Solving Problems in

FLUID MECHANICS

Volume 1

J F Douglas

Solving Problems in Fluid Mechanics

Volume 1

J. F. Douglas MSc PhD DIC ACGI CEng MICE MIMechE MIStructE

Longman Scientific & Technical

Copublished in the United States with
John Wiley & Sons, Inc., New York

Longman Scientific & Technical
Longman Group UK Limited
Longman House, Burnt Mill, Harlow
Essex CM20 2JE, England
and Associated Companies throughout the world

Copublished in the United States with
John Wiley & Sons, Inc., 605 Third Avenue, New York, NY 10158.

First published by Pitman Publishing Limited in 1970 under the title
Solution of Problems in Fluid Mechanics Part 1
All metric edition first published 1975
Ninth impression 1984
This edition published by Longman Scientific & Technical in 1986 under the title
Solving Problems in Fluid Mechanics Volume 1
Second impression 1987

British Library Cataloguing in Publication Data
Douglas, J.F.
[Solution of problems in fluid mechanics]
Solving problems in fluid mechanics. – All-metric
ed. – (Solving problems)
Vol. 1
1. Fluid mechanics
I. [Solution of problems in fluid mechanics]
II. Title
620.1'06 TA357

ISBN 0-582-28641-7

Produced by Longman Singapore Publishers (Pte) Ltd.
Printed in Singapore.

Solving Problems in Fluid Mechanics

Volume 1

Other titles in the Series

Solving Problems in Vibrations, *J.S. Anderson and M. Bratos-Anderson*

Solving Problems in Structures Volume 1, *P.C.L. Croxton and L.H. Martin*

Solving Problems in Fluid Mechanics Volume 2, *J.F. Douglas*

Solving Problems in Soil Mechanics, *B.H.C. Sutton*

Solving Problems in Solid Mechanics Volume 1, *S.A. Urry and P.J. Turner*

Solving Problems in Solid Mechanics Volume 2, *S.A. Urry and P.J. Turner*

Preface

Some students find it difficult to learn from an ordinary textbook because they tend to read it as one might read a novel and fail either to appreciate what is being set out in each section or to study each mathematical step in detail. The result is that the student ends his reading with a glorious feeling of knowing it all and with, in fact, no understanding of the subject whatsoever. To avoid this undesirable end I have adopted for this book an older but traditional presentation of the subject in the form of a catechism with question and answer.

Even if the reader does achieve an understanding of the principles of the subject, there still remains that familiar gap between knowing and actually doing that everyone knows so well. It is one thing to know how a job should be done but it needs experience to do it with confidence. I have therefore provided for each chapter exercises with answers giving an opportunity for plenty of practice before the readers go off to solve their own problems.

This is definitely a textbook, not a book of worked examples, and in its pages the reader will find all the definitions and theory needed presented in question and answer form with selected problems fully worked out and suitable exercise questions to test the understanding of each section.

The material included in this volume covers the elementary work in Fluid Mechanics for engineering students in Universities, Polytechnics and Colleges of Higher Education. More advanced work is covered in the subsequent volume.

In writing for a wide readership it is impossible to satisfy everyone. For instance in the simple matter of density, some swear that mass density is the only true density and that all else is an abomination, others find specific weight a useful concept, yet others say that specific gravity should be abolished. Again while SI units are preferred by many, other systems are still in use and perhaps merit a reference. While I may prefer mass density and am committed to SI units, I have tried to meet the requirements of others where possible.

It has been suggested that some attention should be given to the use of computers in the solving of problems in fluid mechanics. For those who want them, examples will be found in *Fluid Mechanics* by Douglas, Gasiorek and Swaffield (Pitman, 2nd Edn. 1985). I have preferred to concentrate here on the principles of fluid mechanics which do not change and on the derivation of the corresponding algebraic equations. How these equations are solved will depend on the facilities available – computer, calculator, abacus, slide rule, mathematical tables and pencil and paper. In engineering, success depends on the reliability of the results achieved, not on the method of achieving them.

I would like to express my appreciation of the assistance which I have received from my former colleagues in the teaching profession. I am particularly indebted to Dr. R. D. Matthews for his advice on the preparation of this new text and for the provision of examples and exercises with particular reference to Chapter 14.

