OPTIMAL DESIGN

THEORY and APPLICATIONS to MATERIALS and STRUCTURES

Edited by
Valery V. Vasiliev
Zafer Gürdal

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Valery V. Vasiliev

Corresponding Member of Russian Academy of Sciences
Professor and Head of Department
Mechanics and Optimization of Processes and Structures
MATI—Russian State University of Technology

Zafer Gürdal, Ph.D.

Professor
Departments of Aerospace and Ocean Engineering and
Engineering Science and Mechanics
Virginia Polytechnic Institute and State University

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PREFACE

Optimal structural design can be referred to as one of the most important and promising branches of applied mathematics and mechanics. The basic problem of optimal design is to construct a structure that satisfies a system of given constraints and provides the best quality and performance. Although this problem is quite natural and has been known for a long time, development of a consistent theory of optimal design has matured only recently. This delay is associated with three reasons. First, the most important applications of optimal design come from such modern fields of industry as the aerospace engineering. Second, actual problems of optimal design for complicated spatial structures can be efficiently solved only with the aid of modern analytical and numerical methods of applied mathematics and mechanics. And third, realization of optimal structures has become possible only with development of sophisticated manufacturing processes and computer-controlled machines.

Another important contribution to the theory and application of optimal structural design is associated with the maturity of modern composite material technology. Composite structures, as a rule, can efficiently work only having the optimal shape and material distribution corresponding to specified loading and operational conditions. Spreading use of composite materials in many fields and the need to provide maximum performance at a least cost and material usage provides an important stimulus to the acceptance of the methods of optimization in structural design.

Because of possible applications that allow us to modify existing structures and develop novel structural concepts with improved performance, optimal structural design is currently under intensive study in many countries. This book reflects the culmination of Russian activity in the field of optimal structural design. It consists of nine chapters, eight of which represent the recent Russian achievements in the theory and application of optimal structural design.

Chapter 1 is written by the editors; it presents the fields of their interest in structural optimization and contains general formulation of the design problem and discussions concerning the role of optimization in engineering and natural processes and presents methods of optimization that are derived from the natural phenomena. Chapter 2 is devoted to the formulation of the design problem and classical methods of structural and shape optimization. Chapter 3 contains a description of an efficient numerical method of optimization involving simultaneous variations of design and field variables. Chapter 4 presents practical approaches and methods of optimal design for airplane frame structures. Chapter 5 is concerned with the theory of multicriteria and multiparameter structural optimization, which is now under intensive study. Chapter 6 covers a specific problem of optimal design of laminates whose operational characteristics should be accomplished with a finite number of layers having specified properties. A local variation method is developed to solve this problem. Chapter 7 deals with optimal design of weight and cost-efficient lattice composite cylindrical shells, made by filament winding, which are currently considered for use in various aerospace structures. Optimal winding patterns for composite shells of revolution are studied in Chapter 8. Composite bars for space truss systems, as well as the general formulation of the design problem for composite structures, are discussed in Chapter 9.

The book chapters cover a wide class of theoretical and applied problems of optimal structural design. The book is designed to be used by specialists working in the fields of solid mechanics and structural design. It can also be useful for graduate students in engineering.

V. V. VASILIEV
Z. GÜRDAL

CONTENTS

CHAPTER 1

Optimal Structural Design

1.1. INTRODUCTION

Optimal design implies determination of the values of design parameters that control shape, materials properties, and dimensions of a structure which must meet a set of specified constraints and improve some measure of quality to achieve the best possible design. In general, optimal design is a natural part of the activities of any design engineer whose challenge is to develop a proper structure saving as many resources (material, energy, labor, etc.) as possible. However, in the majority of practical cases optimal design pursues a more specific objective with the aim of improving the operational characteristics of the structure. For aerospace applications, design for minimum weight/mass is formulated primarily to reach the best structural performance rather than to save the material. This chapter is devoted to the discussion of some general aspects of optimal design problems.

