

LECTURE NOTES  
IN PHYSICS

J. Asch  
A. Joye  
(Eds.)

# Mathematical Physics of Quantum Mechanics

Selected  
and Refereed Lectures  
from QMath9



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Joachim Asch Alain Joye (Eds.)

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Selected and Refereed Lectures from QMath9



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## Editors

Joachim Asch  
Université du Sud Toulon Var  
Centre de physique théorique  
Département de Mathématiques  
BP 20132  
F-83957 La Garde Cedex  
France  
E-mail: asch@univ-tln.fr

Alain Joye  
Institut Fourier  
Université Grenoble 1  
BP 74  
38402 Saint-Martin-d'Hères Cedex  
France  
E-mail: alain.joye@ujf-grenoble.fr

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# Preface

The topics presented in this book were discussed at the conference “QMath9” held in Giens, France, September 12th-16th 2004. QMath is a series of meetings whose aim is to present the state of the art in the Mathematical Physics of Quantum Systems, both from the point of view of physical models and of the mathematical techniques developed for their study. The series was initiated in the early seventies as an attempt to enhance collaboration between mathematical physicists from eastern and western European countries. In the nineties it took a worldwide dimension. At the same time, due to engineering achievements, for example in the mesoscopic realm, there was a renewed interest in basic questions of quantum dynamics.

The program of QMath9, which was attended by 170 scientists from 23 countries, consisted of 123 talks grouped by the topics: *Nanophysics*, *Quantum dynamics*, *Quantum field theory*, *Quantum kinetics*, *Random Schrödinger operators*, *Semiclassical analysis*, *Spectral theory*. QMath9 was also the frame for the 2004 meeting of the European Research Group on “Mathematics and Quantum Physics” directed by Monique Combescure. For a detailed account of the program, see <http://www.cpt.univ.mrs.fr/~qmath9>.

Expanded versions of several selected introductory talks presented at the conference are included in this volume. Their aim is to provide the reader with an easier access to the sometimes technical state of the art in a topic. Other contributions are devoted to a pedagogical exposition of quite recent results at the frontiers of research, parts of which were presented in “QMath9”. In addition, the reader will find in this book new results triggered by discussions which took place at the meeting.

Hence, while based on the conference “QMath9”, this book is intended to be a starting point for the reader who wishes to learn about the current research in quantum mathematical physics, with a general perspective. Effort has been made by the authors, editors and referees in order to provide contributions of the highest scientific standards to meet this goal.

We are grateful to Yosi Avron, Volker Bach, Stephan De Bièvre, Laszlo Erdős, Pavel Exner, Svetlana Jitomirskaya, Frédéric Klopp who mediated the scientific sessions of “QMath9”.

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*Joachim Asch*  
*Alain Joye*

# List of Contributors

## **W.K. Abou-Salem**

Institute for Theoretical  
Physics, ETH-Hönggerberg  
CH-8093 Zürich, Switzerland  
walid@phys.ethz.ch

## **M. Aizenman**

Departments of Mathematics  
and Physics  
Jadwin Hall Princeton University  
P. O. Box 708, Princeton  
New Jersey, 08544  
USA

## **A. Avila**

CNRS UMR 7599  
Laboratoire de Probabilités  
et Modèles aléatoires  
Université Pierre  
et Marie Curie-Boite  
courrier 188, 75252-Paris Cedex 05  
France  
artur@ccr.jussieu.fr

## **V. Betz**

Institute for Biomathematics  
and Biometry GSF  
Forschungszentrum Postfach  
1129 D-85758  
Oberschleißheim  
Germany  
volker.betz@gsf.de

## **S. De Bièvre**

Université des Sciences et  
Technologies de Lille, UFR

de Mathématiques et Laboratoire  
Painlevé, 59655 Villeneuve d'Ascq  
Cedex, France  
Stephan.De-Bievre@math.univ-  
lille1.fr

## **O. Bokanowski**

Laboratoire Jacques Louis Lions  
(Paris VI)  
UFR Mathématique B. P. 7012  
& Université Paris VII  
Paris, Paris Cedex 05  
France  
boka@math.jussieu.fr

## **M. Büttiker**

Département de Physique Théorique  
Université de Genève  
CH-1211 Genève 4, Switzerland  
Markus.Buttiker@physics.unige.ch

## **J.-M. Combes**

Département de Mathématiques  
Université de Toulon-Var  
BP 132, 83957 La Garde Cédex  
France  
combes@cpt.univ-mrs.fr

## **H.D. Cornean**

Department of Mathematical  
Sciences  
Aalborg University  
Fredrik Bajers Vej 7G  
9220 Aalborg, Denmark  
cornean@math.aau.dk