When author, printer and publisher have all done their best, some errors may still remain. For these I apologise and I will be glad to receive any correction or constructive criticism.

John Douglas — August 1985

Introduction

Fluid Mechanics is the section of Applied Mechanics concerned with the statics and dynamics of liquids and gases. The same ideas of momentum and energy, etc. used in ordinary mechanics can be applied, but frequently fluid mechanics is concerned with streams of fluid instead of individual bodies or particles.

Hydraulics (from the Greek word for water) is the study of the problems of the flow and storage of water but is often applied to other liquids, as for example in the case of "hydraulic" control gear usually using oil as the operating fluid.

A fluid can offer no permanent resistance to any force causing change of shape. Fluids flow under their own weight and take the shape of any solid body with which they are in contact.

Change of shape is caused by shearing forces; therefore if shearing forces are acting in a fluid it will flow. Conversely, if a fluid is at rest there can be no shearing forces in it, and all forces are perpendicular (normal) to the planes on which they act.

Fluids are divided into liquids and gases. A *liquid* is difficult to compress; a given mass occupies a fixed volume irrespective of the size of the container holding it and a "free surface" is formed as a boundary between the liquid and the air above it. A *gas* is easily compressed; it expands to fill any vessel in which it is contained and it does not form a free surface.

Distinctions between a solid and a fluid are: (1), up to the limit of elasticity the deformation of a solid is such that the strain is proportional to the applied stress; for a fluid the rate of strain is proportional to the stress; (2), the strain of a solid is independent of the time of application of the force and if the elastic limit is not exceeded the deformation disappears when the stress is removed, but a fluid continues to flow as long as the stress is applied and does not recover its original form when the stress is removed.

Units and dimensions

SI Units

The United Kingdom has now adopted the system of metric units known as the Système International d'Unités, abbreviated to SI. In due course these will replace the old British units such as the pound, poundal and foot as the only legal system of measurement.

The Système International (SI) has six basic units which are arbitrarily defined. These are:

length: metre (m)
mass: kilogramme (kg)
time: second (s or sec)
electric current: ampere (A)
absolute temperature: kelvin (K)
luminous intensity: candela (cd)

All other units are derived from these fundamental units, since the SI is a coherent system in which the product or quotient of any two unit quantities within the system is the unit of the resultant quantity. For example, the unit of velocity is obtained by dividing the unit of distance, the metre, by the unit of time, the second, and will therefore be metres per second. The relation between mass and force is established by making the constant of proportionality in Newton's second law equal to unity so that

$$\text{Force} = \text{mass} \times \text{acceleration}$$

The unit of force will be the product of the unit of mass (kilogramme) and the unit of acceleration (metre/sec^2) which is the kilogramme-metre/sec^2 and is known as the newton.
Larger or smaller units are formed by adding a prefix to the basic unit. For example one thousandth part of a metre is a *millimetre* while one thousand metres is a *kilometre*. The following prefixes are currently in use:

Multiples			*Sub-multiples*		
deca	da =	$\times 10$	*deci*	d =	$\times 10^{-1}$
hecto	h =	$\times 10^{2}$	*centi*	c =	$\times 10^{-2}$
kilo	k =	$\times 10^{3}$	milli	m =	$\times 10^{-3}$
mega	M =	$\times 10^{6}$	micro	μ =	$\times 10^{-6}$
giga	G =	$\times 10^{9}$	nano	n =	$\times 10^{-9}$
tera	T =	$\times 10^{12}$	pico	p =	$\times 10^{-12}$
			femto	f =	$\times 10^{-15}$
			atto	a =	$\times 10^{-18}$

The prefixes shown in italics are not preferred but are in common use. Another unit in common use is the metric tonne = 10^3 kg = 2205 lb.

Other systems of units

While SI units are the preferred system, there are other systems which remain of importance in various parts of the world and in particular fields of activity. The foot-pound-second system (fps), the centimetre-gramme-second system (cgs) and the metre-kilogramme-second system (MKS) have been used extensively and no doubt will continue to be used. They are coherent systems based on a constant of unity in Newton's second law and appear in two forms: in the absolute systems the unit of mass is a fundamental unit and the unit of force is derived, whereas in the technical system the unit of force is a fundamental unit and the unit of mass is derived using Newton's second law (Table I).

