
Advanced Level Physics

Fourth edition

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Preface to Fourth Edition

In this edition we have taken account of the revised syllabuses of the Examining Boards, including the new London Advanced level syllabus. Briefly, the main changes in the text are as follows:

Mechanics and Properties of Matter

- (i) Gravitation now includes an account of the potential and kinetic energy of satellites.
- (ii) In molecular theory, the variation of potential energy with molecular separation has been amplified, and properties of solids deduced.
- (iii) An account of the various types of bonds, mainly in solids, has been added.
- (iv) A discussion of dislocation and slip has been included in elasticity.

Heat

- (i) The first law of thermodynamics in the form of $\Delta Q = \Delta U + p.\Delta V$ has been applied to ideal gases.
- (ii) In the kinetic theory of gases, there is now an account of the maxwellian distribution of molecular speeds, mean free path, and viscosity and thermal conductivity.
- (iii) Electrical methods have been given prominence in specific heat capacity and specific latent heat, and a mechanical method added for specific heat capacity.
- (iv) In radiation, the non-equilibrium case has been considered.
- (v) The triple point and its determination have been discussed in thermometry.
- (vi) In accordance with the new syllabuses, the section on the thermal expansion of solids and liquids has been revised.

Geometrical Optics

There is now a more concise account of mirrors, a more direct treatment of lenses, and early consideration of refractor and reflector telescopes. Except for basic definitions, photometry has been omitted.

Waves

This section has been expanded in accordance with the new syllabuses. It contains a general treatment of (i) mechanical and electromagnetic waves; (ii) progressive and stationary waves; (iii) reflection, refraction, interference, diffraction and polarization of waves. Sound waves, and the measurement of their velocity in air, have been fully discussed.

Wave Optics

- (i) A qualitative account of the effect of lenses and mirrors on waves has been given.
- (ii) There are now separate chapters on interference, diffraction and polarization. The principles of holography and of the radio-telescope resolving power have been added, and the section on polarization has been expanded.

Sound

This section has been revised and now includes the case of the reflector in the Doppler effect and a brief account of high-fidelity reproduction.

Electricity

The electrostatics text now contains applications of the electrometer and d.c. amplifier, and the charge and discharge of a capacitor through a resistor. In current electricity: (i) a comparison of ohmic and non-ohmic conductors has been given; (ii) the classical electron theory has been extended; (iii) a fuller account of the e.m.f. of a thermocouple by the potentiometer, and of the absolute method of measuring resistance, have been added; (iv) the chapters on electrolysis and magnetic materials have been revised.

Atomic Physics

(i) The chapter on Electrons includes the cathode ray oscilloscope, and the triode valve is only utilised as an introduction to the field effect transistor. (ii) Junction diodes and Transistors now discusses rectifiers, amplifiers, oscillators, switches, logic gates, the multivibrator and the field effect transistor. (iii) In the section on Energy Levels, there is an account of spontaneous and stimulated emission and its application to the laser. (iv) The chapter on Radioactivity and the Nucleus contains a discussion of absorption by metals, more details of the Rutherford scattering law and the nuclear reactor, and an account of the neutron-proton ratio aspect of unstable nuclei.

Finally, two Multiple Choice papers have been added at the end to assist students in this type of examination, the Summary now contains only SI units, and the Exercises at the end of chapters have been revised to include more recent questions from Examining Boards acknowledged in the Third Edition.

The author is very much indebted to the following for their generous assistance with the new edition: Mrs J. Pope, formerly lecturer, Middlesex Polytechnic, for her valuable work and advice in planning the text; J. H. Avery, senior science master, Stockport Grammar School, for many valuable suggestions throughout the whole text; C. F. Tolman, senior science master, Whitgift School, Croydon, for constructive advice; M. V. Detheridge, senior physics master, William Ellis School, London, for reading many parts of the revised text; and for help throughout to D. Deutsch, Clare College, Cambridge; S. S. Alexander, formerly senior science master, Woodhouse School, London; J. Severn, William Ellis School, London; P. Betts, senior physics master, Woodhouse School, London. He is also grateful to R. Croft, The City University, London, and N. Phillips, Loughborough University, for providing photographs illustrating the section on holography, and to R. D. Harris, Ardingly College, Sussex. I am also indebted to Richard Gale and Trevor Hook of the Publishers for their unfailing courtesy and expert advice in the publication of this edition.

