

STUDY TEXT

QUANTITATIVE STUDIES

Recommended by the ICSA Commended by the ICS

# **QUANTITATIVE STUDIES**

江苏工业学院图书馆 藏 书 章

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#### **PREFACE**

The examinations of the Institute of Chartered Secretaries and Administrators are a demanding test of students' ability to master the wide range of legal, commercial and financial knowledge required of the modern chartered secretary. The Institute's rapid response to the pace of change is shown both in the content of the syllabuses and in the style of examination questions set.

BPP's experience in producing study material for the Institute's examinations is unparalleled. Over the years, BPP's Study Texts and Practice and Revision Kits, now supplemented by the Password series of multiple choice (objective test) question books, have helped students attain the examination success that is a prerequisite of career development.

This Study Text is designed to prepare students for the *Quantitative Studies* paper. It provides comprehensive and targeted coverage of the syllabus (reproduced on page (vii)) in the light of recent examination questions (analysed on pages (viii) to (x)).

BPP's Study Texts are noted for their clarity of explanation. They are reviewed and updated each year. BPP's study material, at once comprehensive and up to date, is thus the ideal investment that the aspiring chartered secretary can make for examination success.

#### The 1991 edition of this text

This text has been reviewed and updated in the light of recent developments in the subject and in the examination. The most significant amendments are outlined below.

- New material has been added to take account of the syllabus changes effective from June 1992.
- The text has been thoroughly revised to make learning easier.
- The illustrative questions have been revised to take into account the latest trends in the examination.

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For details of other BPP titles relevant to your studies for this examination, please turn to page 347. If you wish to send in your comments on this text, please turn to page 348.

#### **Syllabus**

#### Ob jective

To provide an introduction to quantitative techniques which are widely applicable in business administration. In both sections of the syllabus a purely theoretical approach should be avoided and the emphasis should be on applications concerned with the solution of business problems.

The syllabus is divided into two sections: Section A, which has a weighting of approximately two thirds, covers business statistics; section B, which has a weighting of approximately one third, covers business mathematics.

#### Section A: Business Statistics

Collection and presentation of numerical information. Official sources of economic and business data. Survey methods. Questionnaire design, sampling methods, interviews, postal questionnaire. Tabulation. Graphs, charts and diagrams.

Frequency distribution. Measures of location: mean, median and mode; the geometric mean; the harmonic mean. Measures of dispersion: quartile deviation and standard deviation. Skewness: coefficient of variation.

Relationships and forecasting. Correlation and regression. Product moment correlation coefficient. Rank correlation coefficient. Linear regression using the least squares method. Time series analysis. Components of a time series: trend, cyclical, seasonal and random. Moving averages. Simple methods of forecasting.

Index numbers. Price relative methods. Aggregate methods. Problems involved in the use of index numbers.

Probability and statistical inference. Introduction to probability from a practical viewpoint. Equally likely outcomes, combinations of events, union of events, conditional probability, mutually exclusive events. The binomial distribution; the Poisson distribution. Statistical inference. Confidence intervals and hypothesis testing for a single mean and single proportion. Two-sample hypothesis tests.

#### Section B: Business Mathematics

Representation of business situations using models. Simple models to represent algebraically and geometrically accounting, economic and business concepts. Breakeven analysis, cost, revenue and profit functions. Linear programming. Graphical methods of solving optimising problems: formulation, graphical representation and solution. The simplex method

Compound interest and investment appraisal. Compound interest. Sinking funds, annuities and mortgages. Discounted cash flow. Net present value, internal rate of return.

#### A note on the BPP study text

This study text has been designed to cover every topic mentioned in the syllabus reproduced above. The text has been structured to correspond with the order in which the syllabus itself has been laid out. This will assist you by helping you to monitor your progress.

Note that you will not have to commit any formulae to memory for this paper. Discounted cash flow tables, normal distribution tables and a list of statistical formulae will be provided in the examination. The tables provided are reproduced at the end of this introduction.

