}$$

It is seen that the steady state error constants of the system are closely related with the locations of poles and zeros in the s-plane.

State-space Interpretation

Instead of expanding into Laurent series, we expand (3) into Maclaurin's series:

$$T(s) = \underset{\sim}{C}^T(S\underset{\approx}{I}-\underset{\approx}{A})^{-1}\underset{\sim}{b}$$

$$= \underset{\sim}{C}^T(-\underset{\approx}{A}^{-1}-S\underset{\approx}{A}^{-2}-S^2\underset{\approx}{A}^{-3}- \ldots)\underset{\sim}{b}$$

Using the distribution law of matrices, we have

$$T(s)=\underset{\sim}{C}^T(-\underset{\approx}{A}^{-1})\underset{\sim}{b}+\underset{\sim}{C}^T(-S\underset{\approx}{A}^{-2})\underset{\sim}{b}+\underset{\sim}{C}^T(-S^2\underset{\approx}{A}^{-3})\underset{\sim}{b}+\ldots \tag{13}$$

Equating (13) and (8), we obtain the following equalities:

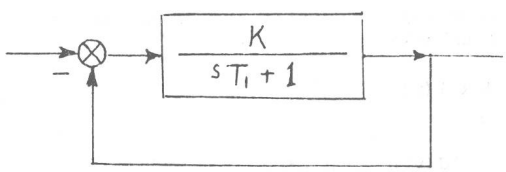
$$\frac{K_p}{1+K_p} = C^T(-A^{-1})b$$

$$-\frac{1}{K_v} = C^T(-A^{-2})b \qquad (14)$$

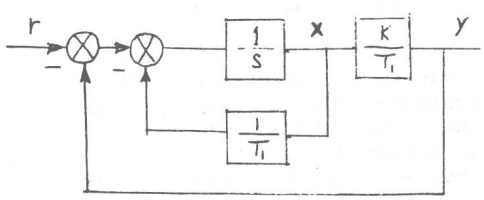
$$-\frac{1}{K_a} = C^T(-A^{-3})b$$

Equations shown in (14) are the state-space interpretation of error constants.

Consider the following illustrative example:



It can be rewritten into a state diagram as shown in Fig. 2.



The state equation description is

$$\dot{x} = -\left(\frac{1}{T_1} + \frac{K}{T_1}\right) x + r \qquad (15)$$

$$y = \frac{K}{T_1} x \qquad (16)$$

we have

$$\underset{\approx}{A} = -\frac{K+1}{T_1}, \quad \underset{\sim}{b} = 1, \quad \underset{\sim}{C}^T = \frac{K}{T_1} \qquad (17)$$

Substituting (17) into (14) yields

$$\frac{K_p}{1+K_p} = \underset{\sim}{C}^T(-\underset{\approx}{A}^{-1})\underset{\sim}{b}$$

$$= \frac{K}{T_1}\left(-\frac{T_1}{-(1+K)}\right)1$$

$$= \frac{K}{1+K}$$

Therefore $K_p = K \qquad (18)$

Similarly, we have

$$\frac{1}{K_v} = C^T(A^{-2})B$$

$$= \frac{SKT_1}{(1+K)^2}$$

and therefore

$$K_v = \frac{(1+K)^2}{ST_1} \qquad (19)$$

And the acceleration error constant is obtained as follows

$$\frac{1}{K_a} = \frac{-T_1^2 K}{(1+K)^3}$$

Therefore

$$K_a = -\frac{(1+K)^3}{T_1^2 K} \qquad (20)$$

Conclusion

A state space interpretation of error constants has been given. Either position, velocity or acceleration error constants in the classical sense can be evaluated from the state space parameters A, B, and C directly. The approach sheds new light on performance analysis and reveals more physical meaning in realization than Ho - Kalman's original approach.

Acknowledgment

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