The MKS absolute units, so far as mechanics is concerned, correspond with SI units and it seems possible that MKS technical units may continue in use for sometime alongside SI units.

In solving problems it is essential to keep to one system of units only. If the data are in different systems they should be converted immediately to the system selected.

Table I

Quantity	fps		cgs		MKS
	Absolute	Technical	Absolute	Technical	Technical
Length	ft	ft	cm	cm	m
Time	sec	sec	sec	sec	sec
Mass	lb-mass	slug	g-mass	981 g	9·81 kg
Force or weight	poundal	lb-force	dyne	g-force	kg-force

1 slug = 32·2 lb-mass, 1 g-force = 981 dynes, 1 lb-force = 32·2 poundals.

Dimensions

The units chosen for measurement do not affect the quantity measured. One kilogramme of water means exactly the same as 2·2046 lb of water. It is sometimes convenient not to use any particular system but to think in terms of mass, length, time, force, temperature, etc.

In mechanics all quantities can be expressed in terms of the fundamental dimension of mass M, length L and time T.

Thus $$\text{acceleration} = \frac{\text{distance}}{(\text{time})^2}$$

so that

$$\text{dimensions of acceleration} = \frac{\text{dimension of distance}}{(\text{dimension of time})^2} = \frac{L}{T^2}$$

Similarly $$\text{Force} = \text{mass} \times \text{acceleration}$$

so that

$$\text{Dimension of Force} = \text{dimension of mass} \times \text{dimension of acceleration}$$

$$= \frac{ML}{T^2}$$

The dimensions and SI units of common quantities are shown in Table II.

Dimensional equations

If an equation is to represent something which is physically real the terms on both sides must be of the same sort (for example, all forces) as well as the two sides being numerically equal, otherwise the equation is meaningless. Every term must have the same dimensions so that like is compared with like.

Table II Dimensions and units of common quantities

Quantity	Defining equation	Dimensions	Unit	Symbol
Geometrical				
Angle	Arc/radius (a ratio)	$[M^0L^0T^0]$	radian	rad
Length	(Including all linear measurement)	$[L]$	metre	m
Area	Length × Length	$[L^2]$	square metre	m^2
Volume	Area × Length	$[L^3]$	cubic metre	m^3
First moment of area	Area × Length	$[L^3]$	metre cubed	m^3
Second moment of area	Area × Length²	$[L^4]$	metre to fourth power	m^4
Strain	Extension/Length	$[L^0]$	a ratio	
Kinematic				
Time		$[T]$	second	s
Velocity, linear	Distance/Time	$[LT^{-1}]$	metre per second	ms^{-1}
Acceleration, linear	Linear velocity/Time	$[LT^{-2}]$	metre per second squared	ms^{-2}
Velocity, angular	Angle/Time	$[T^{-1}]$	radians per second	rad s^{-1}
Acceleration, angular	Angular velocity/Time	$[T^{-2}]$	radians per second squared	rad s^{-2}
Volume rate of discharge	Volume/Time	$[L^3T^{-1}]$	cubic metres per second	m^3s^{-1}
Dynamic				
Mass	Force/Acceleration	$[M]$	kilogramme	kg
Force	Mass × Acceleration	$[MLT^{-2}]$	newton = kilogramme-metre per second²	N = $kgms^{-2}$
Weight	Force	$[MLT^{-2}]$	newton	N
Mass density	Mass/Volume	$[ML^{-3}]$	kilogrammes per cubic metre	kg m^{-3}
Specific weight	Weight/Volume	$[ML^{-2}T^{-2}]$	newtons per cubic metre	N m^{-3}
Specific gravity	Density/Density of water	$[M^0L^0T^0]$	a ratio	–
Pressure (intensity)	Force/Area	$[ML^{-1}T^{-2}]$	newton per square metre = pascal	Nm^{-2} = Pa
Stress	Force/Area	$[ML^{-1}T^{-2}]$	newton per square metre	Nm^{-2}
Elastic modulus	Stress/Strain	$[ML^{-1}T^{-2}]$	newton per square metre	Nm^{-2}
Impulse	Force × Time	$[MLT^{-1}]$	newton seconds	N s
Mass moment of inertia	Mass × Length²	$[ML^2]$	kilogramme-metre squared	kg m^2
Momentum, linear	Mass × Linear velocity	$[MLT^{-1}]$	kilogramme-metre per second	kg m s^{-1}
Momentum, angular	Moment of inertia × Angular velocity	$[ML^2T^{-1}]$	kilogramme-metre squared per second	kg m^2s^{-1}
Work, energy	Force × Distance	$[ML^2T^{-2}]$	newton-metre = joule	Nm = J
Power	Work/Time	$[ML^2T^{-3}]$	joule per second = watt	JS^{-1} = W
Moment of a force	Force × Distance	$[ML^2T^{-2}]$	newton-metre	Nm
Viscosity, dynamic	Shear stress/Velocity gradient	$[ML^{-1}T^{-1}]$	kilogrammes per metre-second (= 10 poise)	kg $m^{-1}s^{-1}$
Viscosity, kinematic	Dynamic viscosity/Mass density	$[L^2T^{-1}]$	metre squared per second	m^2s^{-1}
Surface tension	Energy/Area	$[MT^{-2}]$	newton per metre = kilogrammes per second squared	Nm^{-1} = kgs^{-2}