#### The format of the examination paper

The examination paper has hitherto been in two sections. A total of five questions have had to be attempted. Section A on business statistics has contained six questions worth 20 marks each. Section B on business mathematics has contained three questions worth 20 marks each. Candidates have had to answer at least one question from Section B.

With effect from the June 1992 examination, the paper will contain eight questions, of which candidates must answer four. The questions will be longer than in previous examinations, with the interpretation of results being required.

#### Analysis of past papers

#### June 1991

#### Section A

- 1 Statistics published by governments
- 2 Histograms. The mean and other measures of location. The standard deviation
- 3 Time series using moving averages
- 4 Correlation and regression
- 5 Laspeyre and Paasche index numbers
- 6 Hypothesis testing. The normal distribution

#### Section B

- 7 The effect of inflation. Sinking funds. Loan repayments
- 8 The formulation and graphing of expressions for costs. The minimisation of costs
- 9 Linear programming

#### December 1990

#### Section A

- 1 Definitions of statistical terms
- 2 The mean and the quartile deviation. A comparison of two distributions
- 3 Time series using moving averages
- 4 Scatter diagrams and regression
- 5 Index numbers
- 6 Probability. The normal distribution

#### Section B

- 7 Equivalent annual interest rates. The growth of a deposit
- 8 The formulation and graphing of an expression for profit. Quadratic equations
- 9 Linear programming

#### June 1990

#### Section A

- l Sampling methods
- 2 The mean, the median and the quartile deviation
- 3 Time series using moving averages
- 4 Correlation
- 5 Index numbers
- 6 Probability and significance testing

#### Section B

- 7 Linear programming
- 8 Algebraic models and breakeven analysis
- 9 NPV, internal rate of return and sinking funds

#### December 1989

#### Section A

- I Index numbers
- 2 Time series using moving averages
- 3 Measures of dispersion and significance testing
- 4 The median, quartiles and the quartile deviation
- 5 Lorenz curves
- 6 Correlation

#### Section B

- 7 Annual mortgage payments
- 8 Linear programming
- 9 Algebraic models

#### June 1989

#### Section A

- l Venn diagrams. Probability
- 2 The mean and the standard deviation. 95% confidence intervals
- 3 Index numbers
- 4 Scatter diagrams and regression
- 5 Primary data and secondary data. Economic indicators. Postal questionnaires and interviews
- 6 Time series using moving averages

#### Section B

- 7 Breakeven analysis: the graphical method
- 8 The calculation of the APR. Investment appraisal
- 9 Breakeven analysis: the algebraic approach

#### December 1988

#### Section A

- 1 Time series using moving averages
- 2 Draft a letter and a questionnaire to employees concerning a possible relocation of the company
- 3 The mean, the standard deviation and the coefficient of variation. Significance tests
- 4 The median, quartiles and the quartile deviation
- 5 Rebasing a series of index numbers
- 6 Scatter diagrams and the correlation coefficient

#### Section B

- 7 Linear programming
- 8 The NPV method of ranking two projects
- 9 Breakeven analysis: the graphical method

#### June 1988

#### Section A

- 1 Histograms. Hypothesis testing for a single mean
- 2 Lorenz curves
- 3 Regression analysis
- 4 Methods of sampling and collecting data
- 5 Index numbers
- 6 Tabulation. Probability

#### Section B

- 7 Calculate the annual payment for four financing proposals
- 8 Breakeven analysis. Drawing a graph of hire costs
- 9 Linear programming

#### Tables and formulae

#### FINANCIAL TABLES

#### 1. Compound interest

Year	5%	10%	15%	20%
0	1.00000	1.00000	1.00000	1.00000
1	1.05000	1.10000	1.15000	1.20000
2	1,10250	1.21000	1.32250	1.44000
3	1.15763	1.33100	1.52088	1.72800
4	1,21551	1.46410	1.74901	2.07360
5	1,27628	1.61051	2.01136	2.48832
6	1.34010	1.77156	2.31306	2.98598

