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Preface

With many outstanding properties superior to traditional materials, advanced composite materials have found wide application in various engineering sectors. In most applications, composite structures are subjected to a variety of dynamic loadings. A study of wave propagation in such structures is very helpful for understanding their dynamic characteristics and failure mechanism. Moreover, ultrasonic-based non-destructive evaluation (NDE) plays more and more important roles in the determination of material properties and the detection of defects (cracks and flaws) in composites. An analysis of wave behavior is prerequisite in applying effectively and innovatively NDE techniques using ultrasonic and elastic waves.

Analysis of elastic waves in anisotropic media is much more complicated than that for isotropic media. Composite structures in engineering practices are made of a stack of plies, each of which is reinforced by fibers. The ply orientation, ply material, stacking sequence, and the number of layers vary with the different requirements of design. This results in an anisotropic and inhomogeneous nature in the material properties for the laminate. The governing equations for the anisotropic laminates are coupled with each other, and general solutions to these equations are usually very difficult to obtain by conventional methods of analysis. With the rapid development of computers, numerical methods of full domain discretization, such as finite element method (FEM) and finite difference method (FDM), have evolved as flexible and powerful computational tools for a wide range of engineering practical problems. These numerical methods are very often both extravagant and time-consuming in the treatment of waves propagating in anisotropic laminates. The results obtained using entirely numerical methods are usually in the form of a vast volume of data, from which important phenomena and characteristics of wave propagation are extremely difficult to extract and reveal in an explicit manner. This creates obstacles between the physics of wave propagation phenomenon and the analysts (students, researchers, and engineers) who are striving to learn and to understand these important phenomena. Methods of analytical nature with minimum domain discretization are therefore preferred for studying and investigating wave phenomena in composite laminates, as they can provide insightful results revealing important characteristics. This book provides a set of high-performance analytical-numerical methods for elastic wave analysis of anisotropic layered structures. At present, despite certain existing books on wave propagation in anisotropic media, there is no text available that is devoted entirely to modern analytical-numerical methods on this subject.

The material covered in this text is developed from the research work of the first author and co-workers in the past 15 years. The salient feature of this work is that the methods presented are novel and efficient in the treatment of wave propagation in anisotropic layered media. A comprehensive introduction, theoretical development, formulation and applications of these methods are provided. Key techniques discussed are

1. Layered media are discretized only in the thickness direction, so as to take advantage of the FEM to handle difficulties caused by the inhomogeneity of the media in the thickness direction. The use of the FEM leads to a set of dimension-reduced partial differential equations (not algebraic equations).
2. Analytic techniques, such as the Fourier transform technique and modal superimposition method, are used to deal with the dimension-reduced partial differential equations.
3. Strip element method (SEM) is introduced to deal with laminates with delaminations and flaws. Clear advantages of the SEM are demonstrated through examples by comparing the results obtained by the SEM and the FEM. SEM provides also the Green's function for anisotropic laminates. Green's function is essential for the BEM, and it is difficult to obtain for anisotropic media using conventional means.
4. Techniques for problems in both time and frequency domains are treated in great detail. Complex path method is introduced for wavenumber-space domain transformation.
5. A set of six characteristic wave surfaces is introduced to clearly visualize the characteristics of waves propagating in anisotropic layered media.
6. Application of these techniques to smart materials, plates, and shells composed of functionally graded and piezoelectric materials is discussed.
7. Methods for inverse problems related to wave propagation are presented, including material property characterization, impact-loading identification, and crack and flaw detection for anisotropic laminates using elastic waves.
8. Methods proposed for inverse problems are conjugate gradient methods, genetic algorithms, and conventional curve fitting techniques. These methods are effectively combined with forward solvers of wave propagation problems.

The general philosophy governing the writing of the book is to make all the topics insightful but simple, informative but interesting, and theoretical but practical. It attempts to show the reader that the topic of wave propagation and its inverse problems are actually not that difficult.

The book is written primarily for senior university students, postgraduate students, and engineers in civil, mechanical, geographical, aeronautical engineering and engineering mechanics. The book is written in an easy-to-understand manner, using simple and common terminology, so that anyone with an *elementary* knowledge of matrix algebra, the Fourier transform, complex variables, and the linear theory of elasticity should be able to understand the contents fairly easily.

A picture tells a thousand words. Numerous drawings and charts are used to describe important concepts, theories, and results. This is very important for readers coming from non-engineering backgrounds.