Example. The equation $v^2 = u^2 + 2as$ gives the final velocity v of a body which started with an initial velocity u and received an acceleration a for a distance s. When dimensions are substituted for the quantities each term must have the same dimensions if the equation is true.

The dimensions of the quantities are $v = LT^{-1}$, $u = LT^{-1}$, $a = LT^{-2}$, $s = L$

Dimensions of v^2 are $(LT^{-1})^2 = L^2T^{-2}$

Dimensions of u^2 are $(LT^{-1})^2 = L^2T^{-2}$

Dimensions of $2as$ are $(LT^{-2}) \times L = L^2T^{-2}$

All three terms have the same dimensions and the equation is dimensionally correct and could represent a real event.

A check on dimensions will not show whether any pure numbers in the equation are correct since pure numbers are ratios and have the dimension of unity.

Note that some practical formulae used by engineers do not appear to be dimensionally correct. For example, a formula for the volume in m^3/s per second Q flowing over a rectangular weir of width B metres when the depth over the sill is H metres is $Q = 1{\cdot}79BH^{3/2}$. The dimensions of the left-hand side (m^3/s) are L^3T^{-1}. The dimensions of the right-hand side are apparently $L^{5/2}$. The reason for the difference is that the coefficient 1·79 is not a pure number but is the numerical value in SI units of $0{\cdot}57\sqrt{g}$ which has dimensions $L^{1/2}T^{-1}$, giving dimensional agreement between the two sides.

Use of dimensions for finding conversion factors

This is shown in the following example.

Example. The coefficient of dynamic viscosity μ of water at 95°F is $1{\cdot}505 \times 10^{-5}$ ft slug sec units; what is the value in (*a*) poises, (*b*) SI units.

From Table II the dimensions of μ are M/LT.

$$(a)\ \therefore \quad \frac{\mu_{(\text{poises})}}{\mu_{(\text{ft slug sec})}} = \frac{(M/LT)\text{ in cgs absolute units}}{(M/LT)\text{ in ft slug sec units}}$$

$$= \frac{\text{Mass (cgs abs)}}{\text{Mass (ft slug sec)}} \times \frac{\text{length (ft slug sec)}}{\text{length (cgs)}} \times \frac{\text{time (ft slug sec)}}{\text{time (cgs)}}$$

$$1\text{ slug} = 32{\cdot}2\text{lb mass} = 32{\cdot}2 \times 453{\cdot}6\text{g mass}$$

$$1\text{ft} = 30{\cdot}48\text{cm}$$

The time unit is 1 sec in both systems.

