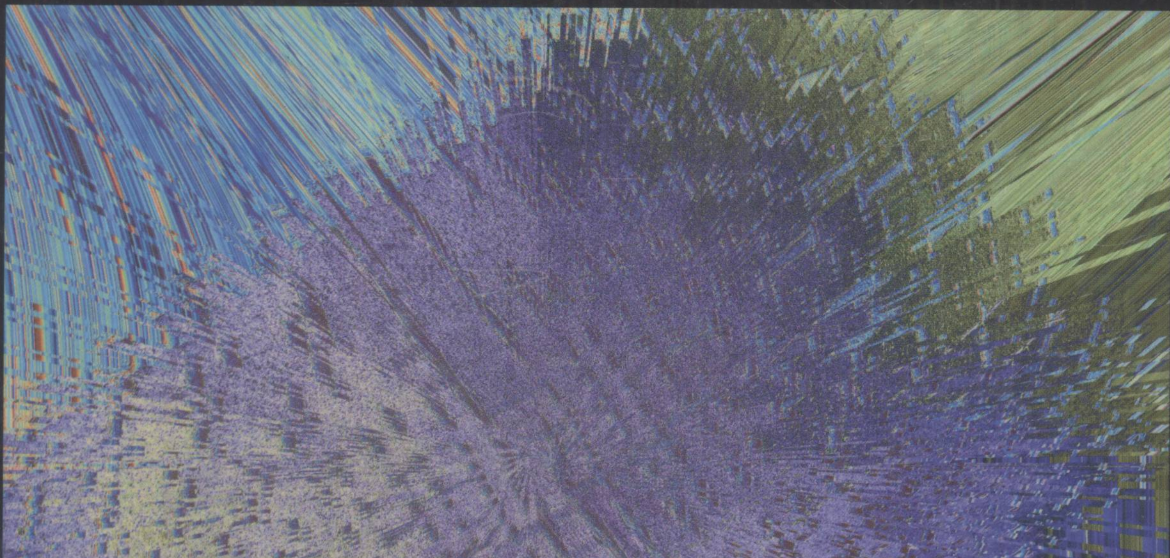
CONCEPTS IN SYSTEMS AND SIGNALS

SECOND EDITION



John D. Sherrick

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PEARSON
Prentice
Hall

Upper Saddle River, New Jersey
Columbus, Ohio

Library of Congress Cataloging-in-Publication Data

Sherrick, John D.
Concepts in systems and signals / John D. Sherrick.—2nd ed.
p. cm.
ISBN 0-13-178271-1 (alk. paper)
1. Signal processing—Mathematics. 2. System analysis—Data processing. 3. MATLAB.
I. Title.

TK5102.9.S5325 2005
621.382'2—dc22 2003062425

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This book was set in New Century Schoolbook by Carlisle Communications, Ltd. It was printed and bound by R.R. Donnelley & Sons Company. The cover was printed by The Lehigh Press, Inc.

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10 9 8 7 6 5 4 3 2 1
ISBN 0-13-178271-1

PREFACE

The topic of digital signal processing (DSP) started entering undergraduate engineering programs in the early 1980s, just as the age of the personal computer was dawning. It has since become increasingly clear that many low-power analog circuits can be implemented better and more cheaply with these digital techniques. The ability to reprogram a digital circuit to perform a new task, and the higher packaging density of digital over analog chips, favor the digital approach. At the same time, and for the same reasons, digital control systems have become dominant. Spectrum analyzers, which once were far too expensive for most college labs, have become low-priced oscilloscope options. Clearly, the topics of digital signal processing need to be a part of any electrical/electronics program today.

This text was originally written to support an introductory course required in all upper-division engineering technology programs of the Electrical, Computer, and Telecommunications Department at Rochester Institute of Technology. The potentially diverse backgrounds of students in this course require that the text establish a common base, and that critical theorems be presented without sophisticated mathematics. Spectral analysis is generally limited to periodic signals in this course.

Phasors are used to introduce both s and z domain transfer functions. The sampling theorem, which provides the key between the discrete and continuous-time domains, is presented at two levels of mathematical sophistication. It can be deduced from sampling a sinewave, or derived more formally by defining the ideal sampler. The presentation is balanced between the continuous and discrete-time domains, and between systems and signals. An integrated introduction to MATLAB® is used to support the concepts presented.

