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shape optimization and optimal design

edited by

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shape optimization and optimal design

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Additional Volumes in Preparation

Preface

This volume comprises selected papers from the sessions "Distributed Parameter Systems" and "Optimization Methods and Engineering Design" held within the 19th conference System Modeling and Optimization in Cambridge, England.

Those sessions were organized by the Working Groups 7.2 (Computational Techniques in Distributed Systems) and 7.4 (Discrete Optimization) of the Technical Committee 7 (Modeling and Optimization Techniques) of the International Federation for Information Processing (IFIP).

The aim of these sessions was to present the latest developments and major advances in the fields of passive and active control for systems governed by partial differential equations. Shape analysis and optimal shape design were particularly emphasized during the talks. The active control portion includes exact controllability and nonlinear boundary control/ stabilization.

We would like to acknowledge the contribution of M. J. D. Powell, who was the main organizer of the conference, and of M. Kocvara and K. Zowe, who were the organizers of the session "Optimization Methods and Engineering Design."

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Boundary Variations in the Navier-Stokes Equations and Lagrangian Functionals

Sébastien Boisgérault and Jean-Paul Zolésio

Abstract. We study the shape sensitivity of the stationary Navier-Stokes Equations in the general case of non-homogeneous and shape-dependent forces and boundary conditions. Under an assumption of non-singularity of the equations, the shape differentiability of the velocity and the pressure are obtained in some Sobolev spaces. The influence of the regularity of the geometrical and functional data on the best space for which the result holds is stressed. We apply these results on a class of shape functionals where a high regularity is required: the Lagrangian functionals. Their main characteristic is to take into account the paths of the fluid particles. The usual shape calculus is extended to take into account such features. We determine the shape derivative of a shape-dependent flow and develop the methods to achieve the explicit calculation of the shape gradient.

1 Introduction

1.1 The Navier-Stokes Equations

We consider the stationary incompressible Navier-Stokes Equations (NSE) in some smooth enough open and bounded sets $\Omega \subset \mathbb{R}^3$ of boundary Γ

$$\begin{aligned} -\nu\Delta u + (u \cdot \nabla)u + \nabla p &= f \quad \text{in } \Omega \\ \operatorname{div} u &= 0 \quad \text{in } \Omega \\ u &= g \quad \text{on } \Gamma \end{aligned} \tag{1.1}$$

The shape analysis will therefore include as special cases the situation where the flow is uniquely driven by the force field (with homogeneous Dirichlet boundary conditions, cf. [3]), as well as the one where the flow is induced by a body moving at a constant velocity (cf. [1]).

Moreover, for a greater generality, we assume that the data (f, g) may explicitly depend on the shape Ω. For a given set Ω, the corresponding value of f, denoted f_Ω, is *a priori* defined only on Ω (and g_Γ only on Γ). We assume that $f_\Omega \in H^{-1}(\Omega; \mathbb{R}^3)$, $g_\Gamma \in H^{1/2}(\Gamma; \mathbb{R}^3)$ and moreover that for any admissible set Ω, and for any connected component Λ of Γ, we have

$$\int_\Lambda \langle g_\Gamma, n \rangle_{\mathbb{R}^3} \, d\mathcal{H}^2 = 0 \tag{1.2}$$

(n_Ω, or simply n when no doubt is possible, is the unit outer normal to Ω).

We recall briefly the corresponding abstract setting. We denote by $V^1(\Omega)$ the space $\{u \in H^1(\Omega; \mathbb{R}^3),\ \operatorname{div} u = 0\}$, we set $V_0^1(\Omega) = V^1(\Omega) \cap H_0^1(\Omega; \mathbb{R}^3)$ and $V^{1/2}(\Gamma) = \{g \in H^{1/2}(\Gamma; \mathbb{R}^3),\ \int_\Gamma \langle g, n \rangle \, d\mathcal{H}^2 = 0\}$. The linear operator $\pi : H^{-1}(\Omega; \mathbb{R}^3) \to (V_0^1(\Omega))'$ is Leray's projector: $\pi(f)$ is the restriction of the linear form f on $H_0^1(\Omega; \mathbb{R}^3)$ to $V_0^1(\Omega)$. For any u and $v \in V^1(\Omega)$, we set $Au = -\pi(\Delta u)$ and $B(u, v) = \pi((u \cdot \nabla)v)$. The (nonlinear) Navier-Stokes operator is the mapping

$$F : \begin{array}{ccc} V^1(\Omega) & \to & V_0^1(\Omega)' \times V^{1/2}(\Gamma) \\ u & \mapsto & (\nu A u + B(u, u), u|_\Gamma) \end{array} \tag{1.3}$$

1.2 Shape Analysis Framework

The shape sensitivity of this equation is studied in the framework of the *Speed Method.* The first step is to generate some of the geometries around the reference set Ω while staying in a given design region. In that purpose, we associate to a smooth, open and bounded set D (designed later on as the *hold-all*), a velocity space $\mathcal{V}$, chosen among the $\mathcal{V}_k$ for a $k \geq 1$

$$\mathcal{V}_k = \{V \in C^0([-T; T]; C^k(\overline{D}; \mathbb{R}^3)),\ \langle V, n_D \rangle_{\mathbb{R}^3} = 0 \text{ on } \partial D\} \tag{1.4}$$

Then for any $V \in \mathcal{V}$, a one-parameter family of deformations $T_s : \overline{D} \to \overline{D}$ is given by the following initial-value problem

$$\begin{array}{l} \partial_s T_s = V(s) \circ T_s \\ T_0 = I \end{array} \tag{1.5}$$

We denote $\Omega_s := T_s(\Omega)$ the corresponding transported sets generated by Ω and V. We also set $\Gamma_s := T_s(\Gamma)$.

The regularity of shape-dependent mappings such as f and g are defined in the following way. Let $W(\Omega)$ be either one of the Sobolev spaces $W^{m,p}(\Omega; \mathbb{R}^m)$ ($m \geq 0$, $p \geq 1$) or one of the spaces $C^k(\overline{\Omega}; \mathbb{R}^m)$ ($k \geq 0$).