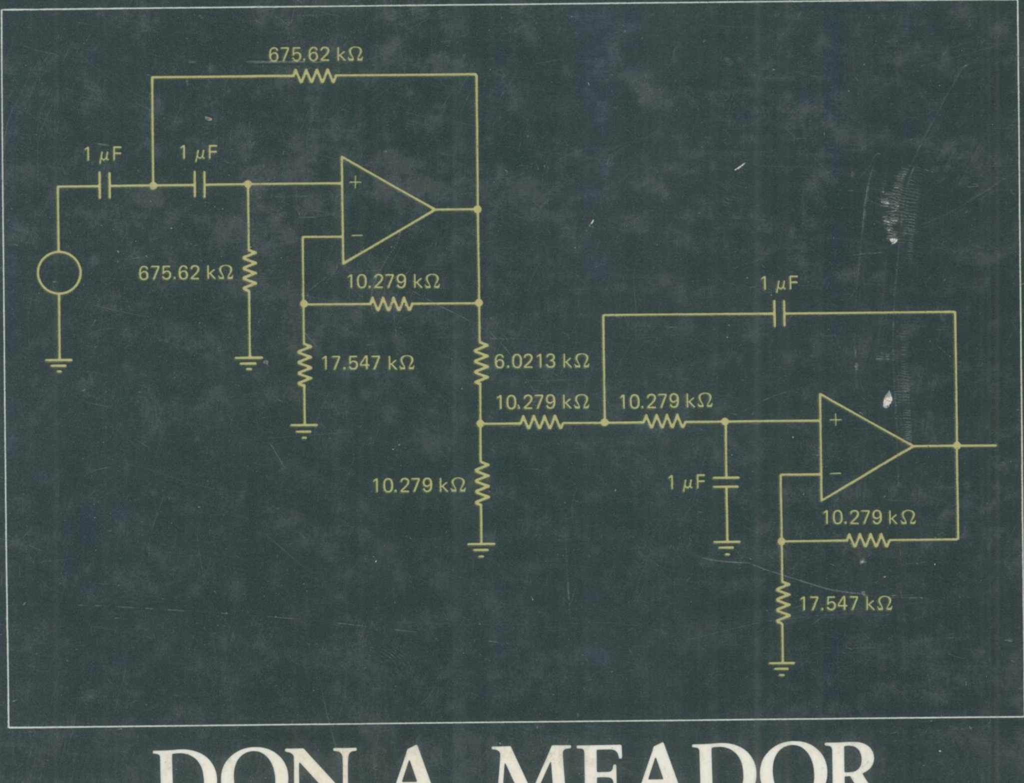
LAPLACE CIRCUIT ANALYSIS AND ACTIVE FILTERS



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LAPLACE CIRCUIT ANALYSIS AND ACTIVE FILTERS

to Joan, Angie, and Abbie

Preface

The student is assumed to have a working knowledge of dc and ac steady-state time-domain circuit analysis, and to have had a beginning course in calculus. (Since calculus is not crucial to understanding the book, it is possible to skip around those parts without adverse problems.) The student should also have a working knowledge of op-amps for the filter section.

The book is intended for either engineers or technologists. The material is appropriate for a one- or two-semester course in a mid-level engineering or an upper-level technology program.

Laplace transforms are presented as a circuit analysis tool rather than as a math course. Although the mathematical equations are included, the proofs and theorems are minimized, and their use and applications are emphasized.

The book is self-contained, so other sources are unnecessary when going into greater depth than the text presentation—advanced equation and derivations are given in this book. This makes it possible to teach at many levels. The chapters do not go through long dissertation on the equations, but are thorough in examples.

Special attention to the details of how to work a problem are used to eliminate a weakness found in other books. Most texts give you an idea of how to work a problem, but avoid showing the details of more complex problems.

Some textbooks show several ways to work special case problems. This book uses techniques that apply equally well to typical and special case problems. For example, the derivative of $tu(t)$ and $u(t)$ in most texts are handled as uniquely different problems. Here the derivatives are handled in the same way for both cases.

A major goal of this book is to use a minimum of formulas that will apply to a maximum of cases. The student should not be barraged with special cases when first confronted with Laplace transforms. The student should have a holistic understanding of all cases.

