ADVANCED STRENGTH AND APPLIED ELASTICITY

THIRD EDITION

Ansel C. Ugural ▪ Saul K. Fenster

Advanced Strength and Applied Elasticity

Third Edition

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Preface to the Third Edition

This text is a development of classroom notes prepared in connection with advanced undergraduate and first-year graduate courses in elasticity and the mechanics of solids. It is designed to satisfy the requirements of courses subsequent to an elementary treatment of the strength of materials. In addition to its applicability to aeronautical, civil, and mechanical engineering and to engineering mechanics curricula, the authors have endeavored to make the text useful to practicing engineers. Emphasis is given to *numerical techniques* (which lend themselves to computerization) in the solution of problems resisting *analytical treatment*. The stress placed on numerical solutions is not intended to deny the value of classical analysis, which is given a rather full treatment. It instead attempts to fill what the authors believe to be a void in the world of textbooks.

An effort has been made to present a balance between the theory necessary to gain insight into the mechanics, but which can often offer no more than crude approximations to real problems because of simplifications related to geometry and conditions of loading, and numerical solutions, which are so useful in presenting stress analysis in a more realistic setting. The authors have thus attempted to emphasize those aspects of theory and application that prepare a student for more advanced study or for professional practice in design and analysis.

The theory of elasticity plays three important roles in the text: it provides exact solutions where the configurations of loading and boundary are relatively simple; it provides a check on the limitations of the mechanics of materials approach; and it serves as the basis of approximate solutions employing numerical analysis.

To make the text as clear as possible, attention is given to the presentation of the fundamentals of the mechanics of materials. The physical significance of the solutions and practical applications are given emphasis. The authors have made a special effort to illustrate important principles and applications with numerical examples. Consistent with announced national policy, problems are included in the text in which the physical quantities are expressed in the International System of Units (SI). All important quantities are defined in both SI and U.S. Customary System of units. A sign convention, consistent with vector mechanics, is employed throughout for loads, internal forces, and stresses. This convention conforms to that used in most classical strength of materials and elasticity texts, as well as to that most often employed in the numerical analysis of complex structures.

Because of the extensive subdivision into a variety of topics and the employment of alternative methods of analysis, the text should provide flexibility in the choice of assignments to cover courses of varying length and content. Most chapters are substantially self-contained. Hence, the order of presentation can be smoothly altered to meet an instructor's preference. It is suggested, however, that Chapters 1 and 2, which address the analysis of basic concepts, should be studied first. The emphasis placed on the treatment of two-dimensional problems in elasticity (Chapter 3) may differ according to the scope of the course.

This third edition of *Advanced Strength and Applied Elasticity* seeks to preserve the objectives and emphases of the previous editions. Every effort has been made to provide a more complete and current text through the inclusion of new material dealing with the fundamental principles of stress analysis: an essential review of mechanics of materials theory; three-dimensional stress and strain transformations; contact stresses; stress concentration; energy and variational methods extended to beams on elastic foundations and to plates; failure criteria; and the elastic–plastic analysis of thick-walled cylinders. Tables covering computer programs for principal stresses and area properties, deflection of beams, material properties, and conversion factors have been added to Appendixes B, C, and D.

The entire text has been reexamined and many major improvements have been made throughout by a process of elimination and rearrangement. Some sections have been expanded to improve on previous expositions. The references, provided as an aid to the student who wishes to pursue further certain aspects of a subject, have been updated and identified at the end of the text. We have resisted the temptation to increase the material covered except where absolutely necessary. However, it was considered desirable to add a number of illustrative examples and a large number of problems important in engineering practice and design. Most changes in subject-matter coverage were prompted by the suggestions of faculty familiar with earlier editions.

As before, it is hoped that we have maintained clarity of presentation, simplicity as the subject permits, unpretentious depth, an effort to encourage intuitive understanding, and a shunning of the irrelevant. In this context, as throughout, emphasis is placed on the use of fundamentals in order to build student understanding and an ability to solve the more complex problems. Answers to selected problems are given at the end of the book.