#### 2. Discount factors

Year	5%	10%	15%	20%
0	1.00000	1.00000	1.00000	1.00000
1	0.95238	0.90909	0.86957	0.83333
2	0.90703	0.82645	0.75614	0.69444
3	0.86384	0.75131	0.65752	0.57870
4	0.82270	0.68301	0.57175	0.48225
5	0.78353	0.62092	0.49718	0.40188
6	0.74622	0.56447	0.43233	0.33490

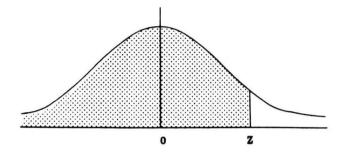
### 3. $s_{n}$ = Future Value of an Annuity

Year	5%	10%	15%	20%
1	1.00000	1.00000	1.00000	1.00000
2	2.05000	2.10000	2.15000	2.20000
3	3.15250	3.31000	3.47250	3.64000
4	4.31013	4.64100	4.99338	5.36800
5	5.52563	6.10510	6.74238	7.44160
6	6.80191	7.71561	8.75374	9.92992

## 4. $a_{nl}$ = Present Value of an Annuity

Year	5%	10%	15%	20%
1	0.95238	0.90909	0.86957	0.83333
2	1.85941	1.73554	1.62571	1.52778
3	2.72325	2.48685	2.28323	2.10648
4	3.54595	3.16987	2.85498	2.58873
5	4.32948	3.79079	3.35216	2.99061
6	5.07569	4.35526	3.78448	3.32551

Table of Areas of the Normal Distribution



Standardised Deviate Z	Area to left of Z	Standardised Deviate Z	Area to left of Z
0.00	0.5000	1.96	0.9750
0.25	0.5987	2.00	0.9772
0.50	0.6915	2.25	0.9878
0.75	0.7734	2.33	0.9901
1.00	0.8413	2.50	0.9938
1.25	0.8944	2.58	0.9951
1.50	0.9332	2.75	0.9970
1.64	0.9495	3.00	0.9987
1.75	0.9599	3.09	0.9990

#### **BASIC FORMULAE**

These formulae are supplied for assistance only. Their use is not compulsory.

Arithmetic mean and standard deviation

$$\overline{x} = \frac{\Sigma f x}{\Sigma f}$$
  $s = \int \frac{\Sigma f x^2}{\Sigma f} - \frac{(\Sigma f x)^2}{(\Sigma f)^2}$ 

Geometric mean

$$^{n}/(x_{1} \times x_{2} \times .... \times x_{n})$$

Harmonic mean

$$n/[(1/x_1) + (1/x_2) + .... + (1/x_n)]$$

Quartile deviation = 
$$\frac{Q_3 - Q_1}{2}$$

Skewness

Quartile measure = 
$$\frac{Q_3 + Q_1 - 2(Median)}{Q_3 - Q_1}$$

Pearson measure = 
$$\frac{3(\bar{x} - Median)}{s}$$

Coefficient of variation = 
$$\frac{100s}{\overline{x}}$$

Spearman rank correlation coefficient

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

Product moment correlation coefficient

$$r = \frac{n\Sigma xy - \Sigma x\Sigma y}{\sqrt{(n\Sigma x^2 - (\Sigma x)^2)(n\Sigma y^2 - (\Sigma y)^2)}}$$

Regression. Regression line of y on x: y = a + bx

$$b = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \qquad a = \frac{\Sigma y}{n} - \frac{b\Sigma x}{n}$$

$$a = \frac{\sum y}{n} - \frac{b\sum x}{n}$$

Binomial probabilities

P(x out of n)

$$= \frac{n!}{x!(n-x)!} p^{x}q^{(n-x)}$$

Poisson probabilities

$$P(x) = \frac{m^x e^{-m}}{x!}$$

Standard error of a sample mean

= 
$$\frac{\sigma}{\sqrt{n}}$$
 or  $\frac{s}{\sqrt{n}}$  when  $\sigma$  is unknown.