In this text a new teaching/learning technique is used. The different topics are approached in a similar or parallel manner. This allows the student to apply techniques learned on one topic to another possibly unrelated topic. This is most clearly seen in the approach of complex waveforms in Chapter 2 and Bode plots in Chapter 5.

We may call this the "parallel learning curve" method. This method significantly reduces the learning curve. Instead of starting a new learning curve with each new topic, we use part of the previous learning curve on the next topic. With the explosion of information we must somehow become capable of learning more information faster. This method allows us to do that.

The inverse Laplace transform in this book has a new formula that eliminates the need for extensive inverse tables. This formula finds the inverse of multiple-order complex-conjugate roots. This finally makes it possible to write a computer program to find the inverse Laplace of any practical electronic circuit.

Following is a brief summary of the book.

Chapter 1 is a brief overview of the book and some study hints.

Chapter 2 defines complex waveforms. It goes step by step from the basic parts of waveforms with discontinuities to complex waveforms. This chapter also prepares the student for Bode plots in Chapter 5 since both chapters use the same technique.

Chapter 3 covers the Laplace and the inverse Laplace transform. The Laplace transform technique in this book makes extensive use of Laplace operations, reducing the amount of material to memorize. Only equations that work on the general and special cases are used. This chapter is unique due to the development of a new inverse transform formula. This formula makes it possible to find the inverse Laplace of any practical electronic problem with only two techniques.

Chapter 4 is written to show the student how to use Laplace transforms to solve electronic circuits. It is organized so that the student will be given a review of circuit analysis at the same time. Most texts hop around to different topics of circuit analysis as if trying to humiliate the student into reading other texts to brush up on the topics. Here the circuit analysis rules are clearly stated and used in the examples. Even the very basics of circuit analysis (what is series and parallel, Ohm's law, etc.) are included in an appendix.

Chapter 5 is written so that the drawing of Bode plots are similar to the way complex waveforms are drawn in Chapter 2. This reduces the difficulty of teaching this topic since the student will have an insight to the drawing technique. Most other texts use the gosh-and-by-golly technique to find x-axis crossing of these graphs. In this book everything has a formula to calculate any point on the Bode plot. This allows the student to start seeing the construction of Bode plots as a whole instead of as a collection of special cases. The construction of the Bode plot for breaks occurring at the same frequency has been simplified and explained so that the student will not be in the dark on this subject any more.

Chapter 6 is an introduction to filters which takes the student from the general use of Laplace transforms to a specific use. This chapter builds on Chapter 5 to show how frequency plots can be used to define filter specifications. The different types of filters (LP, HP, BP, and notch) are defined. Also, in this chapter the approximate op-amp is defined to refresh the student's memory before the op-amp is used as an active filter.

Chapter 7 show the student how to build normalized low-pass filters. First the admittance Laplace transfer function is found from basic topologies. The admittance transfer functions are then transformed to circuit values using coefficient matching. At this point the Butterworth, Chebyshev, and elliptic filters are defined. Here, unlike other textbooks, the Butterworth filter is normalized in the same way as the Chebyshev and elliptic. This makes frequency shifting of all three filters the same process.

Chapter 8 shows the student how to shift the normalized filter's frequency, gain, and impedance to make a practical filter. This chapter also shows how to design high-pass, band-pass, and notch filters by transforming the equations in Chapter 7.

An IBM compatible disk that is described in Appendix G can be obtained through:

Millennial Marketing
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P.O. Box 2123
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I would like to thank my colleagues and friends for their encouragement and help, my students for what they taught me about learning, but most of all my thanks to my wife, daughters, and family for their unquestioning understanding and support in the long hours required to create this child.

Don A. Meador

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CHAPTER 1

Introduction

1-1 WHAT'S IN THIS BOOK

This book covers the mathematical tools required to work electronic circuits having complex waveforms and reactive components. At the center of these mathematical tools are Laplace transforms. Using Laplace transforms, we are able to analyze a circuit having complex waveforms at multiple frequencies for the complete response (from $t = 0$ to $t = \infty$). In addition, these tools apply to steady-state responses of single-frequency sinusoidal circuits.

