DIGITAL CONTROL AND ESTIMATION

A UNIFIED APPROACH

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Preface

Many books already exist on the topics of digital control and estimation. The prospective reader of this book might well then ask, "why another book in this area?"

One problem with the existing literature is that it emphasizes the differences between discrete and continuous theory. This dichotomy is largely historical in nature and may not be the best approach from a pedagogical viewpoint. For example, shift operators and Z-transforms, which form the basis of most discrete time analyses, are inappropriate when used with fast sampling and have no continuous time counterpart. Our philosophy, as presented in this book, is that the continuous and discrete cases can, and should, be understood under a common framework. We show that this is facilitated if the shift operator is augmented with alternative forms including one which we call the delta operator. Using the latter operator, it becomes evident that all discrete time theory converges smoothly to the appropriate continuous results as the sampling rate increases. An additional, and somewhat unexpected, bonus arising from the use of the alternative operators is that numerical properties can be substantially improved relative to the more traditional shift operator.

Thus, this book presents continuous and discrete control and estimation theory in a unified fashion, highlighting the interrelationships between the two cases. Our firm belief is that this unified view of discrete and continuous theory is much richer and more informative than when either of the two are studied in isolation.

Another thrust of the book is to unify practical considerations with theoretical analysis. This is achieved by discussing implementation issues in detail and by presenting an industrial case study.

The book has a dual audience. Part of the book would be suitable for a first undergraduate course in digital control. The remainder would form the basis of one or more graduate courses in advanced control and estimation.

The prerequisite for the "undergraduate" portion of the text is an elementary mathematical background in Linear Algebra, Differential Equations, Calculus and Complex Numbers. The more advanced material depends upon additional background normally available to graduate students.

The authors would like to acknowledge those who assisted with the preparation of this book. First, the book would not have been possible without the support and understanding of our wives Ruth (Middleton) and Rosslyn (Goodwin). The book has also been used in both undergraduate and graduate courses at the University of Newcastle, Australia, and we would like to thank the students in these courses for their advice as the book took shape. A detailed solutions manual was prepared by Changyun Wen and Youyi Wang. Very helpful feedback was also obtained from Robert Bitmead, Robin Evans, Arie Feuer, Art Harvey, Peter Hippe, Konrad Hitz, Michel Kinnaert, Bengt Lennartson, Mario Salgado and Bjorn Wittenmark. The authors are also grateful for very helpful suggestions made by several anonymous reviewers.

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Introduction

This book presents a fresh approach to the topics of digital control and estimation. A central theme is to unify continuous and discrete theory. Our belief is that it is instructive to view continuous time results as a suitable limiting case of the corresponding discrete results. This represents a divergence from previous presentations of discrete theory, which tended to highlight the differences between the two cases.

Typically, the topics of digital control and estimation are described in a different framework than that used for continuous time systems. As an example of the divergence between the approaches, consider the problem of stability. In the usual continuous time theory, we say that a linear system is stable if its poles lie in the left half-plane. However, in the traditional discrete time theory, we say that a linear system is stable if its poles lie inside the unit circle. It is difficult to see the connection between these results. We argue in this book that the apparent schism that has arisen between analog and digital theory can be traced to the widespread use of shift operators and Z-transforms in the discrete case. In this book, we introduce an additional operator, which we call the delta operator. This operator offers the same flexibility as does the shift operator in the description of discrete systems yet has several advantages over the latter, including:

- It highlights the similarities, rather than the differences, between discrete and continuous systems, thus allowing continuous insights to be applied to the discrete case.
- It introduces the sampling period as an exhibit parameter, thus allowing the effect of different choices for this parameter to be readily assessed.
- It allows a unified systems theory to be developed without needing to run a separate line of development for continuous and discrete.

- It allows most continuous time results to be obtained as a simple special case of the discrete results (by setting the sampling period to zero).
- It offers substantial numerical advantages in most cases of practical interest.

A brief outline of the content of the book is as follows. We begin in Chapter 2 by giving a brief overview of how simple models of physical systems can be derived from the laws of nature. This modeling typically involves the use of state space ideas. The important question of linearization of nonlinear models is also discussed in some detail, including such techniques as input–output transformations, feedback compensation, and linear approximation.

Chapter 3 describes the sampling process, including choice of sampling rate, and aliasing. We also show how models for sampled data systems can be derived from an underlying continuous time representation. The notion of the shift operator is introduced, and this is then extended to more general representations, including the delta operator. We also briefly discuss numerical issues leading to the conclusion that the delta operator has significant advantages over the shift form.

One of the most useful tools to have evolved for the analysis of linear estimation and control systems is that of transform techniques. The traditional techniques of Laplace and Z-transforms are discussed in Chapter 4. This is followed by the development of a unified transform theory, which allows both continuous and discrete systems to be treated simultaneously.

This leads us, in Chapter 5, to the topic of transfer function analysis. Particular emphasis is given to the notions of poles and zeros and their relationships for discrete and continuous systems. A distinctive feature of the chapter is the insightful discussion of the zeros that arise when a continuous time system is sampled.

Chapter 6 builds on the notion of transfer functions and introduces the important concept of system frequency response. We examine the interrelationship of continuous and discrete frequency responses. We also study systems having pure time delays.

Chapter 7 explores the analysis of control system performance using classical (that is, frequency domain) techniques. The discussion includes elementary notions of stability, the motivation for the use of feedback, as well as performance specifications in both the time and frequency domain for control systems.

In Chapter 8, we extend the methods of systems analysis to time domain techniques based on state space representations and matrix fraction descriptions. The notions of controllability, observability, minimal realizations, per unit values, canonical forms, and balanced realizations of systems are discussed.

This leads naturally to the discussion of state observers and state variable feedback in Chapter 9. We examine the interrelationship with frequency domain control systems analysis. We also introduce the ideas of fractional representations and their use in control systems analysis.

Chapter 10 discusses optimal state estimation. The Kalman filter is derived and its properties analyzed. Also, the class of all stable unbiased state estimators is described and used to motivate various robustness issues.