Standard error of the difference between two means

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

(xiii)

Standard error of a sample proportion

$$= \frac{\pi(1-\pi)}{n} \quad \text{or} \quad \frac{p(1-p)}{n} \quad \text{when } \pi \text{ is unknown.}$$

Index numbers

$$\frac{\Sigma wI}{\Sigma w}$$
 where  $w = Weight$ ;  $I = Price relative$ 

	Price	Quantity
Laspeyre	$\frac{\Sigma PnQo}{\Sigma PoQo} \times 100$	$\frac{\Sigma PoQn}{\Sigma PoQo} \times 100$
Paasche	ΣPnQn × 100 ΣPoQn	$\frac{\Sigma PnQn}{\Sigma PnQo} \times 100$

Compound interest  $Pn = Po(1 + i)^n$ 

Present value of an annuity:  $a_{n\bar{l}} = \frac{1 - (1 + i)^{-n}}{i}$ 

Sinking fund or future value of an annuity:  $s_{nl} = \frac{(1+i)^n - 1}{i}$ 

Discounted values

$$\frac{A_1}{(1+r)} + \frac{A_2}{(1+r)^2} + \frac{A_3}{(1+r)^3} + \dots \frac{A_n}{(1+r)^n}$$

#### Poisson probabilities

Y
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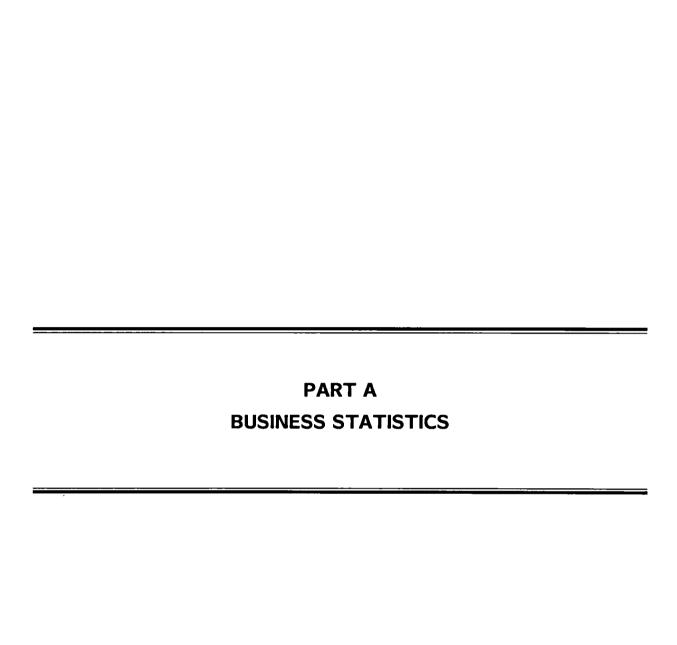
Mean m	0	I	2	3	4	5	6	7	8
0.1	0.9048	0.0905	0.0045	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
0.2	0.8187	0.1637	0.0164	0.0011	0.0001	0.0000	0.0000	0.0000	0.0000
0.3	0.7408	0.2222	0.0333	0.0033	0.0003	0.0000	0.0000	0.0000	0.0000
0.4	0.6703	0.2681	0.0536	0.0072	0.0007	0.0001	0.0000	0.0000	0.0000
0.5	0.6065	0.3033	0.0758	0.0126	0.0016	0.0002	0.0000	0.0000	0.0000
0.6	0.5488	0.3293	0.0988	0.0198	0.0030	0.0004	0.0000	0.0000	0.0000
0.7	0.4966	0.3476	0.1217	0.0284	0.0050	0.0007	0.0001	0.0000	0.0000
0.8	0.4493	0.3595	0.1438	0.0383	0.0077	0.0012	0.0002	0.0000	0.0000
0.9	0.4066	0.3659	0.1647	0.0494	0.0111	0.0020	0.0003	0.0000	0.0000
1.0	0.3679	0.3679	0.1839	0.0613	0.0153	0.0031	0.0005	0.0001	0.0000
1.1	0.3329	0.3662	0.2014	0.0738	0.0203	0.0045	0.0008	0.0001	0.0000
1.2	0.3012	0.3614	0.2169	0.0867	0.0260	0.0062	0.0012	0.0002	0.0000
1.3	0.2725	0.3543	0.2303	0.0998	0.0324	0.0084	0.0018	0.0003	0.0001