The first half of the book is devoted to learning these mathematical tools, the second half to applying them to active filters. In Chapter 2 we will learn how to describe complex waveforms. These complex waveforms will not be emphasized in later chapters, so the mathematical tools will be clearly seen, but the complex waveforms will apply equally. Although we are learning Laplace transforms, we will be using impractical circuits simply to learn the techniques—playing games with electronic circuits. In the second half we apply these tools to active filter design.

1-2 LAPLACE TRANSFORMS

The Laplace transform is a technique that transforms a differential equation from the time domain (equations expressed as a function of time) to the s-domain (equations expressed as a function of a complex variable s). This makes the process of solving the differential equation an algebraic process. Solving a differential equation is somewhat difficult, and in electronics, simultaneous differential equations are common. When the process is algebraic, simultaneous equations become much easier to solve.

In the beginning, for electronics, it is not important to understand exactly what Laplace transforms are. We are more interested in how to use the tool than how the tool was "developed and manufactured." However, when Laplace transforms are introduced in the following chapters, we begin with how the

Laplace transforms are derived but we should not be overly concerned with this. Understanding Laplace transforms is much easier when we know how they are used.

1-3 NOTATIONS

In this book we must be very careful to recognize function notation. We will be concerned primarily with two types of functions, $f(t)$ and $F(s)$. These are read "f of t" and "F of s." They are not "f times t" or "f times s," but represent a function of time and a function of s. These two functions are the same except that one is in the time domain and the other is in the s-domain (or Laplace domain). The functions "$f(t)$ and $F(s)$" and "$h(t)$ and $H(s)$" are generic names and do not refer to a specific function. The function "$e(t)$ and $E(s)$" usually refers to a source voltage, and the function "$v(t)$ and V(s)" usually refers to a voltage drop across a component.

The typical electronic notations are used in this book. These notations are typical of most textbooks, but some may seem unusual, depending on the texts used for basic circuit analysis. We should use the notations presented in this book while learning the technique to avoid confusion and the use of "translation sheets."

1-4 CALCULATIONS

An important aspect of this book is the way the examples are calculated. Full computer/calculator accuracy is used to calculate the answers, rounded to five significant digits. If an intermediate result is shown, subsequent calculation will be based on the full computer/calculator number rather than the five significant digits shown. For calculations based on tabulated values, only the values in the tables will be used, but after that point full computer/calculator accuracy will be maintained.

We should develop the habit of carrying the full-digit accuracy of our computer/calculator to prevent becoming obsolete in our accuracy. As time progresses, the accuracy of manufactured components increases and the need for carrying more significant digits in our calculations increases. If we learn to carry the full accuracy of our computer/calculator, our calculations will never become obsolete.

1-5 HOW TO STUDY

The purpose of this book is learning. Therefore, this is the most important section in the book. If we fail here, we will learn nothing in the following chapters. The rules are simple to follow, but must be followed consistently. Learning Laplace transforms enough to work the problems in this book means very little. The main idea is to know Laplace transforms as well as we know Ohm's law. Then a door to many new worlds will be opened.

RULE 1: Study in short, frequent intervals of time.

Laplace transforms are easy to watch and understand when someone else is working the problems, but difficult when we wait too long to start working problems. We must work problems as soon as we are exposed to any new idea, no matter how small. More can be learned in one hour spread over several days than in two hours of unbroken time. The longer hours at a stretch will only make us feel noble—we won't retain much.

RULE 2: Always write the equation being used in its original form with variables instead of numbers.

There are always equations that must be memorized. The easiest way is to write the equation in its original form first (without copying from the text or notes). Second, use algebra to solve for the variable required. Last, substitute in the numbers. The second and last step may be reversed, depending on personal preference. Many of the equations will be similar, and this will help to keep us from combining different equations.

RULE 3: Work problems without the text or notes.

When we have to use references to work a problem, the only thing we learn is that we do not know how to work the problem. We must work the problems without the text. When we are not sure what to do, try anything, but try something. (A good thing to try is to make a list of known and unknown variables and values.) When we have an answer, then and only then should we use our notes and the text to determine if we did something wrong.