LECTURE NOTES IN PHYSICS

J. Asch
A. Joye
(Eds.)

Mathematical Physics of Quantum Mechanics

Selected and Refereed Lectures from QMath9

Springer

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Editors

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Preface

The topics presented in this book were discussed at the conference "QMath9" held in Giens, France, September 12th-16th 2004. QMath is a series of meetings whose aim is to present the state of the art in the Mathematical Physics of Quantum Systems, both from the point of view of physical models and of the mathematical techniques developed for their study. The series was initiated in the early seventies as an attempt to enhance collaboration between mathematical physicists from eastern and western European countries. In the nineties it took a worldwide dimension. At the same time, due to engineering achievements, for example in the mesoscopic realm, there was a renewed interest in basic questions of quantum dynamics.

The program of QMath9, which was attended by 170 scientists from 23 countries, consisted of 123 talks grouped by the topics: *Nanophysics, Quantum dynamics, Quantum field theory, Quantum kinetics, Random Schrödinger operators, Semiclassical analysis, Spectral theory.* QMath9 was also the frame for the 2004 meeting of the European Research Group on "Mathematics and Quantum Physics" directed by Monique Combescure. For a detailed account of the program, see http://www.cpt.univ.mrs.fr/~qmath9.

Expanded versions of several selected introductory talks presented at the conference are included in this volume. Their aim is to provide the reader with an easier access to the sometimes technical state of the art in a topic. Other contributions are devoted to a pedagogical exposition of quite recent results at the frontiers of research, parts of which were presented in "QMath9". In addition, the reader will find in this book new results triggered by discussions which took place at the meeting.

Hence, while based on the conference "QMath9", this book is intended to be a starting point for the reader who wishes to learn about the current research in quantum mathematical physics, with a general perspective. Effort has been made by the authors, editors and referees in order to provide contributions of the highest scientific standards to meet this goal.

We are grateful to Yosi Avron, Volker Bach, Stephan De Bièvre, Laszlo Erdös, Pavel Exner, Svetlana Jitomirskaya, Frédéric Klopp who mediated the scientific sessions of "QMath9".

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Toulon — *Joachim Asch*
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January 2006

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Contents

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