M.N.

Note to 1977 Reprint

In this reprint, the mechanism of thermal conduction in solids, a laboratory experiment in photoelectricity and an additional example on electron motion have been added, and some diagrams clarified and misprints amended. I am indebted to Dr. L. S. Julien, University of Surrey, S. S. Alexander, The Mount School for Girls, London, and M. V. Detheridge, William Ellis School, London, for helpful comments.

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Preface to First Edition

This text-book is designed for Advanced Level students of Physics, and covers Mechanics and Properties of Matter, Heat, Optics, and Sound. Electricity and Atomic Physics to that standard. It is based on the experience gained over many years of teaching and lecturing to a wide variety of students in schools and polytechnics.

In the treatment, an Ordinary Level knowledge of the subject is assumed. We have aimed at presenting the physical aspect of topics as much as possible, and then at providing the mathematical arguments and formulae necessary for a thorough understanding. Historical details have also been given to provide a balanced perspective of the subject. As a help to the student, numerous worked examples from past examination papers have been included in the text.

It is possible here to mention only a few points borne in mind by the authors. In Mechanics and Properties of Matter, the theory of dimensions has been utilized where the mathematics is difficult, as in the subject of viscosity, and the 'excess pressure' formula has been extensively used in the treatment of surface tension. In Heat, the kinetic theory of gases has been fully discussed, and the experiments of Joule and Andrews have been presented in detail. The constant value of $n \sin i$ has been emphasized in refraction at plane surfaces in Optics, there is a full treatment of optical treatment of optical instruments, and accounts of interference, diffraction and polarization. In Sound, the physical principles of stationary waves, and their applications to pipes and strings, have been given prominence. Finally, in Electricity the electron and ion have been used extensively to produce explanations of phenomena in electrostatics, electromagnetism, electrolysis and atomic physics; the concept of e.m.f. has been linked at the outset with energy; and there are accounts of measurements and instruments.

We acknowledge our gratitude to the following for their kindness in reading sections of the work before the complete volume was compiled: Mr. J. H. Avery, Stockport Grammar School; Dr. J. Duffey, formerly of Watford Technical College; Mr. J. Newton, The City University, London; Mr. A. W. K. Ingram, Lawrence Sheriff School, Rugby; Mr. O. C. Gay, College of Technology, Hull; Mr. T. N. Littledale, Gunnersbury Grammar School; Mr. C. R. Ensor, Downside School, Bath; Mr. L. S. Powell, Garnett College, London; Dr. D. W. Stops, The City University, London; and Professor H. T. Flint, formerly of London University.

Publisher's Note

Since the first publication of *Advanced Level Physics*, the revisions for reprints and new editions have been undertaken by Mr. Nelkon owing to the death of Mr. Parker.

Preface to Second and Third Editions

The following changes are made in these editions:

In the new text of the Electricity section, magnetic flux density B and electric intensity E are now used in discussions of magnetic and electric fields and their associated phenomena.

The subject of Waves is extended, and the treatment expanded, to cover matter and electromagnetic waves. In Optics, interference and diffraction are more fully discussed. In Heat, the joule is now used throughout as the unit of heat and the van der Waals' equation is emphasized. Intermolecular forces and their applications are treated further in Properties of Matter and there is a more extensive treatment of angular momentum and the dynamics of a rigid body in Mechanics.

