PROCEEDINGS

INTERNATIONAL SYMPOSIUM ON NEW ASPECTS ON SHEET METAL FORMING (NASMF 1981)

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MAY 14 (Thursday) and 15 (Friday) 1981

THE IRON AND STEEL INSTITUTE OF JAPAN

THE JAPAN SHEET METAL FORMING RESEARCH GROUP

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This Symposium was first planned according to the kind advice of Dr. Pomey, Dr. Pearce, Dr. Parniere and the other friends in Europe. Last summer, Prof. Marciniak from Poland, Dr. Pearce from England, Dr. Jalinier from France and the members of the Japanese group got together in Tokyo. We discussed and decided the purpose and themes of the Symposium.

The purpose is to learn the past research again, analyze the present status, ---- thus after reviewing the already-established knowledge, finally forecast the social, economic, engineering and scientific problems in sheet properties and press forming in 1980s.

In particular, as such a periodic change in sheet steel and press forming has taken place in Japan almost every ten years after the World War Two, we are now at the transition point of sheet metal forming. I feel, it is especially important for us to make more active the mutual assistance between steel sheet properties and press forming techniques.

The IDDRG activities have played a very important rôle for the development of mutual understanding at the impact between steel and autobody forming technologies.

This year, the IDDRG Working Group meetings are held in Japan, so very many people who are very active in the first line have gathered together here. It is a very nice chance to exchange a valuable information with each other.

Taking this chance, we asked the outstanding scientists and engineers to give the lecture. For our requests, they kindly gave a wonderful reply, to complete such an interesting program today.

By the participation of outstanding speakers, chairmen and many of foreign participants, this Symposium is more fulfilled than we expected before.

We sincerely hope the proceedings of this Symposium will benefit the international exchange of scientific and technological knowledge as well as the renewing of our international friendship.

Kiyota Yoshida
Chairman of the Executive Committee

C O N T E N T S

GREETINGS

Takitsu MITSUI
Managing Director
The Iron and Steel Institute of Japan

Ladies and gentlemen:

On behalf of the Iron and Steel Institute of Japan, I would like to express my sincere gratitude to all participants and the leaders of the IDDRG and the Japan Steel Metal Forming Research Group which have given us the honor of co-sponsoring this meeting. One of the most important policies of our institute is to enhance and promote closer cooperation with steel users' technical staff, because we are now living in a highly advanced age when steel engineers are unable to develop their technology effectively without knowing what developments are going on in those of steel users. As the matter of fact researches in sheet metal forming technology are being carried out quite strenuously by steel makers as well as by steel users. Although this overlapping research activities are in principle mutually beneficial, there must be some catalystic agent to make them really beneficial。 In this point of view we admire a great deal of efforts which Dr. Yoshida and his colleagues are making in this country. In the same sense I also should like to pay great respect to the role played by the IDDRG on a world-wide basis. Consequently I am fully convinced that this meeting will turn out to be a great success for furthering close cooperation between steel makers and steel users on a national as well as on an international basis. Again, on behalf of our institute, I should like to express our warmest feeling of welcome and wish your stay here in Japan will remain a happy memory.

Recent Trends in the Fundamental Study of Material Plasticity in Europe

Bernard BAUDELET* and Jean-Michel JALINIER*

Among the different fields of research in material plasticity in Europe, the study of plastic damage is certainly one of the most important and original topics developped these last years. Both the fields of Mechanics and Physical Metallurgy were brought to bear in a combined effort to get a more complete and powerful description of the damage process and its influence on the plastic properties of materials.

This damage is characterized by the formation of voids. This presentation will be confined to the case of cold working where these voids initiate at inclusions or second phase particles, either by decohesion of the particle-matrix interface or by failure of the particle.

Recent studies show that the damage process not only controls the ductile fracture but also has a great influence from the very start of the plastic deformation of the material.