1.2. FORMULATION OF THE DESIGN PROBLEM

Typical formulation of a design problem includes a set of constraints and quality criteria or objective (merit) functions that should be maximized or minimized by a proper choice of design variables. In general the design variables directly determine the geometry and the properties of the struc-

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V. V. Vasiliev, Department of Mechanics and Optimization of Processes and Structures, Moscow State University of Aviation Technology, Moscow, Russia; Z. Gürdal, Departments of Aerospace and Ocean Engineering, and Engineering Science and Mechanics, Virginia Tech, Blacksburg, VA.

ture. However, one of the main features of many optimization problems for load-bearing structures is that the design variables do not appear explicitly in the set of constraints which typically describe the appropriate strength and stiffness requirements for the structure. Instead, such constraints are often written in terms of field variables, i.e., stresses, strains, and displacements which more often than not cannot be written explicitly in terms of the design variables (with the exception of a few simple design problems). In order to express the constraints in terms of the design variables we need to attack the equations of solid mechanics which, for realistic structures, can be typically solved only with numerical methods. To overcome this problem, two main approaches are used in optimal design of structures. The first approach is to use the so-called structural optimality criteria methods which replace the original problem with conditions that are described in terms of the field variables rather than the design variables. For example, the condition of minimum mass, which is expressed in terms of materials densities and geometric parameters of the structures, is often changed to the condition of uniform strength which includes stresses. Structural optimality criteria methods are described in detail elsewhere [1] and their implementation can be found in Chapters 2, 4, 7, and 9 of this book. Optimality criteria methods are usually efficient for relatively simple structures such as columns, beams, plates, and membrane shells for which analytical solutions for the field variables can be obtained in terms of the design variables. As mentioned earlier, this is not the case for complicated structures which are usually designed on the basis of the second approach which involves iterative numerical mathematical optimization process. There exist numerous iterative procedures [1] that search the optimal solution starting from some initial set of design variables. The general feature of all such methods is the necessity to calculate the stress-strain state of the structure at each step of the iteration process in order to check strength and stiffness constraints. Thus, the design problem, which is the inverse problem of mechanics, is reduced to a set of direct analysis problems. Despite its widespread application, this approach does not seem to be natural because the inverse problem is usually less complicated than the set of direct ones.

1.3. FORMULATION OF THE OPTIMALITY CRITERION

Consider for example the optimization problem of an isotropic homogeneous plate loaded with transverse pressure, p. The plate is to be designed to have a thickness distribution which provides either the maximum stiffness for a specified mass or the minimum mass for specified stiffness requirements. The governing equation for a plate of variable stiffness D can be written in the form

$$\frac{\partial^2}{\partial x^2}\left(D\frac{\partial^2 w}{\partial x^2}\right)+\nu\left[\frac{\partial^2}{\partial x^2}\left(D\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2}{\partial y^2}\left(D\frac{\partial^2 w}{\partial x^2}\right)-2\frac{\partial^2}{\partial x\partial y}\left(D\frac{\partial^2 w}{\partial x\partial y}\right)\right]$$

$$+2\frac{\partial^2}{\partial x\partial y}\left(D\frac{\partial^2 w}{\partial x\partial y}\right)+\frac{\partial^2}{\partial y^2}\left(D\frac{\partial^2 w}{\partial y^2}\right)=p$$

where $D = Eh^3/[12(1 - \nu^2)]$. With respect to the field variable, the deflection w, this governing equation is fourth order, while it is only second order with respect to the design variable, the bending stiffness D or the thickness h. So, the inverse problem of determining the thickness for specified plate deflection is indeed simpler than the direct problem of determining the deflection for a specified plate thickness. However, to solve the second order equation for a preassigned plate deflection, we need to insure that the plate will satisfy the imposed boundary conditions and the resulting equation will have a feasible solution, i.e., the plate thickness will not be negative. This means that, in contrast to the condition of minimum mass (being traditionally expressed in terms of the plate thickness) which specifies what we would like to have, the new formulation will provide the same final result and includes the entire plate characteristic specifying the behavior of the optimal structure. Thus, we have arrived at the central problem of the theory of optimal design which we have not constructed yet—the problem of formulation of the optimality criterion in terms of the field variables.

To illustrate the features of this formulation we will consider one more example. A laminated plate, shown in Figure 1.1, is loaded by in-plane forces N_x, N_y, and N_{xy} which are uniformly distributed along its edges. The laminate consists of k monothropic layers (layers that are capable of carrying loads only along the fiber direction) and has no transverse stiffness such that the i-th layer is characterized with its thickness h_i and the fiber orientation angle φ_i. For such layers the equilibrium equations can be written as

$$N_x = \sum_{i=1}^{k}\sigma_i h_i \cos^2\varphi_i \qquad N_y = \sum_{i=1}^{k}\sigma_i h_i \sin^2\varphi_i$$

$$N_{xy} = \sum_{i=1}^{k}\sigma_i h_i \sin\varphi_i \cos\varphi_i \tag{1.1}$$

where σ_i is the stress acting along the fiber direction in the i-th layer. Forces

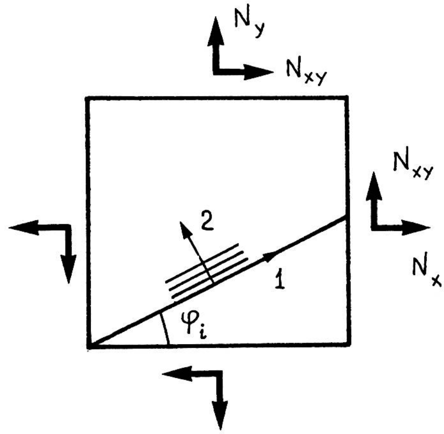

FIGURE 1.1. Reinforced plate in a plane stressed state.