**J. Dereziński**

Department of Mathematics  
and Methods in Physics  
Warsaw University Hoża 74  
00-682 Warsaw  
Poland  
Jan.Derezinski@fuw.edu.pl


**A. Elgart**

Department of Mathematics  
Stanford University  
Stanford, CA 94305-2125  
USA  
elgart@math.stanford.edu

**L. Erdős**

Institute of Mathematics  
University of Munich  
Theresienstr.  
39 D-80333 Munich  
Germany  
lerdos@mathematik.uni-muenchen.  
de

**P. Exner**

Department of Theoretical Physics  
Nuclear Physics Institute  
Academy of Sciences  
25068 Řež near Prague  
Czechia  
and   
Doppler Institute  
Czech Technical University  
Břehová 7, 11519  
Prague, Czechia  
exner@ujf.cas.cz

**A. Fedotov**

Department of Mathematical  
Physics  
St Petersburg State University 1  
Uljanovskaja 198904  
St Petersburg-Petrodvorets, Russia  
mailto:fedotov@svs.ru  
fedotov@svs.ru

**J. Fröhlich**

Institute for Theoretical  
Physics  
ETH-Hönggerberg  
CH-8093 Zürich  
Switzerland  
juerg@itp.phys.ethz.ch

**F. Germinet**

Département de Mathématiques  
Université de Cergy-Pontoise  
Site de Saint-Martin  
2 avenue Adolphe Chauvin  
95302 Cergy-Pontoise Cédex  
France  
germinet@math.u-cergy.fr

**S.D. Głazek**

Institute of Theoretical Physics  
Warsaw University  
00-681 Hoża 69  
Poland  
stglazek@fuw.edu.pl

**B. Helffer**

Département de Mathématiques  
Bât. 425, Université Paris  
Sud UMR CNRS 8628, F91405  
Orsay Cedex  
France  
Bernard.Helffer@math.u-psud.fr

**P.D. Hislop**

Department of Mathematics  
University of Kentucky  
Lexington KY 40506-0027  
USA  
hislop@ms.uky.edu

**A. Jensen**

Department of Mathematical  
Sciences  
Aalborg University  
Fredrik Bajers Vej 7G  
9220 Aalborg, Denmark  
matarne@math.aau.dk

**S. Jitomirskaya**

University of California  
Irvine Department of Mathematics  
243 Multipurpose Science  
& Technology Building  
Irvine, CA 92697-3875  
USA  
szhitomi@uci.edu

**F. Klopp**

LAGA, Institut Galilée  
U.M.R 7539 C.N.R.S  
Université Paris-Nord  
Avenue J.-B. Clément  
F-93430 Villetaneuse, France  
klopp@math.univ-paris13.fr

**D. Lenz**

Fakultät für Mathematik  
Technische Universität  
09107 Chemnitz, Germany  
d.lenz@mathematik.tu-  
chemnitz.de

**E.H. Lieb**

Departments of Mathematics  
and Physics  
Jadwin Hall Princeton University  
P. O. Box 708, Princeton  
New Jersey, 08544  
USA

**J.L. López**

Departamento de Matemática  
Aplicada Facultad de Ciencias  
Universidad de Granada  
Campus Fuentenueva  
18071 Granada  
Spain  
jllopez@ugr.es

**M. Măntoiu**

“Simion Stoilow” Institute of  
Mathematics, Romanian Academy  
21, Calea Grivitei Street  
010702-Bucharest, Sector 1  
Romania  
Marius.Mantoiu@imar.ro

**K.A. Meissner**

Institute of Theoretical Physics  
Warsaw University  
Hoża 69 00-681 Warsaw  
Poland  
Krzysztof.Meissner@fuw.edu

**M. Merkli**

Department of Mathematics and  
Statistics, McGill University  
805 Sherbrooke W., Montreal  
QC Canada, H3A 2K6  
and  
Centre des Recherches  
Mathématiques, Université de  
Montréal, Succursale  
centre-ville Montréal  
QC Canada, H3C 3J7  
merkli@math.mcgill.ca

**V. Moldoveanu**

National Institute of Materials  
Physics, P.O. Box MG-7  
Magurele, Romania  
valim@infim.ro

**J.S. Møller**

Aarhus University Department of  
Mathematical Sciences  
8000 Aarhus C, Denmark  
jacob@imf.au.dk

**M. Moskalets**

Department of Metal  
and Semiconductor Physics  
National Technical University  
“Kharkiv Polytechnic Institute”  
61002 Kharkiv, Ukraine  
moskalets@kpi.kharkov.ua

**B. Nachtergaele**

Department of Mathematics  
University of California  
Davis One Shields Avenue  
Davis, CA 95616-8366, USA  
bxn@math.ucdavis.edu

