

Fuzzy-Reliability Engineering

Concepts and Applications



Narosa

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Preface

There has been tremendous research and development in the field of fuzzy sets and fuzzy logic after the pioneering paper by Lotfi Zadeh in 1965. In fact, it made a revolution in the field of uncertainty, which challenged the probability theory. Instead of two-valued logic i.e. success and failure (0 and 1) as used in probability theory, fuzzy set theory advocated whole range of values between 0 and 1. The proposition like *x is a member of A* is not necessarily true in fuzzy set theory. It may be true to some degree. This is described by degree of membership function, which lies between 0 and 1.

As an engineering discipline, reliability is relatively old, but its relevance to modern world and society is still making a great impact. Its growth has been motivated by several factors like complexity and sophistication of systems, organizational awareness to produce and sell high reliability products, apparently conflicting requirement of low cost and high performance product/service by the customer, global competitiveness etc. Reliability theory is inter-disciplinary in nature and it studies the dependability of engineering products under specific operating conditions when put into service. Reliability is a measure of the expected capability of an engineering product to operate without failures under specific conditions for a given period of time.

Fuzzy-reliability is comparatively a new area of research and interest and still it is in its incubation period. This is an alternative reliability theory, which is rooted in fuzzy sets and possibility theory. At any given time, the product may be in operating state to some degree and in failed state in another degree. Also, the behavior of the product with respect to two fuzzy states can be characterized using possibility theory. Fuzzy-reliability theories are more meaningful than probabilistic reliability when the number of data or samples available is small or if the data/ information is ambiguous, inexact or subjective. It is based on the assumption of fuzzy states and possibility. This book is an excellent attempt to popularize a recent field of fuzzy-reliability in the systems engineering field.

Fuzzy-reliability is the novel concept in systems engineering as fuzzy sets can capture subjective, uncertain and ambiguous information. The book focuses on fuzzy set based concepts and its applications in the area of reliability engineering and almost all outcomes in this book are the research

findings of Reliability Engineering Group at Indian Institute of Technology Bombay. Theoretical aspects of reliability and fuzzy set theory are covered in the initial chapters. The other topics are application topics of fuzzy-reliability to maintenance engineering, software and power systems. Various concepts related to fuzzy-reliability such as probist, profust, fuzzy event based method, fuzzy fault tree analysis, transformations and hybrid approaches have been described. Application in systems reliability, availability and maintainability, software reliability and power system reliability has been discussed in depth. Maintenance issues relate to fuzzy availability modeling for Markov and semi-Markov models, fuzzy dynamic reliability evaluation under imperfect repair, and maintenance strategy selection using fuzzy linguistics. Software reliability using fuzzification of model parameters in cost models with penalty, warranty and risk costs are applied to existing growth models. Composite power system reliability modeling using fuzzy sets and fuzzy confidence intervals with fuzzy linear programming for minimization of load curtailment has demonstrated the flexibility and capability of using fuzzy sets in reliability engineering. This book incorporates illustrative examples to emphasize fuzzy-reliability engineering concepts.

The book covers a wide range of interests of the academic and research community as well as practicing engineers and managers in manufacturing, power and IT sectors. This book is suitable both as a textbook and as a reference material. The book would certainly be an asset to researchers, as this area of reliability is yet not fully explored.

Writing a book in an upcoming field of fuzzy-reliability is not an easy job. Though utmost care has been taken to come out with this edition, authors claim the book may not be entirely free from errors and mistakes. Readers are therefore advised to confirm the research findings from this book before their actual further usage by them. Publishing of this book has been only possible due to financial support from CDP, Indian Institute of Technology Bombay. We are grateful to I. I. T. Bombay for providing research and related facilities throughout the development of this book. Research scholars from reliability engineering group too contributed a lot by way of their work, which helped bringing this book out. We are grateful to them as well. We also thank Narosa Publishing House Pvt. Ltd., New Delhi for their kind cooperation and all their help in publishing this book.

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Chapter 1

Introduction

1.1 Introduction

Probability theory is being used extensively in engineering and management for reliability analysis. Conventional reliability theory is based on the probability/ Bayesian concepts [1,2] and binary state structure. Results based on probability theory do not always provide useful results to the practitioners due to its limitation of being able to handle only quantitative information. The subjective information is not captured during reliability analysis. The results obtained are therefore not of much practical value. This is primarily due to the fact that there is significant impact of subjective information on reliability in relation to the available quantitative information. It has therefore become inevitable to consider subjective information along with quantitative databases to arrive at useful results in the reliability analysis. One way to handle subjective information is the usage of fuzzy set theory. Research in last few decades, however, has shown that probability theory is not the only possible way to represent imprecision and uncertainty. Fuzzy set theory provides a significant alternative to the probabilistic approach of reliability. In recent times, the use of fuzzy sets has been gaining popularity and is playing an important role in the areas of engineering and management disciplines.

