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Finite Element Procedures

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To my students

. . . *Progress in design of new structures seems to be unlimited.*

Last sentence of article: "The Use of the Electronic Computer in Structural Analysis," by K. J. Bathe (undergraduate student), published in *Impact, Journal of the University of Cape Town Engineering Society*, pp. 57–61, 1967.

Preface

Finite element procedures are now an important and frequently indispensable part of engineering analysis and design. Finite element computer programs are now widely used in practically all branches of engineering for the analysis of structures, solids, and fluids.

My objective in writing this book was to provide a text for upper-level undergraduate and graduate courses on finite element analysis and to provide a book for self-study by engineers and scientists.

With this objective in mind, I have developed this book from my earlier publication *Finite Element Procedures in Engineering Analysis* (Prentice-Hall, 1982). I have kept the same mode of presentation but have consolidated, updated, and strengthened the earlier writing to the current state of finite element developments. Also, I have added new sections, both to cover some important additional topics for completeness of the presentation and to facilitate (through exercises) the teaching of the material discussed in the book.

This text does not present a survey of finite element methods. For such an endeavor, a number of volumes would be needed. Instead, this book concentrates only on certain finite element procedures, namely, on techniques that I consider very useful in engineering practice and that will probably be employed for many years to come. Also, these methods are introduced in such a way that they can be taught effectively—and in an exciting manner—to students.

An important aspect of a finite element procedure is its reliability, so that the method can be used in a confident manner in computer-aided design. This book emphasizes this point throughout the presentations and concentrates on finite element procedures that are general and reliable for engineering analysis.

Hence, this book is clearly biased in that it presents only certain finite element procedures and in that it presents these procedures in a certain manner. In this regard, the book reflects my philosophy toward the teaching and the use of finite element methods.

While the basic topics of this book focus on mathematical methods, an exciting and thorough understanding of finite element procedures for engineering applications is achieved only if sufficient attention is given to both the physical and mathematical characteristics of the procedures. The combined physical and mathematical understanding greatly enriches our confident use and further development of finite element methods and is therefore emphasized in this text.

These thoughts also indicate that a collaboration between engineers and mathematicians to deepen our understanding of finite element methods and to further advance in the fields of research can be of great benefit. Indeed, I am thankful to the mathematician Franco Brezzi for our research collaboration in this spirit, and for his valuable suggestions regarding this book.

I consider it one of the greatest achievements for an educator to write a valuable book. In these times, all fields of engineering are rapidly changing, and new books for students are needed in practically all areas of engineering. I am therefore grateful that the Mechanical Engineering Department of M.I.T. has provided me with an excellent environment in which to pursue my interests in teaching, research, and scholarly writing. While it required an immense effort on my part to write this book, I wanted to accomplish this task as a commitment to my past and future students, to any educators and researchers who might have an interest in the work, and, of course, to improve upon my teaching at M.I.T.

I have been truly fortunate to work with many outstanding students at M.I.T., for which I am very thankful. It has been a great privilege to be their teacher and work with them. Of much value has also been that I have been intimately involved, at my company ADINA R & D, Inc., in the development of finite element methods for industry. This involvement has been very beneficial in my teaching and research, and in my writing of this book.

A text of significant depth and breadth on a subject that came to life only a few decades ago and that has experienced tremendous advances, can be written only by an author who has had the benefit of interacting with many people in the field. I would like to thank all my students and friends who contributed—and will continue to contribute—to my knowledge and understanding of finite element methods. My interaction with them has given me great joy and satisfaction.

I also would like to thank my secretary, Kristan Raymond, for her special efforts in typing the manuscript of this text.

Finally, truly unbounded thanks are due to my wife, Zorka, and children, Ingrid and Mark, who, with their love and their understanding of my efforts, supported me in writing this book.

K. J. Bathe

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CHAPTER ONE

An Introduction to the Use of Finite Element Procedures

1.1 INTRODUCTION

Finite element procedures are at present very widely used in engineering analysis, and we can expect this use to increase significantly in the years to come. The procedures are employed extensively in the analysis of solids and structures and of heat transfer and fluids, and indeed, finite element methods are useful in virtually every field of engineering analysis.

The development of finite element methods for the solution of practical engineering problems began with the advent of the digital computer. That is, the essence of a finite element solution of an engineering problem is that a set of governing algebraic equations is established and solved, and it was only through the use of the digital computer that this process could be rendered effective and given general applicability. These two properties—effectiveness and general applicability in engineering analysis—are inherent in the theory used and have been developed to a high degree for practical computations, so that finite element methods have found wide appeal in engineering practice.

As is often the case with original developments, it is rather difficult to quote an exact "date of invention," but the roots of the finite element method can be traced back to three separate research groups: applied mathematicians—see R. Courant [A]; physicists—see J. L. Synge [A]; and engineers—see J. H. Argyris and S. Kelsey [A]. Although in principle published already, the finite element method obtained its real impetus from the developments of engineers. The original contributions appeared in the papers by J. H. Argyris and S. Kelsey [A]; M. J. Turner, R. W. Clough, H. C. Martin, and L. J. Topp [A]; and R. W. Clough [A]. The name "finite element" was coined in the paper by R. W. Clough [A]. Important early contributions were those of J. H. Argyris [A] and O. C. Zienkiewicz and Y. K. Cheung [A]. Since the early 1960s, a large amount of research has been devoted to the technique, and a very large number of publications on the finite element method is available (see, for example, the compilation of references by A. K. Noor [A] and the *Finite Element Handbook* edited by H. Kardestuncer and D. H. Norrie [A]).