The author is indebted to the following for their generous assistance: Professor L. Pincherle, Bedford College, London University, and Professor J. Yarwood, the Polytechnic of Central London, for their advice on atomic physics; Professor M. L. McGlashan, Exeter University, and M. Sayer, Keele University, for advice on SI units; M. V. Detheridge, William Ellis School, London, for his valuable co-operation in the preparation of the new electricity section; Rev. M. D. Phillips, O.S.B., Ampleforth College, Dr R. P. T. Hills and Dr S. Freake, formerly Cambridge University, S. S. Alexander, formerly Woodhouse School, London, and C. A. Boyle, formerly William Ellis School, for reading parts of the work; and C. J. Macdonald, St Paul's School, London, F. Anstis, Reed's School, Surrey, D. J. Behrens, formerly Roedean School, Sussex, G. Ulliyott, Charterhouse School, and L. G. Mead, Wellington School, Somerset, for constructive suggestions.

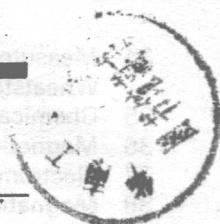
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1 Dynamics

PART ONE

Mechanics and Properties of Matter

We are concerned here with the motion of objects in a straight line. The most displacement is in a constant direction. For example, from 1 to 10 m. The displacement is 9 m. The rate of change of displacement is the velocity.

Velocity is a vector quantity. It has a magnitude and a direction. If an object is moving in a straight line, its velocity is constant. If it is not, its velocity is changing. If an object is moving in a straight line, its velocity is constant. If it is not, its velocity is changing.

If an object moving in a straight line has equal distances in equal times, no matter how small these distances may be, the object is said to be moving with uniform velocity. The velocity of a falling stone increases continuously, and so is a non-uniform velocity.

If, at any point of a journey, Δs is the small change in displacement in a small time Δt , the velocity v is given by $v = \Delta s / \Delta t$. In the limit, using calculus notation,

$$v = \frac{ds}{dt}$$

We call $\frac{ds}{dt}$ the instantaneous velocity at the time or place concerned. The term 'mean velocity' refers to both mean and instantaneous (see above).

Vectors

Displacement and velocity are examples of a class of quantities called vectors which have both magnitude and direction. They may therefore be represented in scale by a line drawn in a particular direction. Thus Cambridge is 50 km from London in a direction 20° E. of N. We can therefore represent the displacement between the two cities in magnitude and direction by a straight line 10 cm long 20° E. of N. where 1 cm represents 5 km. Fig. 1.1 (a). Similarly, we can



Fig. 1.1. (a) Displacement

1 Dynamics

Motion in a Straight Line. Velocity

If a car travels in a constant direction and covers a distance s in a time t , then its *mean* or *average velocity* in that direction is defined as s/t . It therefore follows that

$$\text{distance } s = \text{average velocity} \times t.$$

We are concerned here with motion in a constant direction. The term 'displacement' is given to the distance moved in a constant direction, for example, from L to C in Fig. 1.1 (i). Velocity may therefore be defined as the *rate of change of displacement*.

Velocity can be expressed in *metre per second* (m s^{-1}) or in *kilometre per hour* (km h^{-1}). By calculation, $36 \text{ km h}^{-1} = 10 \text{ m s}^{-1}$. We shall see shortly that if complete information about a velocity is required, we must state its direction: in addition to its magnitude.

If an object moving in a straight line travels equal distances in equal times, no matter how small these distances may be, the object is said to be moving with *uniform velocity*. The velocity of a falling stone increases continuously, and so is a *non-uniform velocity*.

If, at any point of a journey, Δs is the small **change** in displacement in a small time Δt , the velocity v is given by $v = \Delta s / \Delta t$. In the limit, using calculus notation,

$$v = \frac{ds}{dt}.$$

We call ds/dt the *instantaneous velocity* at the time or place concerned. The term 'mean velocity' refers to finite times and finite distances (see above).