1.4	0.2466	0.3452	0.2471	0.1128	0.0395	0.0111	0.0026	0.0005	0.0001
1.5	0.2231	0.3347	0.2510	0.1255	0.0471	0.0141	0.0035	0.0008	0.0001
1.6	0.2019	0.3230	0.2584	0.1378	0.0551	0.0176	0.0047	0.0011	0.0002
1.7	0.1827	0.3106	0.2640	0.1496	0.0636	0.0216	0.0061	0.0015	0.0003
1.8	0.1653	0.2975	0.2678	0.1607	0.0723	0.0260	0.0078	0.0020	0.0005
1.9	0.1496	0.2842	0.2700	0.1710	0.0812	0.0309	0.0098	0.0027	0.0006
2.0	0.1353	0.2707	0.2707	0.1804	0.0902	0.0361	0.0120	0.0034	0.0009
2.1	0.1225	0.2572	0.2700	0.1890	0.0992	0.0417	0.0146	0.0044	0.0011
2.2	0.1108	0.2438	0.2681	0.1966	0.1082	0.0476	0.0174	0.0055	0.0015
2.3	0.1003	0.2306	0.2652	0.2033	0.1169	0.0538	0.0206	0.0068	0.0019
2.4	0.0907	0.2177	0.2613	0.2090	0.1254	0.0602	0.0241	0.0083	0.0025
2.5	0.0821	0.2052	0.2565	0.2138	0.1336	0.0668	0.0278	0.0099	0.0031
2.6	0.0743	0.1931	0.2510	0.2176	0.1414	0.0735	0.0319	0.0118	0.0038
2.7	0.0672	0.1815	0.2450	0.2205	0.1488	0.0804	0.0362	0.0139	0.0047
2.8	0.0608	0.1703	0.2384	0.2225	0.1557	0.0872	0.0407	0.0163	0.0057
2.9	0.0550	0.1596	0.2314	0.2237	0.1622	0.0940	0.0455	0.0188	0.0068
3.0	0.0498	0.1494	0.2240	0.2240	0.1680	0.1008	0.0504	0.0216	0.0081
3.1	0.0450	0.1397	0.2165	0.2237	0.1733	0.1075	0.0555	0.0246	0.0095
3.2	0.0408	0.1304	0.2087	0.2226	0.1781	0.1140	0.0608	0.0278	0.0111
3.3	0.0369	0.1217	0.2008	0.2209	0.1823	0.1203	0.0662	0.0312	0.0129
3.4	0.0334	0.1135	0.1929	0.2186	0.1858	0.1264	0.0716	0.0348	0.0148
3.5	0.0302	0.1057	0.1850	0.2158	0.1888	0.1322	0.0771	0.0385	0.0169
3.6	0.0273	0.0984	0.1771	0.2125	0.1912	0.1377	0.0826	0.0425	0.0191
3.7	0.0247	0.0915	0.1692	0.2087	0.1931	0.1429	0.0881	0.0466	0.0215
3.8	0.0224	0.0850	0.1615	0.2046	0.1944	0.1477	0.0936	0.0508	0.0241
3.9	0.0202	0.0789	0.1539	0.2001	0.1951	0.1522	0.0989	0.0551	0.0269
4.0	0.0183	0.0733	0.1465	0.1954	0.1954	0.1563	0.1042	0.0595	0.0298
4.1	0.0166	0.0679	0.1393	0.1904	0.1951	0.1600	0.1093	0.0640	0.0328

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