ANALYSIS OF

MULTICONDUCTOR TRANSMISSION LINES

SECOND EDITION

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CLAYTON R. PAUL

ANALYSIS OF MULTICONDUCTOR TRANSMISSION LINES

Second Edition

CLAYTON R. PAUL

Professor of Electrical and Computer Engineering
Mercer University
Macon, GA 31207

and

Emeritus Professor of Electrical Engineering
University of Kentucky
Lexington, KY 40506

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Analysis of Multiconductor Transmission Lines

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We can judge the heart of man by his treatment of animals.

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PREFACE

This is the second edition of a textbook that is intended for a senior or graduate-level course in an electrical engineering (EE) curriculum on the subject of the analysis of multiconductor transmission lines (MTLs). It will also serve as a useful reference for industry professionals. The term MTL typically refers to a set of $n + 1$ parallel conductors that serve to transmit electrical signals between two or more points, for example, a source and a load. The dominant mode of propagation on an MTL is the transverse electromagnetic (TEM) mode of propagation, where the electric and magnetic fields surrounding the conductors lie solely in the transverse plane orthogonal to the line axis. This structure is capable of guiding waves whose frequencies range from dc to where the line cross-sectional dimensions become a significant fraction of a wavelength. At higher frequencies, higher order modes coexist with the TEM mode, and other guiding structures such as waveguides and antennas are more practical structures for transmitting the signal between a source and a load. There are many applications for this wave-guiding structure. High-voltage power transmission lines are intended to transmit 60-Hz sinusoidal waveforms and the resulting power. In addition to this low-frequency power signal, there may exist other higher frequency components of the transmitted signal such as when a fault occurs on the line or a circuit breaker opens and recloses. The waveforms on the line associated with these events have high-frequency spectral content. Cables in modern electronic systems such as aircraft, ships, and vehicles serve to transmit power as well as signals throughout the system. These cables consist of a large number of individual wires that are packed into bundles for neatness and space conservation. The electromagnetic fields surrounding these closely spaced individual wires interact with each other and induce signals in all the other adjacent circuits. This is unintended and is referred to as *crosstalk*. This crosstalk can cause functional degradation of the circuits at the ends of the cable. The prediction of crosstalk will be one of the major objectives in this text.

There are numerous other similar transmission-line structures. A printed circuit board (PCB) consists of a planar dielectric board on which rectangular cross-section conductors (lands) serve to interconnect digital devices as well as analog devices. Crosstalk can be a significant functional problem with these PCBs as can the degradation of the intended signal transmission through attenuation, time delay, and other effects. Signal degradation, time delay, and crosstalk can create significant functional problems in today's high-speed digital circuits so that it is important to understand and predict this effect. Digital systems constitute the primary type of electronic system today and will no doubt dominate in the future. Clock speeds and data transfer rates are accelerating at an astounding rate. Personal computers today have clock speeds on the order of 3GHz. The spectral content of these high-speed digital waveforms extends well into the lower GHz frequency range. This means that crosstalk as well as unintended radiated emissions are presenting performance problems in digital systems that were unheard of a decade ago. This will no doubt increase in the future. As recent as 15 years ago, many of the signal connection conductors such as wires and PCB lands were electrically short enough at the highest significant frequency of the digital signals that the effect of those interconnection conductors could generally be ignored. Today, most of the PCB lands must be treated as a transmission line, and the matching of those lines is no longer an option in the design of a digital system. Distributed-parameter properties of these conductors can no longer be ignored. Hence, there is a growing need for digital design engineers to be able to understand the concepts and design principles of a transmission line. The objective of this text is to examine various methods for solving the transmission-line equations for multiconductor lines as well as to develop an understanding of the general properties of wave propagation on the lines.

The analysis of transmission lines consisting of two parallel conductors of uniform cross section has traditionally been a fundamental subject in electrical engineering. Prior to the introduction of digital computer engineering topics into the undergraduate EE curricula, all EE undergraduates were required to complete a course on two-conductor transmission lines. However, because of the introduction of computer engineering courses into an already crowded 4-year undergraduate degree program, the transmission-line course(s) in many EE programs has been relegated to a senior technical elective if offered at all. Unfortunately, the increasing use of high-speed digital technology requires that all EEs must have a working knowledge of transmission lines. In addition, the use of MTLs consisting of more than two conductors is becoming more widespread because of the increasing need for high-volume and high-speed data and signal transmission. Signal integrity (the effect of the transmission line on the signal transmission) is becoming a critical aspect of high-speed digital system performance. This text is intended to fill the need for a structured course on transmission lines in an EE program.

