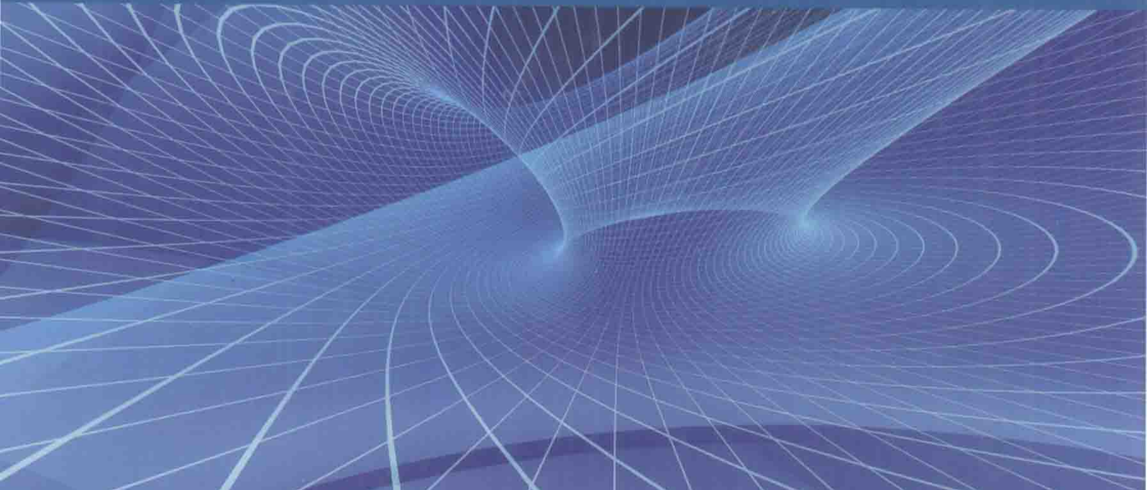
Concepts of Combinatorial Optimization

Edited by Vangelis Th. Paschos



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Vangelis Th. Paschos

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Concepts of Combinatorial Optimization

Preface

What is combinatorial optimization? There are, in my opinion, as many definitions as there are researchers in this domain, each one as valid as the other. For me, it is above all the art of understanding a real, natural problem, and being able to transform it into a mathematical model. It is the art of studying this model in order to extract its structural properties and the characteristics of the solutions of the modeled problem. It is the art of exploiting these characteristics in order to determine algorithms that calculate the solutions but also to show the limits in economy and efficiency of these algorithms. Lastly, it is the art of enriching or abstracting existing models in order to increase their strength, portability, and ability to describe mathematically (and computationally) other problems, which may or may not be similar to the problems that inspired the initial models.

Seen in this light, we can easily understand why combinatorial optimization is at the heart of the junction of scientific disciplines as rich and different as theoretical computer science, pure and applied, discrete and continuous mathematics, mathematical economics, and quantitative management. It is inspired by these, and enriches them all.

This book, *Concepts of Combinatorial Optimization*, is the first volume in a set entitled *Combinatorial Optimization*. It tries, along with the other volumes in the set, to embody the idea of combinatorial optimization. The subjects of this volume cover themes that are considered to constitute the hard core of combinatorial optimization. The book is divided into three parts:

– Part I: Complexity of Combinatorial Optimization Problems;

– Part II: Classical Solution Methods;

– Part III: Elements from Mathematical Programming.

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In the first part, Chapter 1 introduces the fundamentals of the theory of (deterministic) complexity and of algorithm analysis. In Chapter 2, the context changes and we consider algorithms that make decisions by "tossing a coin". At each stage of the resolution of a problem, several alternatives have to be considered, each one occurring with a certain probability. This is the context of probabilistic (or randomized) algorithms, which is described in this chapter.

In the second part some methods are introduced that make up the great classics of combinatorial optimization: branch-and-bound and dynamic programming. The former is perhaps the most well known and the most popular when we try to find an optimal solution to a difficult combinatorial optimization problem. Chapter 3 gives a thorough overview of this method as well as of some of the most well-known tree search methods based upon branch-and-bound. What can we say about dynamic programming, presented in Chapter 4? It has considerable reach and scope, and very many optimization problems have optimal solution algorithms that use it as their central method.

The third part is centered around mathematical programming, considered to be the heart of combinatorial optimization and operational research. In Chapter 5, a large number of linear models and an equally large number of combinatorial optimization problems are set out and discussed. In Chapter 6, the main simplex algorithms for linear programming, such as the primal simplex algorithm, the dual simplex algorithm, and the primal–dual simplex algorithm are introduced. Chapter 7 introduces some classical linear programming methods, while Chapter 8 introduces quadratic integer optimization methods. Chapter 9 describes a series of resolution methods currently widely in use, namely column generation. Chapter 10 focuses on polyhedral methods, almost 60 years old but still relevant to combinatorial optimization research. Lastly, Chapter 11 introduces a more contemporary, but extremely interesting, subject, namely constraint programming.

This book is intended for novice researchers, or even Master's students, as much as for senior researchers. Master's students will probably need a little basic knowledge of graph theory and mathematical (especially linear) programming to be able to read the book comfortably, even though the authors have been careful to give definitions of all the concepts they use in their chapters. In any case, to improve their knowledge of graph theory, readers are invited to consult a great, flagship book from one of our gurus, Claude Berge: *Graphs and Hypergraphs*, North Holland, 1973. For linear programming, there is a multitude of good books that the reader could consult, for example V. Chvátal, *Linear Programming*, W.H. Freeman, 1983, or M. Minoux, *Programmation mathématique: théorie et algorithmes*, Dunod, 1983.

Editing this book has been an exciting adventure, and all my thanks go, firstly, to the authors who, despite their many responsibilities and commitments (which is the lot of any university academic), have agreed to participate in the book by writing chapters in their areas of expertise and, at the same time, to take part in a very tricky exercise: writing chapters that are both educational and high-level science at the same time.

This work could never have come into being without the original proposal of Jean-Charles Pomerol, Vice President of the scientific committee at Hermes, and Sami Ménascé and Raphaël Ménascé, the heads of publications at ISTE. I give my warmest thanks to them for their insistence and encouragement. It is a pleasure to work with them as well as with Rupert Heywood, who has ingeniously translated the material in this book from the original French.

Vangelis Th. PASCHOS
June 2010

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PART I

Complexity of Combinatorial Optimization Problems

Chapter 1

Basic Concepts in Algorithms and Complexity Theory

1.1. Algorithmic complexity

In algorithmic theory, a problem is a general question to which we wish to find an answer. This question usually has parameters or variables the values of which have yet to be determined. A problem is posed by giving a list of these parameters as well as the properties to which the answer must conform. An instance of a problem is obtained by giving explicit values to each of the parameters of the instanced problem.

An algorithm is a sequence of elementary operations (variable affectation, tests, forks, etc.) that, when given an instance of a problem as input, gives the solution of this problem as output after execution of the final operation.

The two most important parameters for measuring the quality of an algorithm are: its *execution time* and the *memory space* that it uses. The first parameter is expressed in terms of the number of instructions necessary to run the algorithm. The use of the number of instructions as a unit of time is justified by the fact that the same program will use the same number of instructions on two different machines but the time taken will vary, depending on the respective speeds of the machines. We generally consider that an instruction equates to an elementary operation, for example an assignment, a test, an addition, a multiplication, a trace, etc. What we call the *complexity in time* or simply the *complexity* of an algorithm gives us an indication of the time it will take to solve a problem of a given size. In reality this is a function that associates an order of

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