A large number of examples are included in the text. The examples are mostly generated using the methods discussed. Software codes of some of these methods are available on the website http://www.nus.edu.sg/ACES. Instruction on the usage of these software codes is also provided there.

The chapters are written in a relatively independent manner. After reading the first chapter, readers can jump to any chapter for the problem or method of his or her interest. Cross-references are provided to link relevant information or materials in other chapters. Readers are advised to read Chapter 1 before reading other chapters. For advanced readers, the material in Chapter 1 might be familiar and too simple, but quickly going through the material could be helpful in the use of terminologies used in the book. The following table gives a concise summary of the content of the book.

Chapters	Topics	Methods Introduced
1	Preliminaries and fundamentals of waves in solids; P wave in a bar	Common terminologies, Analytic procedure, Fourier transform, Cauchy's theorem
2	Waves in functionally graded materials (FGM), P wave in inhomogeneous media	Exact method, Confluent hypergeometric functions, Adaptive integrals
3	P wave, SV wave, SH wave, Characteristics of Lamb waves in laminates, Dispersion of wave velocity, Anisotropy of wave velocity, Wave energy distribution in laminates	Exact method, classic formulation, Method of matrix transfer
4	Harmonic wave in laminates, Transient wave in laminates	Exact method, Matrix formulation, Complex path method, Exponential window method, adaptive integral schemes
5	Characteristics of Lamb waves in laminates, Dispersion relation, Group velocity, six wave surfaces	Layer element method, Matrix formulation, Rayleigh quotient
6	Characteristics of waves in laminated bars	Finite strip element method
7	Characteristics of waves in laminated bars, Edge wave	Semi-exact method
8	Transient wave in laminates	Hybrid numerical method (HNM), Integral schemes for oscillatory integrands

9	Transient wave in FGM plates	HNM, Layer element method
10	Transient wave in Piezoelectric FGM plates, Mechanical excitation, Electric excitation	HNM, Layer element method
11	Wave in solids, Rayleigh wave	Strip element method (SEM)
12	Wave scattering by cracks in laminates, Crack characterization	SEM
13	Wave scattering by flaw in laminates and sandwich laminates, SH wave, Flaw characterization	SEM
14	Bending deformation of laminated plates, Bending wave in laminated plates	SEM
15	Characteristics of helical waves in laminated cylinders, Dispersion relation, Group velocity, six wave surfaces	Layer element method, Rayleigh quotient,
16	Wave scattering by cracks in laminated cylinders, Crack characterization	SEM
17	Inverse reconstruction of impact loading	HNM, Conjugate gradient method
18	Inverse characterization of material property of laminates	HNM, Genetic algorithms

A chapter-by-chapter description of the book is given below.

Chapter 1 provides fundamentals and terminologies for wave propagation in elastic media. One-dimensional longitudinal waves are analyzed in both frequency and time domains. This chapter familiarizes the reader with standard procedures in dealing with wave propagation problems.

Chapter 2 deals with harmonic and transient longitudinal waves in plates made of functionally grade materials. An idealized one-dimensional problem is studied. We formulate an inhomogeneous element method and apply it to a functionally grade plate subjected to harmonic or transient excitations. Waves propagating across the plate are revealed numerically. A novel integral technique is introduced for evaluating the confluent hypergeometric functions with complex valued argument.

Chapter 3 inquires into Lamb wave propagation in anisotropic laminates by an exact method based on classic formulation. The method of matrix transfer is employed. The dispersion and anisotropy of phase velocities for fundamental modes are computed and discussed in detail. The energy distributions in the thickness direction of the laminate are calculated for various modes of Lamb waves.

Chapter 4 formulates an exact method for analyzing elastic waves (both harmonic and transient) propagating in laminates. The dynamic equilibrium equation in the wave number domain is developed by the Fourier transform technique. The solutions in the spatial domain are obtained with the inverse Fourier transform technique. A complex path technique for evaluating the inverse Fourier integration is introduced. The transient solutions are found by applying the Fourier superposition method. The exponential window method is introduced in order to avoid the singularity of integration. Examples are presented to demonstrate the effectiveness and efficiency of the method.

Chapter 5 studies the frequency and group velocity dispersions and characteristic wave surfaces in laminated composite plates. The layer element method is introduced in detail. Dispersion curves for composite laminates are computed. A formula for group velocity is established using the Rayleigh's quotient. Group velocity spectra for various composite laminates are calculated. Six characteristic wave surfaces are defined, formulated, and visualized for composite laminates. These characteristic surfaces can illustrate efficiently the qualitative properties of Lamb waves propagating in anisotropic laminates.