This text is the second edition of a previous text concerning the analysis of MTLs. The text has undergone a significant reorganization with the emphasis given toward a university textbook for a senior/graduate textbook on transmission lines. In the first edition the two-conductor and MTL discussions were combined into a single chapter. In this second edition, each broad analysis topic, for example, per-unit-length parameters, frequency-domain analysis, time-domain analysis, and incident field excitation, now has a chapter concerning two-conductor lines followed immediately by a chapter on MTLs for that topic. This allows the instructor to choose his or her emphasis either on two-conductor lines or on MTLs, or on both. This organization also makes it easier for the reader to understand the analysis of MTLs. The analysis of MTLs is very similar to that of two-conductor lines except that the detail is considerably increased. However, with the aid of matrix symbology and techniques, most of the MTL topics are straightforward extensions of the corresponding two-conductor line topics. Hence, by devoting a separate chapter to each two-conductor line topic and then following that with a chapter on the corresponding MTL topic, the MTL material can be more readily understood.

In addition to this significant reorganization of the material, the text now contains important developments in analysis methods that have been developed in the intervening 13 years since the first edition was published. Digital technology has virtually taken over the field of electronics. The clock and data speeds in those digital systems have accelerated at an astonishing rate, meaning that the spectral content of those signals that are propagating along the interconnect lands now extends into and, in some high-end servers, above the 20-GHz region. This has caused most of the signal interconnects in those systems, which were inconsequential from a standpoint of transmission-line effects 13 years ago, to now become critical to the functionality of those systems. The transmission-line behavior of those interconnects can no longer be ignored. In addition, the number and density of those interconnects have increased dramatically so that the analysis of their effects is now a serious computational problem. Hence, the current emphasis is on the development of macromodels and on "model order reduction" or (MOR) for the time-domain analysis of these high-density interconnects. Generation of macromodels that compactly describe these interconnects from a port standpoint requires the determination of the transfer functions representing those interconnects. Typical distributed-parameter interconnects have an enormous number of poles such that the transfer functions are ratios of very high order polynomials in the Laplace transform variable s. The current analysis methods focus on MOR methods that seek to determine a highly reduced number of dominant poles of those transfer functions, thereby simplifying the analysis. MOR methods such as recursive convolution, complex frequency hopping (CFH), Pade, asymptotic waveform expansion (AWE), and vector fitting (VF) as well as the synthesis of lumped-circuit models are the current methods of choice. Chapters 8 and 9 , which cover the time-domain analysis of two-conductor lines and MTLs, respectively, have been considerably expanded to now include those topics among many others that have been developed in the intervening years.

The second edition of this text is now divided into 13 chapters, whereas the first edition only had eight chapters. Considerable thought has gone into the reorganization of the text. The author is of the strong opinion that organization of subject material into a logical and well thought-out form is perhaps the most important pedagogical technique in a reader's learning process. This logical organization is one of the important attributes of the text. Chapter 1 discusses the background and rationale for the use of MTLs. The general properties of the TEM mode of propagation are discussed, and the transmission-line equations are derived for two-conductor lines using several methods. The various classifications of MTLs (uniform, lossless, and homogeneous medium) are discussed along with the restrictions on the use of the TEM model. An important addition to this chapter has been made in the second edition: the discussion of the time domain versus the frequency domain. This is a crucial aspect of any EE's ability to design electronic systems. The chapter presents a very useful method for the time-domain analysis of a linear system such as an MTL having linear terminations by computing the frequency-domain transfer function, decomposing the periodic or nonperiodic input signal into its Fourier spectral components, passing those through the system, and recombining in time at the output. This allows a straightforward incorporation of frequency-dependent losses of the conductors and the surrounding medium. These frequency-dependent losses complicate, as we will see, the direct time-domain solution of the transmission-line equations.

Chapter 2 provides a derivation of the two-conductor transmission-line equations along with the general properties of the per-unit-length parameters in those equations. Chapter 3 discusses these topics for MTLs.

Chapter 4 discusses the derivation of the per-unit-length parameters of inductance, capacitance, resistance, and conductance for two-conductor lines, whereas Chapter 5 repeats this for MTLs. In both chapters, numerical methods for the determination of these important per-unit-length parameters are discussed in detail.