CHANGES IN THIS EDITION

Students with a good background in the s domain may find that many of this text's introductory chapters are unnecessary, and will proceed quickly to the Fourier series. Topical additions have been made to discuss the continuous-time Fourier transform more fully, and to demonstrate its power in deriving properties of signals. Sections have been added to include the topics of spectral estimation, convolution, and some correlation. Of course, known errors in the first edition have been corrected, particularly those related to the development of the DFT in Chapter 7.

For those who might want to use the text as an introduction to control systems, a chapter has been added to cover applications of the Laplace and z transforms. Since these topics prepare the way, the impulse invariance approach to converting from continuous-time to discrete-time systems has also been added.

Instructors who have used the text in the past will be happy to see that only two problems had to be renumbered. These are in Chapter 6, and were originally in the Advanced Problems section. This category has been renamed Additional Problems, and may include problems that reinforce new examples or discussions. Naturally, as is the custom, most of the problem answers have been changed.

The MATLAB lessons have been reviewed and no changes were needed to be compatible with Release 13. The *length* command has replaced the *size* command in a few places, since it is more suitable. An educational package that includes the Controls and Signal Processing Toolboxes is required to perform the MATLAB lessons used in this text. Student MATLAB also contains all the required commands. MATLAB is a registered trademark of The MathWorks, Inc., of Natick, MA. For further information, visit their Web site at www.mathworks.com.

ACKNOWLEDGMENTS

Since I retired as the first edition of this text was being published, I have had to rely on feedback from my former colleagues at RIT in preparing this edition. My thanks to Professors Charles Swain and Steven Ciccarelli, from the ECT Department, for their suggestions and corrections. Thanks also to the Department Facility Manager, Ken Garland, and to the people at The MathWorks, Inc., for providing the means for checking the MATLAB lessons with Release 13. It is difficult to overstate the value of MATLAB in demonstrating the DSP concepts.

Having spent most of my career in engineering technology, my main assignments have been teaching, rather than research. The *s* domain was part of my undergraduate and graduate studies (as was the slide rule), and my understanding of it has been influenced by a wide variety of texts over the 40 years of my career. I will mention only the one that first introduced me to the *s* domain: *Network Analysis* by M. E. Van Valkenburg (Prentice Hall, 1955).

A 1981 RIT Electrical Engineering seminar by Professors Edward R. Salem and W. F. Walker was my first introduction to the *z* domain. Assigned to teach a senior elective in DSP, I found *Digital Signal Processing* by Stanley, Dougherty, and Dougherty (Reston Publishing, 1984) to be close to the mathematical level I was seeking. I use its sequential design process for FIR window filter designs.

Two relatively recent texts contributed significantly to Chapter 11. These were *Understanding Digital Signal Processing*, a draft manuscript by Richard G. Lyons (Addison-Wesley, 1997) and *Digital Signal Processing and the Microcontroller*, by Dale Grover and John R. Deller (Prentice Hall, 1998). I assume the contents of these texts reflect common knowledge among the community of DSP experts and researchers, but they are the source of the information.

Finally, thanks to those of my students who shared the adventure of exploring DSP with me and who contributed ideas and corrections to the original manuscript.

JOHN D. SHERRICK, P.E.

CONTENTS

1

NUMBERS, ARITHMETIC, AND MATHEMATICS

OUTLINE

OBJECTIVES

1. Discuss the origin of complex numbers.
2. Represent complex numbers in polar form and rectangular form.
3. Define and interpret Euler's identity.
4. Conduct numerical calculations with complex numbers.
5. Describe properties of a function of a complex variable.
6. Interpret and resolve indeterminate values.
7. Use MATLAB for arithmetic operations.

INTRODUCTION

Electrical, mechanical, hydraulic, and other systems contain components that obey calculus-based laws. Determining the behavior of such a system in response to an input signal involves solving differential equations. There is, however, one particular type of signal, e^{st}, for which the linear differential equations reduce to simple algebraic equations. Fortunately, this exponential function can represent the kinds of signals of interest to us. While this signal is a simple function of time, it unfortunately returns values that are complex numbers.

Since complex numbers are central to this text, a brief review of how they arise and the arithmetic they obey is likely to be helpful. A few other mathematical topics related to functions, and especially complex functions, will also be discussed. Otherwise, we will generally introduce mathematical concepts as they are needed.

1.1 THE NUMBER SYSTEM

The names that have been given to different types of numbers show that it is a common reaction to view new concepts with astonishment and possibly skepticism. After reading this section you should be able to:

- Discuss the origin of complex numbers.
- Explain why "imaginary" numbers are necessary.
- Recognize that you are not alone in finding new concepts difficult.