Vectors

Displacement and *velocity* are examples of a class of quantities called *vectors* which have both magnitude and direction. They may therefore be represented to scale by a line drawn in a particular direction. Thus Cambridge is 80 km from London in a direction 20° E. of N. We can therefore represent the displacement between the cities in magnitude and direction by a straight line LC 4 cm long 20° E. of N., where 1 cm represents 20 km, Fig. 1.1 (i). Similarly, we can

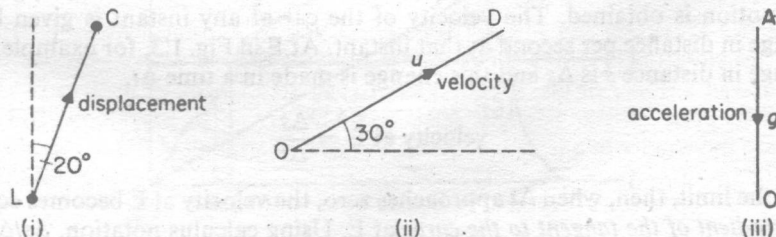


Fig. 1.1 Vectors.

represent the velocity u of a ball initially thrown at an angle of 30° to the horizontal by a straight line OD drawn to scale in the direction of the velocity u , the arrow on the line showing the direction, Fig. 1.1 (ii). The acceleration due to gravity, g , is always represented by a straight line AO to scale drawn vertically downwards, since this is the direction of the acceleration, Fig. 1.1 (iii). We shall see later that 'force' and 'momentum' are other examples of vectors.

Speed and Velocity

A car moving along a winding road or a circular track at 80 km h^{-1} is said to have a *speed* of 80 km h^{-1} . 'Speed' is a quantity which has no direction but only magnitude, like 'mass' or 'density' or 'temperature'. These quantities are called *scalars*.

The distinction between speed and velocity can be made clear by reference to a car moving round a circular track at 80 km h^{-1} say, Fig. 1.2. At every point on the track the *speed* is the same—it is 80 km h^{-1} . At every point, however, the *velocity* is different. At A, B or C, for example, the velocity is in the direction of the particular tangent, AP, BQ or CR, so that even though the magnitudes are the same, the three velocities are all different because they point in different directions. Generally, a vector quantity can be represented by a line drawn in the direction of the vector and whose length represents its magnitude.

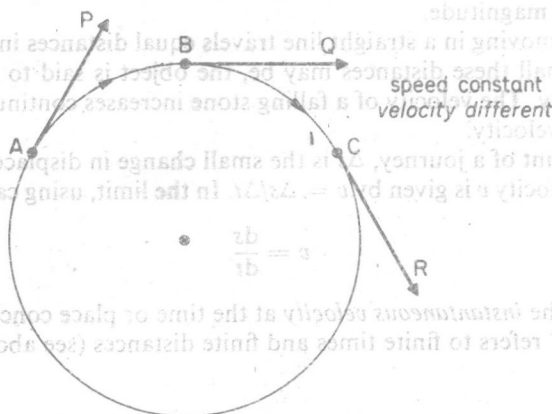


Fig. 1.2 Velocity and speed.

Distance-Time Graphs

When the displacement, or distance, s of a car moving in a constant direction from some fixed point is plotted against the time t , a *distance-time (s - t) graph* of the motion is obtained. The velocity of the car at any instant is given by the change in distance per second at that instant. At E in Fig. 1.3, for example, if the change in distance s is Δs and this change is made in a time Δt ,

$$\text{velocity at E} = \frac{\Delta s}{\Delta t}$$

In the limit, then, when Δt approaches zero, the velocity at E becomes equal to the *gradient of the tangent to the curve* at E. Using calculus notation, $\Delta s/\Delta t$ then becomes equal to ds/dt (p. 3). So the gradient of the tangent at E is the instantaneous velocity at E (p. 3).