Chapter 6 discusses the frequency-domain solution of the transmission-line equations, and Chapter 7 repeats this for MTLs. The discussion of two-conductor transmission lines in Chapter 6 has been expanded considerably over the first edition coverage and now constitutes a traditional undergraduate coverage.

Chapter 8 discusses the time-domain analysis of two-conductor lines, and Chapter 9 repeats this for MTLs. Again, the time-domain solution for two-conductor lines in Chapter 8 is considerably expanded over the first edition coverage and now constitutes a traditional undergraduate coverage. In addition, Chapter 8 now includes an extensive discussion of methods for achieving signal integrity (SI) in high-speed digital interconnects. The finite-difference, time-domain (FDTD) solution method is developed as is the time-domain to frequency-domain transformation (TDFD) method. Both of these allow the inclusion of frequency-dependent losses. Detailed discussions of recursive convolution and MOR techniques such as Pade methods are now included in Chapter 8 . Chapter 9 on the time-domain analysis of MTLs now includes extensive discussion of MOR methods such as the generalized method of characteristics, Pade methods, asymptotic waveform expansion, complex frequency hopping, and vector fitting. In addition, Chapter 9 now includes the development of the FDTD method for dynamic terminations.

Chapter 10 gives the symbolic or literal solution of perhaps the only MTL that admits a closed-form solution in terms of symbols: a three-conductor lossless line in a homogeneous medium. This chapter is virtually the same as in the first edition although it has been revised.

Chapter 11 gives the frequency-domain and time-domain solutions for two-conductor lines with incident field illumination. Chapter 12 repeats that for MTLs. The emphasis is on uniform plane-wave excitation as from a distant antenna or a lightning stroke.

Finally, Chapter 13 discusses the analysis of interconnected transmission-line networks such as branched cables. There is not enough difference between the methods for two-conductor lines and MTLs to warrant splitting this chapter into two chapters.

Appendix A contains the descriptions of numerous FORTRAN computer codes that implement all the techniques in this text. Numerous experimental results are included in this text ranging from cables to PCBs in order to illustrate real-world problems. These codes are used to provide the predictions for these experimental results. The codes can be downloaded from the Wiley ftp site at ftp:\\ftp.wiley.com/public/sci_tech_med/multiconductor_transmission/.

Appendix B is new to this edition and contains a brief but thorough tutorial on the SPICE/PSPICE circuit analysis program.

Many of the author's colleagues have contributed substantially to the advancement of this subject. Professors Antonio Orlandi and Frederick M. Tesche along with the late Albert A. Smith Jr. are among those to whom the author owes a debt of gratitude for many insightful discussions on this subject.

Clayton R. Paul
Macon, Georgia

CONTENTS

Preface **xvii**

1 Introduction **1**

1.1 Examples of Multiconductor Transmission-Line Structures 5
1.2 Properties of the TEM Mode of Propagation 8
1.3 The Transmission-Line Equations: A Preview 18
1.3.1 Unique Definition of Voltage and Current for the TEM Mode of Propagation 19
1.3.2 Defining the Per-Unit-Length Parameters 22
1.3.3 Obtaining the Transmission-Line Equations from the Transverse Electromagnetic Field Equations 28
1.3.4 Properties of the Per-Unit-Length Parameters 30
1.4 Classification of Transmission Lines 32
1.4.1 Uniform versus Nonuniform Lines 33
1.4.2 Homogeneous versus Inhomogeneous Surrounding Media 35
1.4.3 Lossless versus Lossy Lines 36
1.5 Restrictions on the Applicability of the Transmission-Line Equation Formulation 37
1.5.1 Higher Order Modes 38
1.5.1.1 The Infinite, Parallel-Plate Transmission Line 38

1.5.1.2 The Coaxial Transmission Line 43
1.5.1.3 Two-Wire Lines 44

1.5.2 Transmission-Line Currents versus Antenna Currents 45

1.6 The Time Domain versus the Frequency Domain 47

1.6.1 The Fourier Series and Transform 50

1.6.2 Spectra and Bandwidth of Digital Waveforms 52

1.6.3 Computing the Time-Domain Response of Transmission Lines Having Linear Terminations Using Fourier Methods and Superposition 56