Much speculation has occurred on the origins of our numbering system, but it most certainly started with counting. A prehistoric hunter might have recorded the success of his efforts by the number of skins obtained, or the length of his efforts by the number of sunrises that occurred while he was away. Some of his skins might have been bartered away for roots and berries or for tools and weaponry, "I'll give you two of these for one of those" types of transactions. All of these processes required positive integers, a concept that is both natural and easy for us to appreciate.

Manuscripts dating from about 2000 B.C. show that simple fractions were also understood early on. Apparently, questions of how a lord could divide his territory uniformly among his subjects so that they might be equally and fairly taxed were among the most pressing early mathematical issues. Some priorities never change.

If a man had 5 skins and traded 3 of them for a new spear, he could determine how many skins he had left by counting out those needed for the trade and then counting those that remained. No new skills or concepts needed to be developed. It was unthinkable that 5 skins would be traded if only 3 were available. Such a transaction would be impossible! Several millennia would pass before that changed.

It is difficult, today, to fully appreciate some of the discoveries of the early mathematicians. The concepts with which they struggled are now so ingrained in our experience that they are second nature to us. The notations and algebraic laws that allow complicated questions to be addressed systematically were not available to them.

Yet, once the philosophers of the day postulated that no transaction should be impossible, the existence of an entirely new set of numbers was required. Thus negative numbers were discovered, the number system doubled in size, and debt became an element in some transactions.

The concept of the number *zero* was especially difficult to establish. After all, why create a number to count nothing? It is only when man began to philosophize about numbers that meaning could be attached to such an abstraction. The discovery that there were numbers that could not be represented as a ratio of integers, numbers like those special ones we denote today as π or e, did not come from everyday experience. Indeed, the claim that there were such numbers was greeted with much skepticism and disbelief. Such numbers were inconceivable, irrational! We know now that the set of *irrational* numbers is infinitely larger than all those that were known previously.

Finally, but not until around the sixteenth century, it was recognized that there had to be yet again an entirely new set of numbers, in fact a new kind of number, if equations like $x^2 = -1$ were to have solutions. These new numbers were called *imaginary*—a regrettable though understandable label—to distinguish them from the *real* numbers known previously. A j (mathematicians prefer i) is used to indicate the imaginary part of a number, where $j = +\sqrt{-1}$.

Today we conveniently think of the real number system as being represented by a continuous line of numbers running from $-\infty$ to $+\infty$, with every point along the line being a unique number. We think of the imaginary numbers in much the same way, but with two exceptions: (1) Each imaginary number is tagged with a j to distinguish it from the real numbers, and (2) the line of imaginary numbers is independent of, or perpendicular to, the line of real numbers. Together the real and imaginary numbers form a *complex-number plane*, each point of which represents a generalized number that could be used in, or result from, a mathematical calculation. Every equation conceived of by man has yielded a result somewhere in this number plane . . . so far.

It is not necessary to introduce complex numbers into the study of electrical technology. All of our variables—voltage, current, resistance, inductance, etc.—are physical quantities that are described by ordinary real numbers. Ultimately, complex numbers gain their meaning through their application and interpretation, not through the arbitrary tags of “real” or “imaginary” that mathematicians use to distinguish between their two component parts. The fact that these components are independent allows a single complex number to hold information about two separate physical properties. We introduce complex numbers because they can make many of our calculations easier.

1.2 RECTANGULAR/POLAR CONVERSIONS

Complex numbers may be represented in rectangular or polar form. After reading this section you will be able to:

- Represent complex numbers in polar form and rectangular form.
- Convert from one form to the other.

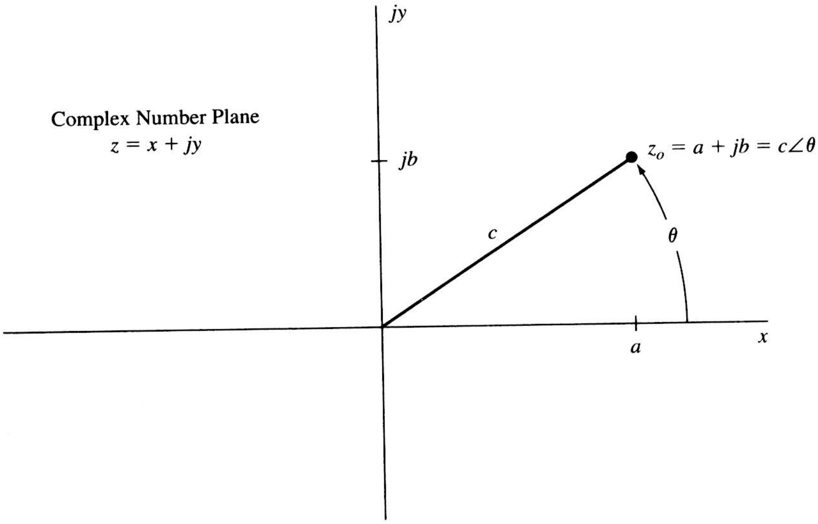


Figure 1.1 A specific complex number may be described either by its rectangular or its polar coordinates in a complex number plane.