This paper will firstly discuss the theoretical concept of damage from the mechanical and physical points of view, secondly the different stages of the process will be analyzed in detail. Finally, the influence of the damage on the plastic properties will be presented in relation with industrial problems.

I. Particle effects

The presence of particles, such as inclusions or precipitates with a behavior different from the matrix leads to an inhomogeneous deformation. In the vicinity of a particle, the local stress and strain fields are different from the overall applied values. Various analysis of this phenomenon have been developped to assess the local values for given limit conditions.

A finite element calculation performed for a two-phase material gives a good representation of the non-homogeneity of the strain field (Fig. 1) /1/.

A first analytical approach has been made by considering the linear elastic problem /2/. The analysis has been completed in the case of a plastic deformation of the matrix

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keeping an elastic accommodation /3/. For a spherical inclusion in a homogeneous isotropic matrix, the following equation is obtained :

$$\underline{\Sigma} = \underline{\sigma} + \mu \left[\underline{\varepsilon}^{p} - \underline{E}^{p} \right] \qquad (1)$$

μ is the elastic shear modulus of the matrix,
$\underline{\Sigma}$, $\underline{E}^{p}$ the stress and plastic strain tensors in the particle,
$\underline{\sigma}$, $\underline{\varepsilon}^{p}$ the overall applied stress and plastic strain tensors.

Recent studies /4,5/ introduce the possibility of a plastic accommodation at the interface between the particle and the matrix. In fact, this assumption is more reasonable since a pure elastic accommodation often gives an unrealistically high accommodation stress. The relation obtained in this case for a spherical particle is :

$$\underline{\Sigma} = \underline{\sigma} + \underline{L} \left[\underline{\varepsilon}^{p} - \underline{E}^{p} \right] \qquad (2)$$

$\underline{L}$ is a fourth order tensor characteristic of the constitutive equation of the matrix.

The application of the general equation (2) is very difficult. A number of simplifications leads to the use of a scalar parameter $\alpha\ \mu$ instead of the tensor $\underline{L}$ $(o < \alpha < 1)$:

$$\underline{\Sigma} = \underline{\sigma} + \alpha\mu \left[\underline{\varepsilon}^{p} - \underline{E}^{p} \right] \qquad (3)$$

In this equation, the introduction of a plastic accommodation instead of an elastic accommodation reduces the shear modulus μ by the factor α. This factor is close to unity for very low strain and it rapidly decreases with the plastic strain. In a uniaxial tension applied stress field σ_1, the value of α is obtained by the relation :

$$\alpha = \left[1 + 1.5\ \mu\ \varepsilon_1^{p}\ /\ \sigma_1 \right]^{-1} \qquad (4)$$

Other analytic approaches have been proposed. Thus, the inhomogeneity of the strain field has been studied by considering the dislocations emitted to accommodate the incompatibility of the deformations in the vicinity of the particle. The dislocations considered in this approach are assumed to be continously distributed /6/. The stress Σ_1 applied by the pile up of dislocations on the particle in a plastic matrix of constitutive equation $\sigma_e = K\ \varepsilon_e^{n}$ (σ_e and ε_e are the equivalent applied stress and strain) is /7,8/ :

$$\Sigma_1 = K\ \varepsilon_1^{n} + 4\ \ell\tau\ /\ \sqrt{2}\ R \qquad (5)$$

τ is the shear stress between the matrix and the cylindrical pile up of dislocations loops ; ℓ, the length of this pile up ; R, the radius of the particle ; ε_1, the maximum principal tensile strain.

Recently a relation was proposed between the length of the pile up and the radius of the particle /8/ :

$$\ell / R = \left[(\sqrt{2}/4)\ (K/\tau)\right]^{1/(n+1)} \left[(1+1/n)\ \varepsilon_1\right]^{n/(n+1)} \qquad (6)$$

This structural approach leads to similar results as the previous macroscopic approach Furthermore, it gives a physical meaning to the plastic accommodation coefficient α.