Problems 61

References 69

2 The Transmission-Line Equations for Two-Conductor Lines 71

2.1 Derivation of the Transmission-Line Equations from the Integral Form of Maxwell's Equations 71

2.2 Derivation of the Transmission-Line Equations from the Per-Unit-Length Equivalent Circuit 77

2.3 Properties of the Per-Unit-Length Parameters 78

2.4 Incorporating Frequency-Dependent Losses 79

2.4.1 Properties of the Frequency-Domain Per-Unit-Length Impedance $\hat{z}(\omega)$ and Admittance $\hat{y}(\omega)$ 81

Problems 85

References 88

3 The Transmission-Line Equations for Multiconductor Lines 89

3.1 Derivation of the Multiconductor Transmission-Line Equations from the Integral Form of Maxwell's Equations 89

3.2 Derivation of the Multiconductor Transmission-Line Equations from the Per-Unit-Length Equivalent Circuit 99

3.3 Summary of the MTL Equations 101

3.4 Incorporating Frequency-Dependent Losses 102

3.5 Properties of the Per-Unit-Length Parameter Matrices **L**, **C**, **G** 103

Problems 108

References 109

4 The Per-Unit-Length Parameters for Two-Conductor Lines 110

4.1 Definitions of the Per-Unit-Length Parameters l, c, and g 111

4.2 Lines Having Conductors of Circular, Cylindrical Cross Section (Wires) 113

4.2.1 Fundamental Subproblems for Wires 113
4.2.1.1 The Method of Images 118
4.2.2 Per-Unit-Length Inductance and Capacitance for Wire-Type Lines 119
4.2.3 Per-Unit-Length Conductance and Resistance for Wire-Type Lines 130

4.3 Lines Having Conductors of Rectangular Cross Section (PCB Lands) 144

4.3.1 Per-Unit-Length Inductance and Capacitance for PCB-Type Lines 145
4.3.2 Per-Unit-Length Conductance and Resistance for PCB-Type Lines 148

Problems 156
References 158

5 The Per-Unit-Length Parameters for Multiconductor Lines 160

5.1 Definitions of the Per-Unit-Length Parameter Matrices **L**, **C**, and **G** 161

5.1.1 The Generalized Capacitance Matrix $\mathscr{C}$ 167

5.2 Multiconductor Lines Having Conductors of Circular, Cylindrical Cross Section (Wires) 171

5.2.1 Wide-Separation Approximations for Wires in Homogeneous Media 171
5.2.1.1 $n+1$ Wires 173
5.2.1.2 n Wires Above an Infinite, Perfectly Conducting Plane 173
5.2.1.3 n Wires Within a Perfectly Conducting Cylindrical Shield 174
5.2.2 Numerical Methods for the General Case 176
5.2.2.1 Applications to Inhomogeneous Dielectric Media 181
5.2.3 Computed Results: Ribbon Cables 187

5.3 Multiconductor Lines Having Conductors of Rectangular Cross Section 189

5.3.1 Method of Moments (MoM) Techniques 190
5.3.1.1 Applications to Printed Circuit Boards 199
5.3.1.2 Applications to Coupled Microstrip Lines 211
5.3.1.3 Applications to Coupled Striplines 219

5.4 Finite Difference Techniques 223
5.5 Finite-Element Techniques 229
Problems 237
References 239

6 Frequency-Domain Analysis of Two-Conductor Lines 240

6.1 The Transmission-Line Equations in the Frequency Domain 241
6.2 The General Solution for Lossless Lines 242
6.2.1 The Reflection Coefficient and Input Impedance 244
6.2.2 Solutions for the Terminal Voltages and Currents 247
6.2.3 The SPICE (PSPICE) Solution for Lossless Lines 250
6.2.4 Voltage and Current as a Function of Position on the Line 252
6.2.5 Matching and VSWR 255
6.2.6 Power Flow on a Lossless Line 256
6.3 The General Solution for Lossy Lines 258
6.3.1 The Low-Loss Approximation 260
6.4 Lumped-Circuit Approximate Models of the Line 265
6.5 Alternative Two-Port Representations of the Line 269
6.5.1 The Chain Parameters 270
6.5.2 Approximating Abruptly Nonuniform Lines with the Chain-Parameter Matrix 273
6.5.3 The Z and Y Parameters 275
Problems 278