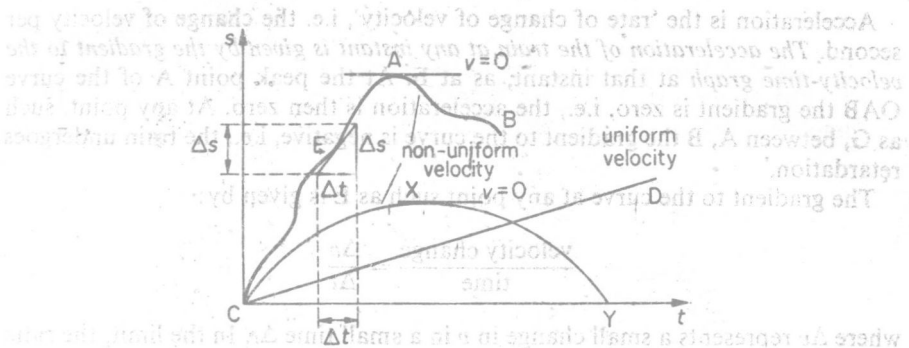


Fig. 1.3 Distance (s)-time (t) graphs (constant direction)

If the distance-time graph is a straight line OD , the gradient is constant at all points; it therefore follows that the car is moving with a **uniform** velocity, Fig. 1.3. If the distance-time graph is a curve CAB , the gradient varies at different points. The car then moves with **non-uniform** velocity. We may deduce that the velocity is zero at the instant corresponding to A , since the gradient at A of the curve CAB is zero.

When a ball is thrown upwards, the height s reached at any instant t is given by $s = ut - \frac{1}{2}gt^2$, where u is the initial velocity and g is the constant equal to the acceleration of free fall (p. 8). The graph of s against t is represented by the parabolic curve CXY in Fig. 1.3; the gradient at X is zero, illustrating that the velocity of the ball at its maximum height is zero.

Velocity-Time Graphs

When the velocity of a moving train is plotted against the time, a 'velocity-time (v - t) graph' is obtained. Useful information can be deduced from this graph, as we shall see shortly. If the velocity is uniform, the velocity-time graph is a straight line parallel to the time-axis, as shown by line (1) in Fig. 1.4. If the train accelerates uniformly from rest, the velocity-time graph is a straight line, line (2), inclined to the time-axis. If the acceleration is not uniform, the velocity-time graph is curved. In Fig. 1.4, the velocity-time graph OAB represents the velocity of a train starting from rest which reaches a maximum velocity at A , and then comes to rest at the time corresponding to B ; the acceleration and retardation are both not uniform in this case.

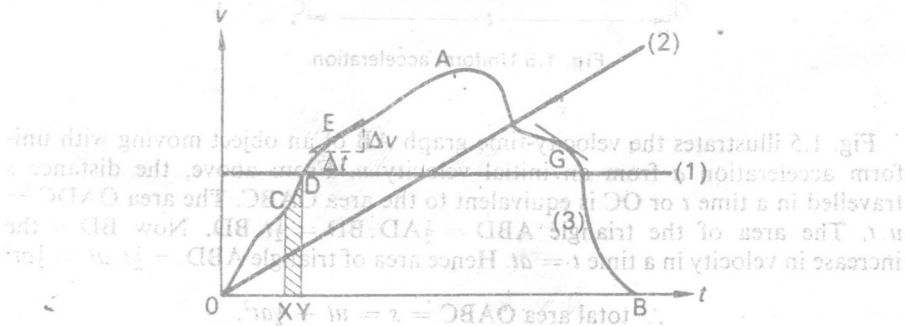


Fig. 1.4 Velocity (v)-time (t) curves.

Acceleration is the 'rate of change of velocity', i.e. the change of velocity per second. The acceleration of the train at any instant is given by the gradient to the velocity-time graph at that instant, as at E. At the peak point A of the curve OAB the gradient is zero, i.e., the acceleration is then zero. At any point, such as G, between A, B the gradient to the curve is negative, i.e., the train undergoes retardation.

