

MODELING and ANALYSIS of DYNAMIC SYSTEMS

Ramin S. Esfandiari
Bei Lu

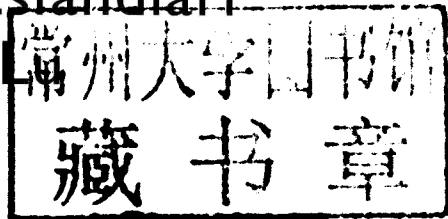


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MODELING
and **ANALYSIS** of
DYNAMIC
SYSTEMS

*To my wife Haleh, my sisters Mandana and Roxana, and my parents to whom
I owe it all.*

Ramin Esfandiari

To my husband Qifu and my parents.

Bei Lu

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Preface

The principal goal of this book is to provide the reader with a thorough knowledge of mathematical modeling and analysis of dynamic systems. MATLAB[®] and Simulink[®] are introduced at the outset and are utilized throughout the book to perform symbolic, graphical, numerical, and simulation tasks. The textbook, written at the junior level, meticulously covers techniques for modeling dynamic systems, methods of response analysis, and an introduction to vibration and control systems.

The book comprises 10 chapters and two appendices. Chapter 1 methodically introduces MATLAB and Simulink to the reader. The essential mathematical background is covered in Chapter 2 (complex analysis, differential equations, and Laplace transformation) and Chapter 3 (matrix analysis). Different forms of system model representation (state-space form, transfer function, input–output equation, block diagram, etc.) as well as linearization are discussed in Chapter 4. Each topic is also handled using MATLAB. Block diagrams are constructed and analyzed by using Simulink.

Chapter 5 treats translational, rotational, and mixed mechanical systems. The free-body-diagram approach is greatly emphasized in the derivation of the systems' equations of motion. Electrical and electromechanical systems are covered in Chapter 6. Also included are operational amplifiers and impedance methods. Chapter 7 discusses pneumatic, liquid-level, and thermal systems.

Chapter 8 deals with the time-domain and frequency-domain analysis of dynamic systems. Time-domain analysis entails the transient response of first-, second-, and higher-order systems. The sinusoidal transfer function (frequency response function) is introduced and utilized in obtaining the system's frequency response, as well as the Bode diagram. An analytical solution of the state equation is also included in this chapter. MATLAB and Simulink play significant roles in determining and simulating system response, and are used throughout the chapter.

Chapter 9 presents an introduction to vibrations and includes free and forced vibrations of single- and multiple-degree-of-freedom systems, vibration suppression including vibration isolators and absorbers, modal analysis, and vibration testing. Also included are some applications of vibrations: logarithmic decrement for experimental determination of the damping ratio, rotating unbalance, and harmonic base excitation.

Chapter 10 gives an introduction to control systems analysis and design in the time and frequency domains. Basic concepts and terminology are presented first, followed by stability analysis, types of control, root locus analysis, Bode plot, and full-state feedback. All these analytical techniques are implemented using MATLAB and Simulink.

APPENDICES

Appendix A gives a summary of systems of units and conversion tables. Appendix B contains useful formulas such as trigonometric identities and integrals.

EXAMPLES AND EXERCISES

Each covered topic is followed by at least one example for better comprehension of the subject matter at hand. More complex topics are accompanied by multiple, painstakingly worked-out examples. Each section of each chapter is followed by several exercises so that the reader can immediately apply the ideas just learned. End-of-chapter review exercises help in learning how a combination of different ideas can be used to analyze a problem.

CHAPTER SUMMARIES

Chapter summaries provide succinct reviews of the key aspects of each chapter.

INSTRUCTOR'S SOLUTIONS MANUAL

A solutions manual, featuring complete solution details of all exercises, is prepared by the authors and will be available to instructors adopting the book.

An ample portion of the material in this book has been rigorously class tested over the past several years. And the valuable remarks and suggestions made by students have greatly contributed to making this book as complete and user-friendly as possible.

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1 Introduction to MATLAB[®] and Simulink[®]

This chapter introduces some fundamental features of MATLAB[®] and Simulink[®]. These include the description and application of several commonly used built-in functions (commands) in MATLAB and the basics of building block diagrams for the purpose of simulation of dynamic systems using Simulink. MATLAB and Simulink are integrated throughout the book, and most of the functions utilized in the upcoming chapters depend on the basic ones presented here.