7 Frequency-Domain Analysis of Multiconductor Lines 282

7.1 The MTL Transmission-Line Equations in the Frequency Domain 282
7.2 The General Solution for An $(n + 1)$-Conductor Line 284
7.2.1 Decoupling the MTL Equations by Similarity Transformations 284
7.2.2 Solution for Line Categories 291
7.2.2.1 Perfect Conductors in Lossy, Homogeneous Media 292
7.2.2.2 Lossy Conductors in Lossy, Homogeneous Media 293
7.2.2.3 Perfect Conductors in Lossless, Inhomogeneous Media 296
7.2.2.4 The General Case: Lossy Conductors in Lossy, Inhomogeneous Media 298
7.2.2.5 Cyclic-Symmetric Structures 298

7.3 Incorporating the Terminal Conditions 305

7.3.1 The Generalized Thevenin Equivalent 305

7.3.2 The Generalized Norton Equivalent 308

7.3.3 Mixed Representations 310

7.4 Lumped-Circuit Approximate Characterizations 312

7.5 Alternative 2*n*-Port Characterizations 314

7.5.1 Analogy of the Frequency-Domain MTL Equations to State-Variable Equations 314

7.5.2 Characterizing the Line as a 2*n*-Port with the Chain-Parameter Matrix 316

7.5.3 Properties of the Chain-Parameter Matrix 318

7.5.4 Approximating Nonuniform Lines with the Chain-Parameter Matrix 322

7.5.5 The Impedance and Admittance Parameter Matrix Characterizations 323

7.6 Power Flow and the Reflection Coefficient Matrix 327

7.7 Computed and Experimental Results 332

7.7.1 Ribbon Cables 332

7.7.2 Printed Circuit Boards 335

Problems 338

References 342

8 Time-Domain Analysis of Two-Conductor Lines 343

8.1 The Solution for Lossless Lines 344

8.1.1 Wave Tracing and the Reflection Coefficients 346

8.1.2 Series Solutions and the Difference Operator 356

8.1.3 The Method of Characteristics and a Two-Port Model of the Line 361

8.1.4 The SPICE (PSPICE) Solution for Lossless Lines 365

8.1.5 The Laplace Transform Solution 368

8.1.5.1 Lines with Capacitive and Inductive Loads 370

8.1.6 Lumped-Circuit Approximate Models of the Line 373

8.1.6.1 When is the Line Electrically Short in the Time Domain? 374

8.1.7 The Time-Domain to Frequency-Domain (TDFD) Transformation Method 375

8.1.8 The Finite-Difference, Time-Domain (FDTD) Method 379

8.1.8.1 The Magic Time Step 385

8.1.9 Matching for Signal Integrity 392
8.1.9.1 When is Matching Required? 398
8.1.9.2 Effects of Line Discontinuities 399
8.2 Incorporation of Losses 406
8.2.1 Representing Frequency-Dependent Losses 408
8.2.1.1 Representing Losses in the Medium 408
8.2.1.2 Representing Losses in the Conductors and Skin Effect 410
8.2.1.3 Convolution with Frequency-Dependent Losses 415
8.2.2 The Time-Domain to Frequency-Domain (TDFD) Transformation Method 421
8.2.3 The Finite-Difference, Time-Domain (FDTD) Method 423
8.2.3.1 Including Frequency-Independent Losses 423
8.2.3.2 Including Frequency-Dependent Losses 427
8.2.3.3 Prony's Method for Representing a Function 431
8.2.3.4 Recursive Convolution 434
8.2.3.5 An Example: A High-Loss Line 439
8.2.3.6 A Correction for the FDTD Errors 443
8.2.4 Lumped-Circuit Approximate Characterizations 447
8.2.5 The Use of Macromodels in Modeling the Line 450
8.2.6 Representing Frequency-Dependent Functions in the Time Domain Using Pade Methods 453
Problems 461
References 467

9 Time-Domain Analysis of Multiconductor Lines 470

9.1 The Solution for Lossless Lines 470
9.1.1 The Recursive Solution for MTLs 471
9.1.2 Decoupling the MTL Equations 476
9.1.2.1 Lossless Lines in Homogeneous Media 478
9.1.2.2 Lossless Lines in Inhomogeneous Media 479
9.1.2.3 Incorporating the Terminal Conditions via the SPICE Program 482
9.1.3 Lumped-Circuit Approximate Characterizations 487
9.1.4 The Time-Domain to Frequency-Domain (TDFD) Transformation Method 488
9.1.5 The Finite-Difference, Time-Domain (FDTD) Method 488
9.1.5.1 Including Dynamic and/or Nonlinear Terminations in the FDTD Analysis 490
9.2 Incorporation of Losses 496
9.2.1 The Time-Domain to Frequency-Domain (TDFD) Method 498