The gradient to the curve at any point such as E is given by:

$$\frac{\text{velocity change}}{\text{time}} = \frac{\Delta v}{\Delta t}$$

where Δv represents a small change in v in a small time Δt . In the limit, the ratio $\Delta v/\Delta t$ becomes dv/dt , using calculus notation.

Area between Velocity-Time Graph and Time-Axis

Consider again the velocity-time graph OAB in Fig. 1.4, and suppose the velocity increases in a very small time-interval XY from a value represented by XC to a value represented by YD. Since the small distance travelled = average velocity \times time XY, the distance travelled is represented by the area between the curve CD and the time-axis, shown shaded in Fig. 1.4. By considering every small time-interval between OB in the same way, it follows that the total distance travelled by the train in the time OB is given by the area between the velocity-time graph and the time-axis. This result applies to any velocity-time graph, whatever its shape.

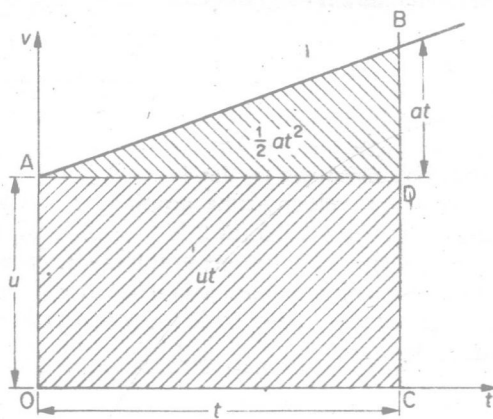


Fig. 1.5 Uniform acceleration.

Fig. 1.5 illustrates the velocity-time graph AB of an object moving with uniform acceleration a from an initial velocity u . From above, the distance s travelled in a time t or OC is equivalent to the area OABC. The area OADC = $u.t$. The area of the triangle ABD = $\frac{1}{2}AD.BD = \frac{1}{2}t.BD$. Now BD = the increase in velocity in a time $t = at$. Hence area of triangle ABD = $\frac{1}{2}t.at = \frac{1}{2}at^2$

$$\therefore \text{total area OABC} = s = ut + \frac{1}{2}at^2$$

This result is also deduced on p. 7.

Acceleration

The *acceleration* of a moving object at an instant is the *rate of change of its velocity* at that instant. In the case of a train accelerating steadily from 36 km h^{-1} (10 m s^{-1}) to 54 km h^{-1} (15 m s^{-1}) in 10 seconds, the uniform acceleration

$$= (54 - 36) \text{ km h}^{-1} \div 10 \text{ seconds} = 1.8 \text{ km h}^{-1} \text{ per second,}$$

or

$$(15 - 10) \text{ m s}^{-1} \div 10 \text{ seconds} = 0.5 \text{ m s}^{-1} \text{ per second.}$$

Since the time element (second) is repeated twice in the latter case, the acceleration is usually given as 0.5 m s^{-2} .

In terms of the calculus, the acceleration a of a moving object is given by

$$a = \frac{dv}{dt}$$

where dv/dt is the velocity change per second.

Distance travelled with Uniform Acceleration. Equations of Motion

If the velocity changes by equal amounts in equal times, no matter how small the time-intervals may be, the acceleration is said to be *uniform*. Suppose that the velocity of an object moving in a straight line with uniform acceleration a increases from a value u to a value v in a time t . Then, from the definition of acceleration,

$$a = \frac{v - u}{t},$$

from which

$$v = u + at \quad (1)$$

Suppose an object with a velocity u accelerates with a uniform acceleration a for a time t and attains a velocity v . The distance s travelled by the object in the time t is given by

$$s = \text{average velocity} \times t$$

$$= \frac{1}{2}(u + v) \times t$$

But

$$v = u + at$$

$$\therefore s = \frac{1}{2}(u + u + at)t$$

$$\therefore s = ut + \frac{1}{2}at^2 \quad (2)$$

If we eliminate t by substituting $t = (v - u)/a$ from (1) in (2), we obtain, on simplifying,

$$v^2 = u^2 + 2as \quad (3)$$

Equations (1), (2), (3) are the equations of motion of an object moving in a straight line with uniform acceleration. When an object undergoes a uniform *retardation*, for example when brakes are applied to a car, a has a *negative* value.