Finally, upper bound methods are being developped to assess the strain field around an elliptical particle in a non-hardening plastic matrix /9/. The principal interest in this method is the possibility for a continuous variation of the particle behavior from the liquid case to the hard undeformable case.

II. Theoretical concept of damage

One aims at defining a mathematical parameter, tensor or scalar, which can describe the damage process.

The evolution of this parameter will certainly depend either on the applied stress or strain field and either on the stress rate or strain rate field. Two main approaches are actually being developped :

- A mechanical approach which consists in introducing a parameter affecting the stresses and (or) the strains applied to the damaged material in order to treat it as an equivalent undamaged material.
- A physical approach which consists in a model description of the material containing voids of a given size, shape and distribution.

II.1 - Mechanical approach

This approach is based upon the search for an imaginary undamaged material whose behavior is similar to that of the damaged material. The damage parameter is generally chosen such that it affects only the stresses. The two materials are compared upon reaching the same strain tensor.

Katchanov and Rabotnov /10,11/ were the first to introduce the concept of an effective area, $\tilde{S}$, smaller than the apparent area, S, in the damaged material :

$$\tilde{S} = S\ (1 - D) \qquad (7)$$

$\tilde{S}$ is different from the geometrical area $\hat{S}$ obtained by simply substracting the voids' area.

In a uniaxial loading it immediately leads to :

$$\tilde{\sigma} = (1 - D)^{-1}\ \sigma \qquad (8)$$

$\tilde{\sigma}$ and σ are respectively the effective and apparent stresses.

In a three-dimensional analysis, this concept can be extended to give : $\underline{\tilde{\sigma}} = \underline{\Delta}\ \underline{\sigma}$ (9)

$\underline{\Delta} = (1 - D)^{-1}$ is a second order tensor /12/.

The concept of effective stress can also be introduced by considering the elastic behavior of the undamaged material : $\underline{\sigma} = \underline{E}\ \underline{\varepsilon}^{e}$ (10) and $\underline{\sigma} = \underline{E_D}\ \underline{\varepsilon}^{e}$ (11) for a damaged one. $\underline{E}$ and $\underline{E_D}$ are the elastic tensors.

A pure material equivalent to the damaged one can be assumed to have the following behavior $\underline{\tilde{\sigma}} = \underline{E}\ \underline{\varepsilon}^{e}$ (12). From the relations (11) and (12), one obtains :

$$\underline{\tilde{\sigma}} = \underline{E}\ \underline{E}_D^{-1}\ \underline{\sigma} = \underline{\Delta}\ \underline{\sigma} \qquad (13)$$

$\underline{\Delta}$ now appears as a fourth order tensor /13,14/. For an isotropic damage behavior $\underline{\Delta}$ can be reduced to a scalar Δ such as : $\Delta = (1 - D)^{-1}$.

The principal interest for this mechanical approach lies in the possibility of describing both damaged and equivalent undamaged materials by the same functional F :

$$\varepsilon = F(\tilde{\sigma}, t) \text{ and } \varepsilon = F(\sigma, t) \text{ respectively} \qquad (14)$$

Another mechanical approach consists in affecting the constitutive equation by a damage function D^* /15/ : $\sigma_e = (1 - D^*)\ c_o \left[\varepsilon_o + \varepsilon_e\right] \left[1 + c_o(\varepsilon_o + \varepsilon_e)\right]^{-1} \sigma_o$ (15)

ε_o, c_o, σ_o are constant parameters.

The evolution of the damage function is assumed to be : $dD^*/d\varepsilon_e = D^{*2}\,p(k + r)^{i}\,\varepsilon_e$

σ_m is the hydrostatical stress ; $k = \sigma_m/\sigma_e$ is a constant for a given linear strain path ; p, i are material constants ; r is characteristic of a strain path along which no damage is developped.

II.2 - Physical approach

This approach consists in making a physical modelization of the voids existing in the material. These voids are assumed to be randomly distributed in the matrix and to be of the same shape and size at each stage. Different parameters can be calculated such as the probability of finding a certain concentration of voids in a region or, equivalently a certain local effective thickness reduction, the relationship between apparent and effective mean stresses and strains ...