This book is a sincere attempt at giving the readers the basic concepts in the field of reliability, and the applications and use of fuzzy set concepts in reliability engineering. The primary purpose of the book is to provide the readers with the background necessary for understanding the concepts of fuzzy set based reliability and its numerous applications.

This book would be of interest both to the academic institutions, research community and industry at large. The first few chapters ensure that the concepts and applications in engineering are well appreciated by beginners, and the detailed applications later in the book would appeal to the advanced students and engineers pursuing research and studies in academic institutes, research laboratories and various industries in manufacturing, process and service sectors.

This introductory chapter presents the overview of fuzzy set applications to reliability engineering as well as other engineering areas.

1.2 Application of Fuzzy Set Theory to Reliability Engineering

There has been a remarkable development in literature on the fuzzy set theory (FST) and its applications since the first paper by Prof. L. A. Zadeh [3] in 1965. FST has proved its credibility in many fields such as linear and nonlinear process control, robotics, automation, tracking, consumer electronics, pattern recognition, image processing, machine vision, decision making, financial systems, information systems, data base management, information retrieval, meteorology, etc. For information that is inherently imprecise or vague, FST is the best choice as others are not well suited for representation and processing by classical binary logic or probabilistic based techniques. Subjectivity and fuzziness are inherent in any system reliability analysis [4-6].

Of late, there have been numerous applications of fuzzy set theory to reliability engineering. This section highlights the important related areas of reliability engineering with the application of fuzzy set theory. These areas are still not fully explored and there is immense scope of fuzzy set theory application in these areas. The different applications of fuzzy set theory to reliability engineering are shown in Figure 1.1.

1.2.1 Quantification of uncertainty in reliability analysis

Reliability analysis of complex systems inevitably involves many uncertainties. Quantification of uncertainty in reliability analysis is very important as it helps for effective decision-making. The conventional approaches for uncertainty analysis assume a probability distribution for the failure probability of system components or basic events to find the uncertainty in the overall system failure probability. Monte-Carlo simulation is the most widely used method for this purpose. However, it is very difficult to make statistical inferences in case of systems where available data is insufficient. These conventional methods are computationally difficult and may require enormous computer time. To overcome some of the difficulties, new methodologies based on fuzzy set theory are also being used in the risk analysis for propagating the basic event uncertainty [7]. In FST, the input parameters are treated as fuzzy number and the variability is characterized by membership function. The fuzzy evaluation of the failure probability of main event in a fault tree/ event tree is carried out using the α -cut method, also known as resolution identity.