The finite element method in engineering was initially developed on a physical basis for the analysis of problems in structural mechanics. However, it was soon recognized that the technique could be applied equally well to the solution of many other classes of problems. The objective of this book is to present finite element procedures comprehensively and in a broad context for solids and structures, field problems (specifically heat transfer), and fluid flows.

To introduce the topics of this book we consider three important items in the following sections of this chapter. First, we discuss the important point that in any analysis we always select a *mathematical model* of a physical problem, and then we solve *that* model. The finite element method is employed to solve very complex mathematical models, but it is important to realize that the finite element solution can never give more information than that contained in the mathematical model.

Then we discuss the importance of finite element analysis in the complete process of computer-aided design (CAD). This is where finite element analysis procedures have their greatest utility and where an engineer is most likely to encounter the use of finite element methods.

In the last section of this chapter we address the question of how to study finite element methods. Since a voluminous amount of information has been published on these techniques, it can be rather difficult for an engineer to identify and concentrate on the most important principles and procedures. Our aim in this section is to give the reader some guidance in studying finite element analysis procedures and of course also in studying the various topics discussed in this book.

1.2 PHYSICAL PROBLEMS, MATHEMATICAL MODELS, AND THE FINITE ELEMENT SOLUTION

The finite element method is used to solve physical problems in engineering analysis and design. Figure 1.1 summarizes the process of finite element analysis. The physical problem typically involves an actual structure or structural component subjected to certain loads. The idealization of the physical problem to a mathematical model requires certain assumptions that together lead to differential equations governing the mathematical model (see Chapter 3). *The finite element analysis solves this mathematical model.* Since the finite element solution technique is a numerical procedure, it is necessary to assess the solution accuracy. If the accuracy criteria are not met, the numerical (i.e., finite element) solution has to be repeated with refined solution parameters (such as finer meshes) until a sufficient accuracy is reached.

It is clear that the finite element solution will solve only the selected mathematical model and that all assumptions in this model will be reflected in the predicted response. We cannot expect any more information in the prediction of physical phenomena than the information contained in the mathematical model. Hence the choice of an appropriate mathematical model is crucial and completely determines the insight into the actual physical problem that we can obtain by the analysis.

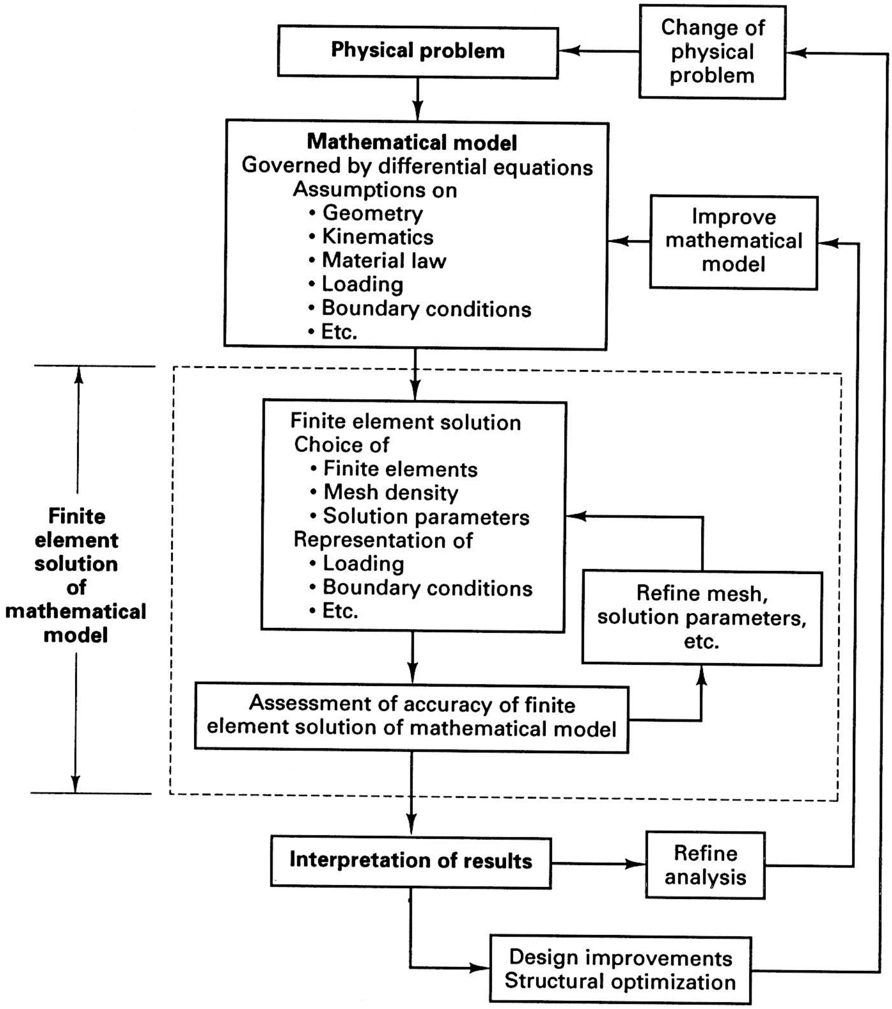

Figure 1.1 The process of finite element analysis

Let us emphasize that, by our analysis, we can of course only obtain *insight* into the physical problem considered: we cannot predict the response of the physical problem *exactly* because it is impossible to reproduce even in the most refined mathematical model all the information that is present in nature and therefore contained in the physical problem.

Once a mathematical model has been solved accurately and the results have been interpreted, we may well decide to consider next a refined mathematical model in order to increase our insight into the response of the physical problem. Furthermore, a change in the physical problem may be necessary, and this in turn will also lead to additional mathematical models and finite element solutions (see Fig. 1.1).

The key step in engineering analysis is therefore choosing appropriate mathematical models. These models will clearly be selected depending on what phenomena are to be