A complex number can be identified by giving it real and imaginary parts to identify its location in the complex-number plane of Figure 1.1. This is called the *rectangular* form of the number. If $z = a + jb$, where a and b are real numbers, a is called *the real part* of z, and b is called *the imaginary part* of z. To emphasize this terminology, whenever we speak of the imaginary part of a complex number, we are talking about a real number that is being multiplied by j.

Some people like to think of j as a *rotational operator*, since it converts a real number into an imaginary number by rotating it 90° to the j axis in the complex-number plane. We prefer simply to define j to be a number, namely, the positive square root of -1, and to treat it like any other number. To us, there is no difference between the numbers $j2$ and $2j$. Some of the products of j are:

$$j = +\sqrt{-1} \qquad j^2 = -1 \qquad j^3 = -j \qquad j^4 = 1 \qquad j^5 = j$$

A complex number may alternatively be specified by giving its distance from the origin along a radius line having a specified angle relative to the positive real axis. This is called the *polar* form of the number, and is often stated using the notation $c\angle\theta$, read as "a magnitude of c at an angle of θ." By convention, θ is taken as positive in the counterclockwise direction. Both c and θ are real numbers.

It is essential to be able to convert between polar and rectangular forms. The required relationships are easily deduced from Figure 1.1. The conversion to polar form is complicated slightly by the fact that the inverse tangent function must be evaluated in the proper quadrant. A sketch is often useful in establishing the proper angle.

Polar/Rectangular Conversions

$P \rightarrow R$ $\qquad$ $a + jb = c\angle\theta$ $\qquad$ $P \leftarrow R$

$a = c\cos\theta$ **(1.1a)**

$b = c\sin\theta$ **(1.1b)**

$c = \sqrt{a^2 + b^2}$ **(1.1c)**

$\phi = \arctan(|b/a|)$ **(1.1d)**

Quadrant	*a*	*b*	θ
1st	+	+	ϕ
2nd	−	+	$\pi - \phi$
3rd	−	−	$\pi + \phi$
4th	+	−	$-\phi$

EXAMPLE 1.1A

Find the rectangular form of the complex number $z_o = 2\angle 140° = 2\angle 2.444$.

Solution

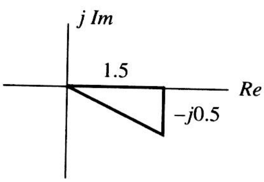
$$a = 2\cos 140° = 2(-0.7664) = -1.533$$
$$b = 2\sin 140° = 2(0.6428) = 1.286$$
$$\therefore z_o = -1.533 + j1.286$$

EXAMPLE 1.1B

Find the polar form of the complex number $z_o = 1.5 - j0.5$.

Solution

A sketch is usually helpful:



$$c = \sqrt{1.5^2 + (-0.5)^2} = \sqrt{2.25 + .25} = 1.581$$
$$\phi = \arctan\left(\left|\frac{-0.5}{1.5}\right|\right) = \arctan\left(\frac{1}{3}\right) = 0.322 = 18.43°$$

A z_o with a positive real part and a negative imaginary part is in the 4th quadrant, so $\theta = -18.43°$.

$$\therefore z_o = 1.581\angle -18.43°$$

Although angles in radians are, strictly speaking, correct, either degrees or radians are generally accepted. It is important, however, to make sure you enter numerical values corresponding to the units your calculator is expecting!

The conversion procedures of Equations 1.1a–d and Examples 1.1a–b are rarely used in practice. Modern technical calculators have built-in rectangular/polar functions that take care of making these conversions for you, including finding the proper quadrant angle information. It is essential that you consult your calculator manual and become proficient in making these conversions.