N_x, N_y, and N_{xy} induce the plate strains ε_x, ε_y, and ε_{xy} which are assumed to be the same for all the layers. The plate strains can be transformed into the layer strains $\varepsilon_1^{(i)}$, $\varepsilon_2^{(i)}$, and $\varepsilon_{12}^{(i)}$ along the principal material coordinates of the layer 1, 2 (see Figure 1.1) with the aid of the following transformation relationships:

$$\varepsilon_1^{(i)} = \varepsilon_x \cos^2 \varphi_i + \varepsilon_y \sin^2 \varphi_i + \varepsilon_{xy} \sin \varphi_i \cos \varphi_i \tag{1.2}$$

$$\varepsilon_2^{(i)} = \varepsilon_x \sin^2 \varphi_i + \varepsilon_y \cos^2 \varphi_i - \varepsilon_{xy} \sin \varphi_i \cos \varphi_i \tag{1.3}$$

$$\varepsilon_{12}^{(i)} = (\varepsilon_y - \varepsilon_x) \sin 2\varphi_i + \varepsilon_{xy} \cos 2\varphi_i \tag{1.4}$$

From the first equation, Eq. (1.2), we can write the constitutive equation for the fibers of the i-th layer

$$\sigma_i = E\varepsilon_1^{(i)} = E(\varepsilon_x \cos^2 \varphi_i + \varepsilon_y \sin^2 \varphi_i + \varepsilon_{xy} \sin \varphi_i \cos \varphi_i) \tag{1.5}$$

where E is the modulus of elasticity, while Eqs. (1.3) and (1.4) specify transverse and shear strains which, in accordance with the assumed material model, are not accompanied by stresses. Equations (1.1), (1.2), and (1.3) readily yield the solution for the direct problem. Indeed, for a given set of design variables h_i and φ_i, substituting σ_i specified by Eq. (1.5) into Eqs. (1.1) we arrive at three equations for strains ε_x, ε_y, and ε_{xy}. Then the strains of all the layers can be found using Eqs. (1.2), (1.3), and (1.4); and Eq. (1.5) gives the stresses.

Consider two formulations of the design problem for the plate with minimum total thickness,

$$\min h = \sum_{i=1}^{k} h_i \tag{1.6}$$

According to traditional approach, we should minimize thickness, Eq. (1.6), with stress constraints of Eq. (1.5) and constraints enforcing Eqs. (1.1). To solve this problem we use Lagrange multipliers method according to which we should introduce multipliers λ and write the following augmented functional:

$$H = \sum_{i=1}^{k} h_i + \lambda_x \left(N_x - \sum_{i=1}^{k} \sigma_i h_i \cos^2 \varphi_i \right)$$

$$+ \lambda_y \left(N_y - \sum_{i=1}^{k} \sigma_i h_i \sin^2 \varphi_i \right) + \lambda_{xy} \left(N_{xy} - \sum_{i=1}^{k} \sigma_i h_i \sin \varphi_i \cos \varphi_i \right)$$

$$+ \sum_{i=1}^{k} \lambda_i [\sigma_i - E(\varepsilon_x \cos^2 \varphi_i + \varepsilon_y \sin^2 \varphi_i + \varepsilon_{xy} \sin \varphi_i \cos \varphi_i)]$$

Minimization of the augmented function with respect to the design variables h_i and φ_i yields

$$\sigma_i (\lambda_x \cos^2 \varphi_i + \lambda_y \sin^2 \varphi_i + \lambda_{xy} \sin \varphi_i \cos \varphi_i) = 1 \tag{1.7}$$

$$h_i \sigma_i [(\lambda_y - \lambda_x) \sin 2\varphi_i + \lambda_{xy} \cos 2\varphi_i]$$

$$= E\lambda_i [(\varepsilon_y - \varepsilon_x) \sin 2\varphi_i + \varepsilon_{xy} \cos 2\varphi_i] \tag{1.8}$$