The damage parameter introduced, depending on the applications can be, for example, the maximum local thickness reduction existing in a sheet or the parameters characteristic of the relation between apparent and effective mean stresses

II.2.1 - Determination of the local thinning /16,18/

This is of particular interest in the analysis of the plastic instability of the material since, in reality, its effective cross-section is non uniform. A local thinning is characterized by $(1 - D') = \hat{e}/e$ (16) ($\hat{e}$ and e are the effective (or real) and apparent thicknesses).

The distribution of the local thinnings is assessed by a statistical analysis of a material containing a random distribution of voids. The probability to find a certain defect D' for a given material is :

$$P(D') = \begin{bmatrix} \nu \\ \nu(1-D') \end{bmatrix} C_v^{\nu(1-D')} \left[1 - C_v\right]^{\nu D'} \qquad (17)$$

C_v mean volume fraction of voids ; $\nu = e/r$ ratio of the apparent thickness to the thickness dimension of the voids.

If the voids' growth equation is known, the statistical calculation can be performed at any instant of the straining. Figure 2 presents the classical shape of the probability curve where the most probable defect D'_m is presented. But necking and subsequently, rupture will occur not on the most probable defect but on interacting extensive local thinnings (i.e., thinnings relatively near each other). With a certain threshold probability of existing P_c. In deep drawing P_c is taken to be equal to 10^{-4} this probability leads to the value of a critical defect D'_c existing in the considered material. The critical defect D'_c will be used for plastic instability calculations later (section V.3)

II.2.2 - Geometrical relation between apparent and effective parameters /19,21/

In a region of void concentration C'_v, the cavities are supposed to be uniformly distributed at octahedron edges. The mean apparent stresses on the volume are given by :

$\sigma_i = F_i \left[\ell_j \ \ell_k\right]^{-1}$ (18) where ℓ_i represents the outside dimension of the considered volume in the i direction. The effective mean stress is given by :

$\hat{\sigma}_i = F_i \left[\ell_j \ \ell_k - 2\pi \ r_j \ r_k\right]^{-1}$ (19) where r_i represents the radial dimension of the voids in the i direction. One easily obtains from (18) and (19) :

$$\hat{\sigma}_i = \sigma_i \left[1 - D''_i\right]^{-1} \quad \text{with} \quad D''_i = 2\pi \ r_j \ r_k \ \ell_j^{-1} \ \ell_k^{-1} \qquad (20)$$

The mean effective strains can be defined with respect to the normal cross-section for an initially cubic element of dimension ℓ_o with spherical voids of radius r_o, assuming the incompressibility of the matrix

$$\hat{\varepsilon}_i = \ell n \left[(\ell_o^2 - 2\pi \ r_o^2) \ (\ell_j \ \ell_k - 2\pi \ r_j \ r_k)^{-1}\right] \quad \text{with} \quad \Sigma_i \ \hat{\varepsilon}_i = 0 \qquad (21)$$

The mean apparent strains are given by : $\varepsilon_i = \ell n \ \ell_o/\ell_i$

Finally it gives :

$$\hat{\varepsilon}_i = \varepsilon_i - 0.5 \ \ell n \left[(1 - D''_o)(1 - D''_i)(1 - D''_j)^{-1}(1 - D''_k)^{-1}\right]$$

$$\text{with } D''_o = 2\pi \ (r_o/\ell_o)^2 \qquad (22)$$

The relation between D''_o and the initial volume fraction of voids is given by :

$$D''_o = 2\pi\left[3 \ C'_{v_o}/16 \ \pi\right]^{2/3}$$

In a statistical approach /7/ the volume fraction of voids C'_{vo} in the plastic controlling zone is greater than the mean concentration C_{vo}. The probability of finding a concentration larger or equal to $C'_v = \beta C_v$ in a material containing a mean volume fraction C_v is given on figure 4 with respect to β. For a critical probability of 10^{-4}, the corresponding value of β is 7.