Chapter 6 analyzes harmonic waves in anisotropic laminated bars. The finite strip element method is formulated and used to investigate wave modes in anisotropic laminated bars. The bar is divided into layer elements in the thickness direction and series expansion is employed in the width direction.

Chapter 7 formulates a semi-exact method for waves in laminated bars. The bar is divided into layer elements in the thickness direction, and the displacement is approximated using shape function only in the thickness direction. The solution in width direction is solved in an exact manner. Wave modes in bars are computed using the semi-exact method and plotted in a number of graphs. Edge waves on the edge of anisotropic laminates are also investigated.

Chapter 8 describes the hybrid numerical method (HNM) used for wave propagation in composite laminates. The laminate is divided into layer elements, and the displacement in the thickness direction of the laminate is approximated using shape functions. The system equations are derived by the principle of virtual work. The transient waves are simulated using the Fourier transform in conjunction with the modal analysis method. Quadrature methods for carrying out the inverse Fourier transform are also introduced.

Chapter 9 applies the layer element method and HNM introduced in the preceding chapters to analyze dispersion and transient waves in plates made of functionally grade materials. Strong surface waves are observed in a number of example problems.

Chapter 10 analyzes waves in plates made of functionally graded piezoelectric materials (FGPM). Detailed formulation for waves in such plates is provided. Piezoelectrical-mechanical coupled system equation is derived using variational principle. Characteristics of waves are investigated, and displacement and electrostatic potential responses are computed for mechanical and electric excitations. Strong surface waves are observed on the softer surface of the plate.

Chapter 11 formulates a strip element method (SEM) for analyzing waves in linearly elastic solids. A number of nonreflection boundary conditions are discussed in detail. The efficiency of the SEM is proved through the comparison with results from the literature for a variety of classical wave propagation problems.

Chapter 12 presents the SEM for dealing with wave scattering by a crack in composite laminates. A laminate with cracks is divided into subdomains to which the SEM is applicable. Procedures of assembling the SEM equations of all the subdomains are detailed. Numerical examples are given for wave scattering by both horizontal and vertical cracks in laminates. Characterization of cracks in composite laminates using the SEM is also studied in detail.

Chapter 13 investigates the scattering of harmonic waves in an anisotropic laminate containing a flaw. A simple rectangular flaw is considered and formulated using SEM. Results indicate that it is possible to detect the presence of a flaw, determine its position, and estimate its size by examining the scattering of the wave response. Characterization of defects in sandwich plates is also studied using SH waves. SEM formulation for SH waves is presented. Based on the wave field scattered by defects, methods for characterizing the size and location of the defects are described.

Chapter 14 applies the strip element method to deal with a bending deformation and bending waves in anisotropic laminates. Detailed SEM formulae are provided. Example problems for plates with various boundary conditions are analyzed.

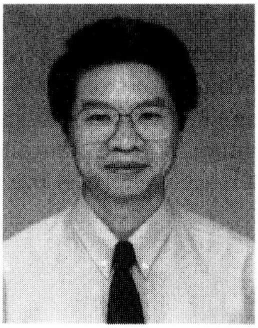
Chapter 15 formulates a layer element method for the dispersion and wave surfaces in laminated composite cylinders. The helical wave in anisotropic hollow cylinders is analyzed. Numerical examples include frequency and group velocity spectra, as well as a set of six characteristic wave surfaces.

Chapter 16 discusses the waves scattered by a crack in laminated composite cylinders. The SEM described in Chapters 11 and 12 is generalized to the case for cracked laminated composite cylinders. The formulation for axisymmetric guided waves is derived. Numerical results are given for wave scattering by axial and radial cracks in both hollow cylinders and circular cylindrical shells.

Chapter 17 introduces techniques for inverse identification of impact loadings on the surface of laminates using elastic waves simulated by HNM. Two- and three-dimensional problems are studied. Both time function and spatial distribution function are reconstructed for a number of example problems.

Chapter 18 introduces techniques for inverse characterization of material properties of laminates using elastic waves simulated by HNM. Two-dimensional problems are studied. A genetic algorithm is employed for minimizing the error function in the determination of the elastic constants of laminates.

Authors



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