**S. Nonnenmacher**

Service de Physique Théorique  
CEA/DSM/PhT, Unité de recherche  
associée au CNRS, CEA/Saclay  
91191 Gif-sur-Yvette  
France  
snonnenmacher@cea.fr

**J. Puig**

Departament de Matemàtica  
Aplicada I  
Universitat Politècnica  
de Catalunya. Av. Diagonal  
647, 08028 Barcelona, Spain

**J.V. Pulé**

Department of Mathematical  
Physics  
University College  
Dublin, Belfield, Dublin 4  
Ireland  
Joe.Pule@ucd.ie

**R. Purice**

“Simion Stoilow” Institute of  
Mathematics, Romanian Academy  
21, Calea Grivitei Street  
010702-Bucharest, Sector 1  
Romania  
Radu.Purice@imar.ro

**G. Raikov**

Facultad de Ciencias  
Universidad de Chile  
Las Palmeras 3425  
Santiago, Chile  
graykov@uchile.cl

**V. Rivasseau**

Laboratoire de Physique Théorique  
CNRS, UMR 8627  
Université de Paris-Sud  
91405 Orsay  
France  
rivass@th.u-psud.fr

**M. Salmhofer**

Theoretical Physics  
University of  
Leipzig Augustusplatz 10  
D-04109 Leipzig  
Germany  
and  
Max-Planck Institute for  
Mathematics, Inselstr. 22  
D-04103 Leipzig  
Germany  
Manfred.Salmhofer@itp.uni-  
leipzig.de

**Ó. Sánchez**

Departamento de Matemática  
Aplicada, Facultad de Ciencias  
Universidad de Granada  
Campus Fuentenueva  
18071 Granada  
Spain  
ossanche@ugr.es

**B. Schlein**

Department of Mathematics  
Stanford University  
Stanford, CA 94305, USA  
schlein@math.stanford.edu

**H. Schulz-Baldes**

Mathematisches Institut  
Friedrich-Alexander-Universität  
Erlangen-Nürnberg Bismarckstr. 1  
D-91054, Erlangen  
schuba@mi.uni-erlangen.de

**R. Seiringer**

Departments of Mathematics  
and Physics, Jadwin Hall  
Princeton University  
P. O. Box 708, Princeton  
New Jersey, 08544  
USA

**J. Soler**

Departamento de Matemática  
Aplicada, Facultad de Ciencias  
Universidad de Granada  
Campus Fuentenueva  
18071 Granada  
Spain  
jsoler@ugr.es

**J.P. Solovej**

Department of Mathematics  
University of Copenhagen  
Universitetsparken 5  
DK-2100 Copenhagen, Denmark  
and  
Institut für Theoretische Physik  
Universität Wien  
Boltzmanngasse 5  
A-1090 Vienna, Austria  
solovej@math.ku.dk

**A. Soshnikov**

University of California at Davis  
Department of Mathematics  
Davis, CA 95616, USA  
soshniko@math.ucdavis.edu

**H. Spohn**

Zentrum Mathematik and  
Physik Department  
TU München D-85747 Garching  
Boltzmannstr 3, Germany  
spohn@ma.tum.de

**S. Starr**

Department of Mathematics  
University of California  
Los Angeles, Box 951555  
Los Angeles, CA 90095-1555  
USA  
sstarr@math.ucla.edu

**P. Stollmann**

Fakultät für Mathematik  
Technische Universität  
09107 Chemnitz, Germany  
p.stollmann@mathematik.tu-  
chemnitz.de

**S. Teufel**

Mathematisches Institut  
Universität Tübingen  
Auf der Morgenstelle 10  
72076 Tübingen  
Germany  
stefan.teufel@uni-tuebingen.de

**A.F. Verbeure**

Instituut voor Theoretische  
Fysika, Katholieke Universiteit  
Leuven Celestijnenlaan  
200D, 3001 Leuven, Belgium  
andre.verbeure@fys.kuleuven.  
ac.be

**H.-T. Yau**

Department of Mathematics  
Stanford University, CA-94305, USA  
yau@math.stanford.edu

**J. Yngvason**

Institut für Theoretische Physik  
Universität Wien  
Boltzmanngasse 5  
A-1090 Vienna, Austria  
and  
Erwin Schrödinger Institute for  
Mathematical Physics  
Boltzmanngasse 9  
A-1090 Vienna, Austria

**V.A. Zagrebnov**

Université de la  
Méditerranée and Centre de  
Physique Théorique, Luminy-Case  
907, 13288 Marseille, Cedex 09  
France  
zagrebnov@cpt.univ-mrs.fr

**S. Zelditch**

Department of Mathematics  
Johns Hopkins University  
Baltimore MD 21218  
USA  
zelditch@math.jhu.edu

# Lecture Notes in Physics

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**Part II Quantum Field Theory and Statistical Mechanics**

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