Examples

1. A car moving with a velocity of 54 km h^{-1} accelerates uniformly at the rate of 2 m s^{-2} . Calculate the distance travelled from the place where acceleration began to that where the velocity reaches 72 km h^{-1} , and the time taken to cover this distance.

(i) $54 \text{ km h}^{-1} = 15 \text{ m s}^{-1}$, $72 \text{ km h}^{-1} = 20 \text{ m s}^{-1}$, acceleration $a = 2 \text{ m s}^{-2}$.

Using

$$\begin{aligned} v^2 &= u^2 + 2as, \\ \therefore 20^2 &= 15^2 + 2 \times 2 \times s \\ \therefore s &= \frac{20^2 - 15^2}{2 \times 2} = 43\frac{1}{4} \text{ m.} \end{aligned}$$

(ii) Using

$$\begin{aligned} v &= u + at \\ \therefore 20 &= 15 + 2t \\ \therefore t &= \frac{20 - 15}{2} = 2.5 \text{ s.} \end{aligned}$$

2. A train travelling at 72 km h^{-1} undergoes a uniform retardation of 2 m s^{-2} when brakes are applied. Find the time taken to come to rest and the distance travelled from the place where the brakes were applied.

(i) $72 \text{ km h}^{-1} = 20 \text{ m s}^{-1}$, and $a = -2 \text{ m s}^{-2}$, $v = 0$.

Using

$$\begin{aligned} v &= u + at \\ \therefore 0 &= 20 - 2t \\ \therefore t &= 10 \text{ s} \end{aligned}$$

(ii) The distance, s , $= ut + \frac{1}{2}at^2$.

$$\therefore s = 20 \times 10 - \frac{1}{2} \times 2 \times 10^2 = 100 \text{ m.}$$

Motion Under Gravity

When an object falls to the ground under the action of gravity, experiment shows that the object has a constant or uniform acceleration of about 9.8 m s^{-2} or 10 m s^{-2} approximately, while it is falling. The numerical value of this acceleration is usually denoted by the symbol g . Suppose that an object is dropped from a height of 20 m above the ground. Then the initial velocity $u = 0$, and the acceleration $a = g = 10 \text{ m s}^{-2}$ (approx.). Substituting in $s = ut + \frac{1}{2}at^2$, the distance fallen s in metres is calculated from

$$s = \frac{1}{2}gt^2 = 5t^2.$$

When the object reaches the ground, $s = 20 \text{ m}$.

$$\therefore 20 = 5t^2, \text{ or } t = 2 \text{ s.}$$

Thus the object takes 2 seconds to reach the ground.

If a cricket-ball is thrown vertically upwards, it slows down owing to the attraction of the earth. The ball is thus retarded. The magnitude of the retardation is 9.8 m s^{-2} , or g . Mathematically, a retardation can be regarded as a negative acceleration in the direction along which the object is moving; and hence $a = -9.8 \text{ m s}^{-2}$ in this case.

Suppose the ball was thrown straight up with an initial velocity, u , of 30 m s^{-1} . The time taken to reach the top of its motion can be obtained from the equation $v = u + at$. The velocity, v , at the top is zero; and since $u = 30 \text{ m s}^{-1}$ and $a = -9.8$ or 10 m s^{-2} (approx.), we have

$$\begin{aligned} 0 &= 30 - 10t \\ \therefore t &= \frac{30}{10} = 3 \text{ s} \end{aligned}$$