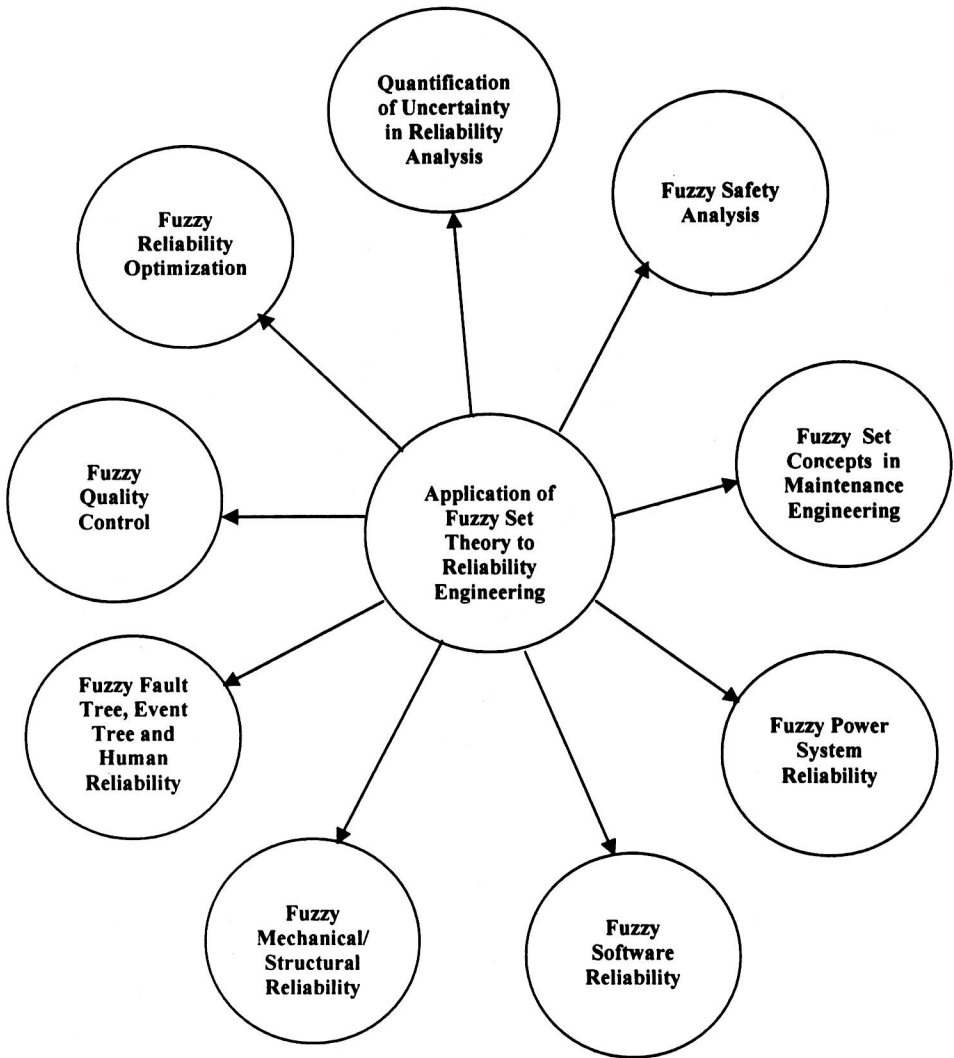


Figure 1.1 Application of Fuzzy Set Theory to Reliability Engineering

1.2.2 Fuzzy safety analysis

Traditionally, safety analysis is carried out on a probabilistic basis. Probability distributions are used to describe a set of states for a system and to deal with uncertainty in order to evaluate potential hazards and assess system safety. In many cases, it is very difficult or even impossible to precisely determine the parameters of a probability

distribution for a given event due to the lack of evidence or due to the inability of the safety engineer to make firm assessments. Therefore, one may have to describe a given event in terms of vague and imprecision descriptors such as likely or impossible - terms that are commonly used by safety analysts. Such judgments are obviously fuzzy and non-probabilistic, and hence non-probabilistic methods such as fuzzy set modeling may be more appropriate to analyze the safety of systems with incomplete information of the kind described above [6,8,9].

Fuzzy set theory is used to describe each failure event and evidential reasoning approach is then employed to synthesis the information thus produced to assess the safety of the whole system. Three basic parameters, namely, failure likelihood, consequence severity and failure consequence probability are used to analyze a failure event. These three parameters are described by linguistic variables that are characterized by a membership function to the defined categories. As safety can also be described by linguistic variables referred to as the safety expressions, the obtained fuzzy safety score can be mapped back to the safety expressions that are characterized by membership functions over the same categories. This mapping results in the identification of the safety of each failure event in terms of degree to which fuzzy safety score belongs to each of the safety expressions. Such degrees represent the uncertainty in safety evaluations and can be synthesized using an evidential reasoning approach so that the safety of the whole system can be evaluated in terms of these safety expressions.

1.2.3 Fuzzy set theory in maintenance engineering

Continued pressure on industries to reduce costs and to improve customer satisfaction has resulted in increasingly detailed examinations of maintenance practices and strategies. The justification of any given maintenance strategy or practice within an organization must consider multiple criteria. It should also be based on the overall objectives of the organization, many of which are intangible and subjective in nature. The use of fuzzy linguistic variables in parameter modeling is a good option that assists decision makers in their evaluation and choice of maintenance strategies and condition monitoring techniques [10]. The optimum maintenance strategy or technique to be employed in a specific situation can be determined based on the assessment of qualitative verbal inputs.

Maintenance of any equipment based on its condition is unavoidable. The condition of the equipment is represented by fuzzy sets. The best type of maintenance, maintenance interval, inspection frequency and replacement time are determined using fuzzy set theory in the literature [11,12]

Fuzzy set theory is effectively used in the transition rates in the Markov process as point estimated values and the system states are considered to exist either in *good* or *bad* states. In reality, the situation is not that simple. The transition from one state to other may not be fixed value and the system states are not that sharp or abrupt. Fuzzy set theory has the capability of dealing with such situations. In fuzzy Markov model [13,14], the system is considered to exist in two states: operating and failed state. The linguistic variables for failure rate as well as repair rate are defined as HIGH (H), MEDIUM (M) and LOW (L). These variables are quantified via subjective expert opinion and translated into triangular membership function. The unavailability of the system is then calculated which is an adequate estimator of the probability that the unit under similar condition will not be available for service in the future.

1.2.4 Fuzzy power system reliability

Reliability evaluation of power systems using probabilistic methods is an important area of concern for system planners and operators. Methods of evaluating the reliability of generating systems have been well established and accepted by the utilities. These techniques do not generally recognize the presence of transmission facilities. Composite system reliability evaluation refers to assessments that consider both generation and transmission facilities. Combined generating unit and transmission line outage events contribute significantly to the unreliability of the composite system, and this is the primary motivation for composite system reliability evaluation. Power system reliability is divided into two basic aspects: adequacy and security. Adequacy and security assessments are usually carried out by either contingency enumeration or Monte Carlo simulation approach.

Fuzzy set theory is used to include uncertainty in power system reliability parameters. Uncertainties that may exist in forecast peak load and, failure and repair rates of system components can easily be included in the normal calculation using fuzzy numbers. The classical linear programming techniques used in power system reliability evaluation methodology can be converted into fuzzy linear