II.3 - Relation between the mechanical and the physical damage parameters

In the physical approach, both the stresses and the strains are affected by the damage parameter. For a matrix with a plastic constitutive equation ($\sigma_e = K\,\varepsilon_e^n$), the relation between the mechanical D parameter (8) and the physical D" parameter (20) for an isotropic damage can be established for a tensile test as follows :

$$\sigma(1 - D)^{-1} = K\,\varepsilon^n \qquad (23)$$

$$\sigma(1 - D'')^{-1} = K\left[\varepsilon - 0.5\,\ell n\,(1 - D''_o)\,(1 - D'')^{-1}\right]^n \qquad (24)$$

which leads to : $(1 - D) = (1 - D'')\,(1 - m)^n$ with $m > 0$ (25)

Thus, the definition of the damage parameters imposes on the mechanical parameter D values greater than those of the physical parameter D", in agreement with experiments where $D \sim (3 - 6)\,D''$.

The presented physical model considers only mean stresses in the material while the mechanical model is very sensitive to the stress distribution in the plastic regime. Subsequently, it will be possible to take into account the stress distribution in the physical model. On the other hand, one can take into account the statistical distribution of voids in the physical model, which is disregarded by the actual mechanical analysis.

III. Measurements of the damage

Different methods are actually used to measure the damage, or more precisely, numerical characteristic values for the amount of damage existing in the material. These methods can be related to a physical or a mechanical definition of the damage to give either an absolute or a relative value which can be a damage tensor or only a scalar parameter

III.1 - Metallographical observations

This method is the most direct assessment of the physical evolution of the damage. Optical observations can be carried out but in most cases, the scanning electron microscope is used ; fewer observations require the transmission electron microscope.

The parameters determined by this method are : the shape, size, distribution and volume fraction of the voids or (and) cracks ; the shape, size, distribution and chemical nature of the particles at the origin of the damage ; the damage mechanism by decohesion and (or) failure of the particles ; the stage of the damage : nucleation, growth or (and) coalescence.

This method appears to be very complete and to give a lot of important basic parameters. Unfortunately, it requires a great number of observations in order to obtain a reasonable accuracy on the numerical values /22/. For example, 300 observations at a magnification of 4000 on the same damaged material lead to a relative accuracy of about 20% on the estimated volume fraction of voids since the total volume of material tested is only 5.10^{-2} mm^3. The accuracy can be seen on figure 5 where the evolution of the surface fraction of voids is presented with respect to the strain for two A.K. steels deformed in a tensile test and in an equibiaxial stretching /22/.

III.2 - Relative density change measurements

The existence of voids affects the density of the material. Generally, the volume fraction of voids remains small in the order of 10^{-3}. In order to obtain a good absolute accuracy lesser than 5.10^{-5}, only relative density changes are determined by the Ratcliffe's technique /23/. Figure 6 shows the evolution of the density change of a deformed sample relative to the density of an undeformed one with respect to the strain for the same two A.K. steels deformed also in a tensile test and in an equibiaxial stretching /24/. This method gives a global measurement of the relative change of void volume fraction. The tested volume is about 250 mm^3 for a steel. One obtains a mean value for damage variations along the material of short wave lengths ($\sim 10^2$ µm) but local measurements can be made for variations of long wave lengths ($\sim 10^4$ µm). It must be noticed that the measurement is mainly a volume change since planar cracks lead to negligible density changes. The measured parameter is related to a physical description of the damage. It is obtained within a reasonable time.

III.3 - Elastic moduli measurements

The existence of voids or (and) cracks affects the distribution of the stresses in the material and this gives rise to an apparent elastic modulus in response to an applied stress. For instance, during unloading-loading cycles in uniaxial tension, the change in Young's modulus can be determined. Taking the initial Young's modulus E_o as reference, the instantaneous value is assumed to be $E_D = E_o(1 - D)$. The parameter D is in complete agreement with a mechanical definition of the damage. On figure 8 is presented the evolution of the parameter D deduced from experiments on tough pit copper /25/.