1.3 EULER'S IDENTITY

In preparing to investigate arithmetic operations involving complex numbers, we need to establish the proper mathematical equivalence of our polar notation $c\angle\theta$. After completing this section you will be able to:

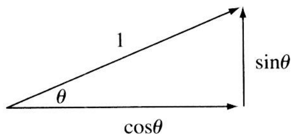
- State and interpret Euler's identity.
- Describe a polar-form complex number using standard math functions.
- Express a cosine as a sum of complex exponentials.
- Express a sine as a sum of complex exponentials.

A very important relationship, known as *Euler's identity*, is

$$e^{\pm j\theta} \equiv \cos\theta \pm j\sin\theta \tag{1.2}$$

If Euler's identity is converted from its rectangular form to polar form, it becomes

$$e^{j\theta} = \cos\theta + j\sin\theta = \sqrt{\cos^2\theta + \sin^2\theta}\angle\arctan(\tan\theta) = 1\angle\theta$$



$$e^{j\theta} \equiv 1\angle\theta \tag{1.3}$$

Thus, the true mathematical interpretation of $A\angle\theta$ is $Ae^{j\theta}$, and the algebraic rules for exponents apply to the angle information of polar-form complex numbers (Table 1.1).

Table 1.1 A Few Laws of Exponents

$e^{jx} e^{jy} = e^{j(x+y)}$	$(e^{jx})^y = e^{jxy} = (e^{jy})^x$	$1/e^{jx} = e^{-jx}$

A few additional special cases follow by direct substitution into Equation 1.2:

$$e^{\pm j0} = 1 \qquad e^{\pm j\pi} = -1 \qquad e^{\pm j2\pi} = 1 \qquad e^{\pm j\pi/2} = \pm j$$

The two signs in Euler's identity give us two equations, which may be solved for the cosine or sine function. For instance,

$$\begin{aligned} e^{j\theta} &= \cos\theta + j\sin\theta \\ e^{-j\theta} &= \cos\theta - j\sin\theta \\ \hline e^{j\theta} + e^{-j\theta} &= 2\cos\theta && \text{(adding the 2 equations)} \\ e^{j\theta} - e^{-j\theta} &= 2j\sin\theta && \text{(subtracting the 2 equations)} \end{aligned}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \tag{1.4}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \tag{1.5}$$

Using these relationships, any sinusoidal signal may be expressed in terms of the even more elementary functions of complex exponentials. This is, in fact, the origin of the *phasor* concept, which is fundamental to our study of alternating current (a-c) circuits. It is difficult to overstate the importance of Euler's identity; we will spend the rest of this text making use of it.

EXAMPLE 1.2

Express $f(x)$ in terms of cosine functions:

$$f(x) = 2e^{j2x} + 4e^{-jx} + 4e^{jx} + 2e^{-j2x}$$

Solution

Regrouping the terms to compare with Equation 1.4, we get

$$f(x) = 2[e^{j2x} + e^{-j2x}] + 4[e^{jx} + e^{-jx}]$$

and

$$f(x) = 4\cos 2x + 8\cos x$$

1.4 COMPLEX-NUMBER ARITHMETIC

Complex numbers are "complex" only in the sense that their arithmetic is more involved than that for real numbers. Adding two complex numbers involves the same amount of effort as adding two sets of two real numbers. This doubles the chances for error as well as the time required to complete the operation. After completing this section you will be able to:

- Conduct numerical calculations with complex numbers.
- Calculate the sum, difference, product, or ratio of complex numbers.
- Raise a complex number to a power.
- Find the conjugate of a complex number.
- Combine a number with its conjugate to find its real or imaginary part, its magnitude, or its angle.

1.4.1 Addition/Subtraction

If $z_1 = x_1 + jy_1$ and $z_2 = x_2 + jy_2$, then $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$

Similarly,

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

The j term serves to remind us that two completely independent types of numbers are present and that they must be handled separately. We also like the final results to be recombined into a single real part and a single imaginary part.

If complex numbers are given in polar form, the usual way to add them is to immediately convert both of them to rectangular form. Most calculators will accept polar-form numbers, convert and add them, and put the result back in polar form for you, all in a single operation. Of course, if two complex numbers have exactly the same angle, their magnitudes may be added directly as if they were vectors.

EXAMPLE 1.3A

Find $z_1 - z_2$ if $z_1 = 2 + j5$ and $z_2 = 2\angle 140°$.

Solution

Since a subtraction is called for, the numbers must be in rectangular form. From Example 1.1a, $z_2 = (2\angle 140°) = -1.533 + j1.286$. Then, $z_1 - z_2 = 2 + j5 - (-1.533 + j1.286) = 3.533 + j3.714$.