Equation (1.8) has the following elementary solution:

$$\lambda_x = E\varepsilon_x \frac{\lambda_i}{h_i \sigma_i} \qquad \lambda_y = E\varepsilon_y \frac{\lambda_i}{h_i \sigma_i} \qquad \lambda_{xy} = E\varepsilon_{xy} \frac{\lambda_i}{h_i \sigma_i}$$

from which it can be concluded that the combination $\lambda_i/(h_i\sigma_i)$ does not depend on i. By substituting this solution into Eq. (1.7) we conclude that the ratio λ_i/h_i also does not depend on i, so that σ_i is constant, and the optimal structure corresponds to the structure of uniform strength. Comparing the result obtained with Eq. (1.5), we can further conclude that for the optimal structure $\varepsilon_x = \varepsilon_y$ and $\varepsilon_{xy} = 0$. In conjunction with Eq. (1.4) these equations yield $\varepsilon_{12}^{(i)} = 0$, so that the fibers in all the layers coincide with the directions of principal strains (for a more general model of material which allows for transverse and shear stresses in the layer, the fibers are directed along the trajectories of maximum principal stresses in the layers [2]).

Now assume that we have the formulation of the optimality criterion in terms of the field variables. According to this formulation, all the fibers should have the same stress and coincide with principal directions, so that

$$\sigma_\iota = \sigma \qquad \varepsilon_{12}^{i} = 0 \qquad (i = 1,2,3,\ldots,k) \tag{1.9}$$

Note that these requirements are quite natural for the structure under consideration and, in contrast to criterion in Eq. (1.6), are expressed in terms of the field variables. Then, summing up the first two equilibrium Eqs. (1.1) and taking into account Eq. (1.6) we obtain the following plate total thickness

$$h = \frac{1}{\sigma}(N_x + N_y) \tag{1.10}$$

Eliminating σ from Eqs. (1.1), we arrive at two optimality conditions in terms of the design variables, i.e.,

$$\begin{aligned} &\sum_{i=1}^{k} h_i (N_x \sin^2 \varphi_i - N_y \cos^2 \varphi_i) = 0 \\ &\sum_{i=1}^{k} h_i [(N_x + N_y) \sin \varphi_i \cos \varphi_i - N_{xy}] = 0 \end{aligned} \tag{1.11}$$

For laminated structures with k layers, the three optimality conditions in Eqs. (1.10) and (1.11) include $2k$ design variables h_i and φ_i, so for $k = 1$ the optimal structure can exist only for a special combination of loads because the set of equations is not consistent for the general loading. For $k \geq 2$ the optimal structure is not unique and can have $2k - 3$ free parameters for the same total thickness in Eq. (1.10). Note that this situation hinders the solution of the optimization problem based on criterion in Eq. (1.6) in conjunction with the iterative method discussed earlier because the result to be obtained will depend on the initial combination of design variables from which the iteration process starts. It should also be emphasized that the optimal combination of the field variables in Eq. (1.9) is the same for all variants of the optimal structure.

Thus, we can conclude that the optimality criterion formulated in terms of the field variables is more efficient and allows us to find the solutions for the optimization problems as inverse problems without the use of variational or iterative methods. Of course, it seems that derivation of such a criterion is not simpler than solving the optimization problem itself using the traditional methods. However, it should also be noted that the formulation of the optimality criterion in terms of the field variables is rather universal and, although derived for a simple problem, can be further used for more complicated ones. Indeed, conditions in Eq. (1.9) established for uniform statically determinate plane-stress problems can be used to find optimal shapes of filament wound composite shells, and optimal fiber patterns in reinforced spinning disks [2]. By now, such formulations exist only for relatively simple structures, i.e., for truss systems, membrane elements, and structures made of perfectly plastic material. By developing the formulation further, this approach may be used to replace existing cumbersome numerical iterative optimization methods by much more efficient analytical or numerical solutions of the inverse problems.

1.4. OPTIMAL SPACE

To proceed with the general formulation of the optimization problem, we return to the foregoing example of bending of a plate with variable thickness. The optimal solution for this plate has a well known solution which is remarkable because it is not continuous. Usually, the fact that the solution for a problem of mechanics of solids is not continuous or has singularities implies that the mathematical model of the structure is not quite adequate. For the problem under consideration, this solution shows that the optimal plate consists of a system of infinitely thin and infinitely high ribs. Accounting for shear deformation, which is ignored in the classical plate theory, or introduction of restrictions for the rib height (or the plate thickness) this solution can be made to look more realistic. Nevertheless, the optimal structure is rather far from a homogeneous plate and can hardly be described by equations of the plate theory.

Thus we can assume that, with respect to the formulation of the optimal-