The change of other elastic parameters can be measured such as Poisson's coefficient One can then evaluate independant parameters (D_1, $D_2 = D_3$) of a diagonal damage tensor

The damage is only measured relative to the initial elastic loading. The method is quite long to be performed. Measurements are obtained on materials whose volumes are of the order of 20 mm^3.

One must note that this damage parameter measures everything that can affect the elastic moduli, not only defects such as voids and cracks but also induced phase transformations, cristalline texture changes, Bauschinger effects, etc ... Furthermore, the plastic damage is defined for an elastic test and it is well-known that plastic and elastic distributions are different.

III.4 - Other techniques

Hydrogen diffusion : the method consists in measuring the time taken by hydrogen to diffuse through a material from one end to the other. This length of time is longer for higher void volume fractions. But the diffusion is also affected by the dislocation configuration, the grain boundary structure ... Figure 8 presents the evolution of the diffusion time with respect to the strain for an A.K. steel deformed in uniaxial tension /26/.

Resistivity measurements : the change of resistivity of a material is followed during testing. This method is commonly used to characterize planar defects such as large cracks in fatigue.

Acoustic emission : this method seems only to be sensitive to the nucleation process of the damage and not on the damage evolution during deformation.

Positron annihilation : this method can be used to detect very small amounts of damage (nucleation and early stages of growth), undetectable by other techniques.

III.5 - Comparison of the different values

A simple model consisting in a pile of cubes containing either a void or a crack has been developped in order to fit with the damage values obtained from both Young's modulus changes and relative density changes /27/. Calculations using this model predict the number of introduced cracks to be 80 times greater than the number of voids. This result is in complete disagreement with the metallurgical observations on materials in which no cracks are detected. This discrepancy can be explained by the fact that the author of this model based his results on the fracture point where there is a severe strain gradient. A physical statistical approach of the phenomenon based on the uniform deformation regime leads to a better agreement between the two measured parameters.

On the determination of the periodic variation of the initial damage along the rolling direction of a sheet metal a comparison between hydrogen diffusion and relative density changes has been made. The results are in qualitative agreement and in particular, the same period is found for the phenomenon.

Finally, the comparison between quantitative metallography and density measurements is easily done (Fig. 9). The agreement between the two methods is very good. Furthermore, the comparison of the relative accuracy of the two methods is clearly shown /22/.

In conclusion, none of these methods is perfect. Each method often involves a lot of experimental effects which require corrections on the exact mathematical treatment of the damage if one is to use the corresponding model.

IV. Evolution of the damage

Three physical stages can be considered for the damage : nucleation, growth and coalescence. They can be totally separated but they can also overlap : nucleation can take place over a large range of strains while the first nucleated voids begin to grow, there can be nucleation but no growth ... But there is always coalescence at the final stage leading to ductile fracture. This fracture can occur before the perfect ductile phenomenon (D = 1) is reached for a critical value of D_ℓ lower than 1.

IV.1 - Nucleation of the damage

The nucleation of the damage occurs when a criterion based on the local values of the stresses and (or) strains is verified. Microscopical models for these local values based either on dislocation configurations or on continuum mechanics have been presented.

A simple model consists in considering a critical normal stress Σ_c to cause the decohesion at the interface (6) or the failure of the particle (8). The main problem in such a criterion lies in obtaining physical values of the critical stress.

To reach this critical stress, pile ups of dislocations have been considered /6,7/ This is suggested by the observed dislocation tangles around particles before decohesion /28/ as well as by the observed dislocation pile ups against fibers before fracture of the fibers in a composite material /29/. In order to assess the fracture stress on hard particles, an interaction mechanism has been proposed between pile ups of two adjacent particles (Fig. 10) /8/. The obtained stress has three physical components ; the overall applied stress ; the stress due to the pile up of dislocations ; an additive stress due to the constraint of the length of the pile up, i.e. more dislocations than in a free pile up for the same length. This criterion has been applied with success to the prediction of the nucleation of the damage by failure of the precipitates in a 3003 aluminium alloy /30/.