1.1 MATLAB COMMAND WINDOW AND COMMAND PROMPT

Once a MATLAB session is opened, commands can be entered at the MATLAB command prompt “>>” (Figure 1.1). For example,

```
>> sqrt(3) + 1
ans =
    2.7321
```

The outcome of a calculation can be stored under a variable name:

```
>> c = cos(pi/4)
c =
    0.7071
```

The result may be suppressed (not displayed) by using a semicolon at the end of the statement:

```
>> c = cos(pi/4);
```

Commands such as `sqrt` (square root) and `cos` (cosine of an angle in radians) are MATLAB built-in functions. Each of these functions is accompanied by a brief but sufficient description through the `help` command.

```
>> help sqrt
SQRT Square root.
SQRT(X) is the square root of the elements of X. Complex
results are produced if X is not positive.
See also sqrtm, realsqrt, hypot.
Overloaded methods:
    sym/sqrt
Reference page in Help browser
    doc sqrt
```

1.2 USER-DEFINED FUNCTIONS

User-defined M file functions and scripts may be created, saved, and edited in MATLAB using the `edit` command. For example, suppose we want to create a function (say, `eqline`) that returns

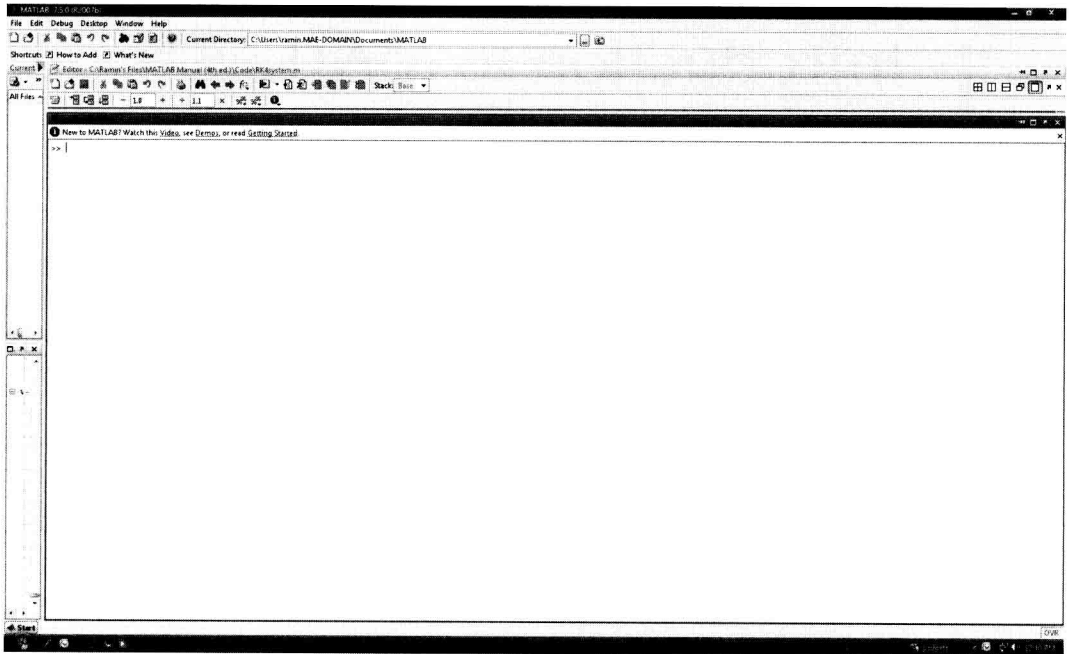


FIGURE 1.1 Screen capture of a MATLAB session.

the equation of a line passing through a point with a given slope. The function can be saved in a folder on the MATLAB path or in the current directory. The current directory can be viewed and/or changed using the drop-down menu at the top of the MATLAB command window. Once the current directory has been properly selected, type

```
>> edit eqline
```

This will open a new window where the function is constructed by typing the following code:

```
function y = eqline(x0, y0, m)
%
% EQLINE finds the equation of a line with slope m going through
% a given point (x0, y0).

syms x    % Declare x as a symbolic variable
y = y0 + m*(x-x0);
```

To execute this successfully, the current directory must be where the function was saved. And the three input arguments (x_0 , y_0 , m) must be supplied. For example, the equation of the line going through the point $(-1, 1)$ with slope 2 can be found as

```
>> y = eqline(-1, 1, 2)
y =
3+2*x
```