It is a particular pleasure to acknowledge the contributions of those who assisted the authors in the evaluation of the text. Thanks, of course, are due to the many readers who have contributed general ideas and to reviewers who have made detailed comments on previous editions. These particularly include the following: F. Freudenstein, Columbia University; R. A. Scott, University of Michigan; M. W. Wilcox and Y. Chan Jian, Southern Methodist University; C. T. Sun, University of Florida; B. Koplik and N. Levy, New Jersey Institute of Technology; H. Smith, Jr., South Dakota School of Mines and Technology; B. P. Gupta, Gannon University; S. Bang, University of Notre Dame; B. Koo, University of Toledo; J. T. Easley, University of Kansas; J. A. Bailey, North Carolina State University; W. F. Wright, Vanderbilt University; R. Burks, SUNY Maritime College; G. E. O. Widera, University of Illinois; R. H. Koebke, University of South Carolina; B. M. Kwak, University of Iowa; G. Nadig, Widener University; R. L. Brown, Montana State University; S. H. Advani, West Virginia University; E. Nassat, Illinois Institute of Technology; R. I. Sann, Stevens Institute of Technology; C. O. Smith, University of Nebraska; J. Kempner, Polytechnic University of New York; and P. C. Prister, North Dakota State University. We are deeply indebted to our colleagues who have found the text useful through the years and who have encouraged the preparation of this edition.

A. C. Ugural
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List of Symbols

A	area
b	width
C	carry-over factor, torsional rigidity
c	distance from neutral axis to outer fiber
D	distribution factor, flexural rigidity of plate
$[D]$	elasticity matrix
d	diameter, distance
E	modulus of elasticity in tension or compression
E_s	modulus of plasticity or secant modulus
E_t	tangent modulus
e	dilatation, distance, eccentricity
F	body force per unit volume, concentrated force
f_s	factor of safety
G	modulus of elasticity in shear
g	acceleration of gravity ($\approx 9.81\ m/s^2$)
h	depth of beam, height, membrane deflection, mesh width
I	moment of inertia of area, stress invariant
J	polar moment of inertia of area, strain invariant
K	bulk modulus, spring constant of an elastic support, stiffness factor, thermal conductivity, fatigue factor, strength coefficient
$[K]$	stiffness matrix of whole structure
k	constant, modulus of elastic foundation, spring constant, stress concentration factor
$[k]$	stiffness matrix of finite element
L	length, span
M	moment

M_{xy}	twisting moment in plates
m	moment caused by unit load
N	fatigue life (cycles), force
n	number, strain hardening index
l, m, n	direction cosines
P	concentrated force
p	distributed load per unit length or area, pressure, stress resultant
Q	first moment of area, heat flow per unit length, shearing force
$\{Q\}$	nodal force matrix of finite element
R	radius, reaction
S	elastic section modulus, shear center
r	radius, radius of gyration
r, θ	polar coordinates
s	distance along a line or a curve
T	temperature, twisting couple or torque
t	thickness
U	strain energy
U_o	strain energy per unit volume
U^*	complementary energy
V	shearing force, volume
v	velocity
W	weight, work
u, v, w	components of displacement
Z	plastic section modulus, curved beam factor
x, y, z	rectangular coordinates

α	angle, coefficient of thermal expansion, form factor for shear
β	numerical factor, angle
γ	shear strain, weight per unit volume or specific weight, angle
δ	deflection, finite difference operator, variational symbol, displacement
$\{\delta\}$	nodal displacement matrix of finite element
Δ	change of a function
ε	normal strain
θ	angle, angle of twist per unit length, slope
ν	Poisson's ratio
λ	axial load factor, Lamé constant
Π	potential energy
ρ	density (mass per unit volume), radius
σ	normal stress
τ	shear stress
ϕ	total angle of twist
Φ	stress function
ω	angular velocity
ψ	stream function

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CHAPTER 1

Analysis of Stress

1.1 INTRODUCTION

The basic structure of matter is characterized by nonuniformity and discontinuity attributable to its various subdivisions: molecules, atoms, and subatomic particles. Our concern in this text is not with the particulate structure, however, and it will be assumed that the matter with which we are concerned is *homogeneous* and *continuously* distributed over its volume. There is the clear implication in such an approach that the smallest element cut from the body possesses the same properties as the body. Random fluctuations in the properties of the material are thus of no consequence. This approach is that of *continuum mechanics*, in which solid elastic materials are treated as though they are continuous media, rather than composed of discrete molecules. Of the states of matter, we are here concerned only with the solid, with its ability to maintain its shape without the need of a container and to resist continuous shear, tension, and compression.

In contrast with rigid-body statics and dynamics, which treat the external behavior of bodies (that is, the equilibrium and motion of bodies without regard to small deformations associated with the application of load), the mechanics of solids is concerned with the relationships of external effect (forces and moments) to internal stresses and strains. Two different approaches used in solid mechanics are the *mechanics of materials* or *elementary theory* (also called the *technical theory*) and the *theory of elasticity*. The mechanics of materials focuses mainly on the more or less approximate solutions of practical problems. On the other hand, the theory of elasticity concerns itself largely with more mathematical analysis to determine the