$$\therefore \quad \mu_{(\text{poises})} = \mu_{(\text{ft slug sec})} \frac{32{\cdot}2 \times 453{\cdot}6}{1} \times \frac{1}{30{\cdot}48} \times \frac{1}{1}$$

$$= \frac{1{\cdot}505 \times 10^{-5} \times 32{\cdot}2 \times 453{\cdot}6}{30{\cdot}48} = 7{\cdot}2 \times 10^{-3}\text{ poises}$$

$$(b) \quad \frac{\mu_{(\text{SI})}}{\mu_{(\text{ft slug sec})}} = \frac{(M/LT)\text{ in ft slug sec units}}{(M/LT)\text{ in SI units}}$$

$$= \frac{\text{Mass (SI units)}}{\text{Mass (ft slug sec)}} \times \frac{\text{length (ft slug sec)}}{\text{length (SI units)}} \times \frac{\text{time (ft slug sec)}}{\text{time (SI units)}}$$

$$1\text{ slug} = 32{\cdot}2\text{lb-mass} = 32{\cdot}2 \times 0{\cdot}4536\text{kg-mass}$$

$$1\text{ft} = 0{\cdot}3048\text{m}$$

The time unit is 1 sec in both systems.

$$\mu_{(\text{SI units})} = \mu_{(\text{ft slug sec})} \frac{32{\cdot}2 \times 0{\cdot}4536}{1} \times \frac{1}{0{\cdot}3048} \times \frac{1}{1}$$

$$= \frac{1{\cdot}505 \times 10^{-5} \times 32{\cdot}2 \times 0{\cdot}4536}{0{\cdot}3048} = 7{\cdot}2 \times 10^{-4}\text{kg/m-s}$$

Properties of fluids

Density

There are three forms of density which must be carefully distinguished.

1. *Mass density* ρ (Gk., rho) is the mass per unit volume. SI unit, kg/m^3 (fps absolute unit, lb-mass/ft^3; technical unit, slug-mass/ft^3).

2. *Specific weight* w is the weight per unit volume. SI unit, N/m^3 (fps absolute unit, poundal/ft^3; technical unit, lb-wt/ft^3).

Since weight = mass × gravitational acceleration

$$w = \rho g$$

3. *Specific gravity*, or relative density s, is the ratio of the weight of a substance to the weight of an equal volume of water at 4°C,

$$s = \frac{w \text{ for substance}}{w \text{ for water}} = \frac{\rho \text{ for substance}}{\rho \text{ for water}}$$

Viscosity

A fluid at rest cannot resist shearing forces but once it is in motion shearing forces are set up between layers of fluid moving at different velocities. The viscosity of the fluid determines its ability to resist these shearing stresses (see Chap. 13).

The *Coefficient of Dynamic Viscosity* μ (Gk., mu) is defined as the shear force per unit area required to drag one layer of fluid with unit velocity past another layer unit distance away from it in the fluid. SI unit, N-s/m^2 or kg/m-s.

(fps absolute unit is lb-mass/ft-sec. Technical unit, slug/ft-sec.)

In the absolute cgs system of units the unit of viscosity is the poise which is divided into 100 centipoises. (1 poise = 1 g/cm-s.)

Kinematic Viscosity ν (Gk., nu) is the ratio of dynamic viscosity to mass density

$$\nu = \frac{\mu}{\rho}$$

Note that, if μ is in kg/m-s, ρ must be in kg/m^3, thus the units of ν are independent of mass. The SI unit is m^2/s. (fps unit, ft^2/s.) In the cgs system the unit is the stoke which is divided into 100 centistokes.

Variation of Viscosity with Temperature. The viscosity μ of liquids decreases with increase of temperature, but the viscosity of gases increases with increase of temperature.

Poiseuille showed that

$$\mu = \mu_0 \left(\frac{1}{1 + at + bt^2} \right)$$

where μ = coefficient of viscosity at t°C, μ_0 = coefficient of viscosity at 0°C, a and b are constants.

For water μ_0 = 0.0179 poise = 0.00179 kg/m-s, a = 0.033368 and b = 0.000221.

Surface tension
σ (Gk., sigma)

Within the body of a liquid a molecule is attracted equally in all directions by the other molecules surrounding it, but at the surface between liquid and air the upward and downward attractions are unbalanced. The liquid surface behaves as if it were an elastic membrane under tension. This surface tension is the same at every point on the surface and acts in the plane of the surface normal to any line in the surface. Surface tension is not affected by the curvature of the surface, and it is constant at a given temperature for the surface of separation of two particular substances. Increase of temperature causes a decrease of surface tension.