An original study of the nucleation process /31/ has been carried out by performing uniaxial tests on notched cylindrical specimens in which the internal states of stresses and strains have been determined by finite element calculations at each stage of deformation. The material contains equiaxe particles of uniform size. The limiting line separating regions with and without particle decohesion is determined by metallurgical observations at different stages of deformation. This border line is well defined by the following criterion of nucleation deduced from a continuum approach (1) /4/ : $\sigma_1 + k(\sigma_e - \sigma_o) = \Sigma_c$ (26)

where σ_1 is the maximum applied principal stress, σ_e is the equivalent stress, σ_o and k are constant parameters.

A similar model is obtained by considering that the density of dislocations in the plastic zone around the particles is proportional to the imposed strain. The imposed strain ε_N to achieve nucleation can be expressed as $\varepsilon_N^{1/2} = H\left[\Sigma_c - \sigma_m\right]$ (27)

σ_m is the hydrostatical stress. Data from the literature are in agreement with (27) and give a correct value of Σ_c for the Fe - Fe_3C system [32].

An energetic criterion has been proposed for nucleation : decohesion occurs when the plastic energy required to deform the matrix without particle decohesion is greater than the plastic energy required when decohesion occurs [2,33]. This criterion leads to a strong influence of the particle size. Figure 11 presents the predicted nucleation strain in uniaxial tension as a function of the dimensions of carbide particles in a mild steel. It must be noticed that this criterion often leads to decohesion in the elastic region for particles greater than 250 Å, in contradiction with metallurgical observations where particles ten times bigger are still stuck to the matrix at 5% plastic strain.

In conclusion, the nucleation of the damage is very difficult to assess. It requires information on the value of the critical stress and observations of very fine phenomena.

IV.2 - Damage growth

IV.2.1 - Mechanical equations

In a mechanical approach, a phenomenological equation is derived from experimental measurements [25,27]

$$dD = \left[(\sigma^* - \sigma_D)(\sigma_u - \sigma_D)^{-1}(1 - D)^{-1} \right]^s (\sigma_u - \sigma_D)^{-1} d\sigma^* \qquad (28)$$

with the conditions dD = 0 if $\sigma^* < \sigma_D$ or $d\sigma^* < 0$; $\sigma^* = 3h\,\sigma_m + (1 - h)\,\sigma_e$ is the damage equivalent stress ; s and h are coefficients characteristic of the material ; σ_D the nucleation stress related to this approach ; σ_u the ultimate stress.

For example, for tough pit copper deformed in tension, the parameters are found to be : h = 0 ; s = 0.7 ; σ_u = 775 MPa ; σ_D = 330 MPa

IV.2.2 - Physical equations

The two damage processes, decohesion and failure of the particle, must lead to different growth behaviors. Thus, for an A.K. steel damaged by decohesion, the amount of damage measured by relative density changes depends on the strain path (Fig. 12) [34] while for a 3003 alloy with damage by particle failure (Fig. 13) [30], it is relatively independent of the strain path. A physical approach of the damage growth has to differentiate between these two mechanisms.

- Damage by decohesion : two principal analyses have been carried out to determine the growth of a single cavity in a plastic matrix under a triaxial stress state : the growth of an infinite cylindrical void in a hardening matrix [35] and the growth of a spherical void in a non hardening matrix [36].

The first analysis leads to the following equations :

$$\ell n\, r/r_o = 0.5\,\sqrt{3}(1 - n)^{-1}\,\varepsilon_e\, sh \left[(\sqrt{3}/2)(1 - n)(\sigma_1 + \sigma_2)/\sigma_e + (\varepsilon_1 + \varepsilon_2)/2 \right] \qquad (29)$$