SECOND EDITION

ELASTICITY IN ENGINEERING MECHANICS

ARTHUR P. BORESI KEN P. CHONG

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Second Edition

ARTHUR P. BORESI
Professor Emeritus
Department of Civil and Architectural Engineering
University of Wyoming
Laramie, Wyoming

KEN P. CHONG
Professional Engineer
Arlington, Virginia

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ELASTICITY IN ENGINEERING MECHANICS

PREFACE

The material presented is intended to serve as a basis for a critical study of the fundamentals of elasticity and several branches of solid mechanics, including advanced mechanics of materials, theories of plates and shells, composite materials, plasticity theory, finite element, and other numerical methods. Chapter 1 includes, for ready reference, certain mathematic preliminaries. Depending on the background of the reader, this material may be used either as required reading or as reference material. The main content of the book begins with the theory of deformation in Chapter 2. The theory of stress is presented in Chapter 3. The theories of deformation and stress are treated separately to emphasize their independence of one another and also to emphasize their mathematical similarity. By so doing, one can clearly see that these theories depend only on approximations related to modeling of a continuous medium, and that they are independent of material behavior. The theories of deformation and stress are united in Chapter 4 by the introduction of three-dimensional stress–strain–temperature relations (constitutive relations). The major portion of Chapter 4 is devoted to linearly elastic materials. However, a brief discussion of nonlinear constitutive relations is presented in Appendix 4B. Chapters 5 and 6 treat the plane theory of elasticity, in rectangular and polar coordinates, respectively. Chapter 7 presents the three-dimensional problem of prismatic bars subjected to end loads. Material on thermal stresses is incorporated in a logical manner in the topics of Chapters 4, 5, and 6.

General solutions of elasticity are presented in Chapter 8. Extensive use is made of appendixes for more advanced topics such as complex variables (Appendix 5B) and stress–couple theory (Appendixes 5A and 6A). In addition, in each chapter,

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examples and problems are given, along with explanatory notes, references, and a bibliography for further study.

As presented, the book is valuable as a text for students and as a reference for practicing engineers/scientists. The material presented here may be used for several different types of courses. For example, a semester course for senior engineering students may include topics from Chapter 2 (Sections 2-1 through 2-16), Chapter 3 (Sections 3-1 through 3-8), Chapter 4 (Sections 4-1 through 4-7 and Sections 4-9 through 4-12), Chapter 5 (Sections 5-1 through 5-7), as much as possible from Chapter 6 (from Sections 6-1 through Section 6-6), and considerable problem solving. A quarter course for seniors could cover similar material from Chapters 2 through 5, with less emphasis on the examples and problem solving. A course for first-year graduate students in civil and mechanical engineering and related engineering fields can include Chapters 1 through 6, with selected materials from the appendixes and/or Chapters 7 and 8. A follow-up graduate course can include most of the Appendix material in Chapters 2–6, and the topics in chapters 7 and 8, with specialized topics of interest for further study by individual students.

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