Surface tension causes drops of liquid to tend to take a spherical shape and is also responsible for capillary action which causes a liquid

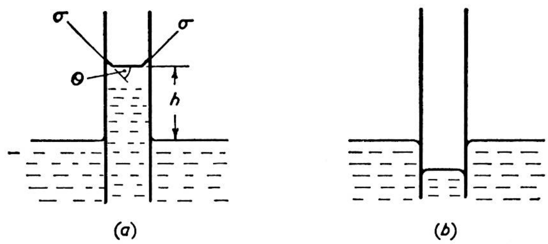

Figure 1

to rise in a fine tube when its lower end is inverted in a liquid which wets the tube (Fig. 1*a*). If the liquid does not wet the tube it will be depressed in the fine tube below the surface outside.

If θ is the angle of contact between liquid and solid, upward pull due to surface tension $= \sigma\pi d \cos\theta$ where $d =$ diameter of tube.

Putting $h =$ height liquid is raised and $w =$ sp. wt of liquid

$$\text{weight of liquid raised} = w\frac{\pi}{4}d^2h$$

so that

$$\sigma\pi d\cos\theta = w\frac{\pi}{4}d^2h$$

$$h = \frac{4\sigma\cos\theta}{wd}$$

Capillary action is a source of error in reading gauge glasses. For water in a tube 6 mm in diameter h will be 4·5 mm, while for mercury the corresponding figure is −1·5 mm.

Compressibility

For liquids the relationship between change of pressure and change of volume is given by the bulk modulus K.

$$\text{Bulk modulus} = \frac{\text{change in pressure intensity}}{\text{volumetric strain}}$$

$$= \frac{\text{change in pressure intensity}}{\text{(change in volume/original volume)}}$$

The relation between pressure and volume for a gas can be found from the gas laws.

For all perfect gases $pv = RT$, where p = absolute pressure, v = specific volume = $1/w$ = $1/\rho g$, T = absolute temperature, R = gas constant.

If changes occur isothermally (at constant temperature) pv = constant.

If changes occur adiabatically (without gain or loss of heat), pv^{γ} = constant, where γ = ratio of specific heat at constant pressure to specific heat at constant volume.

Contents

1

Static pressure and head

A force or pressure is exerted by a fluid on the surfaces with which it is in contact, or by one part of a fluid on the adjoining part. The *intensity of pressure* at any point is the force exerted on unit area at that point and is measured in newtons per square metre (*pascals*) in SI units (pounds per square foot in fps technical units). An alternative metric unit is the bar, which is 10^5 N/m^2. In practice intensity of pressure is abbreviated to *pressure*.

1.1 Pressure intensity

A mass m of 50 kg acts on a piston of area A of 100 cm^2. What is the intensity of pressure on the water in contact with the underside of the piston if the piston is in equilibrium.

Solution.

$$\text{Force acting on piston} = mg$$
$$= 50 \times 9{\cdot}81 = 490{\cdot}5\,\text{N}$$
$$\text{Area of piston } A = 100\,\text{cm}^2 = \frac{1}{100}\,\text{m}^2$$
$$\text{Intensity of pressure} = \frac{\text{Force}}{\text{Area}} = \frac{490{\cdot}5}{0{\cdot}01}\ \text{N/m}^2$$
$$= \mathbf{4{\cdot}905 \times 10^4\,N/m^2}$$

1.2 Pressure and depth

Find the intensity of pressure p at a depth h below the surface of a liquid of specific weight $w = \rho g$ if the pressure at the free surface is zero.

A diver is working at a depth of 18 m below the surface of the sea. How much greater is the pressure intensity at this depth than at the surface? Specific weight of sea water is 10 000 N/m^3.

Solution. The column of liquid (Fig. 1.1) of cross-sectional area A extending vertically from the free surface to the depth h is in equilibrium in the surrounding liquid under the action of its weight acting downwards, the pressure force on the bottom of the column acting upwards,