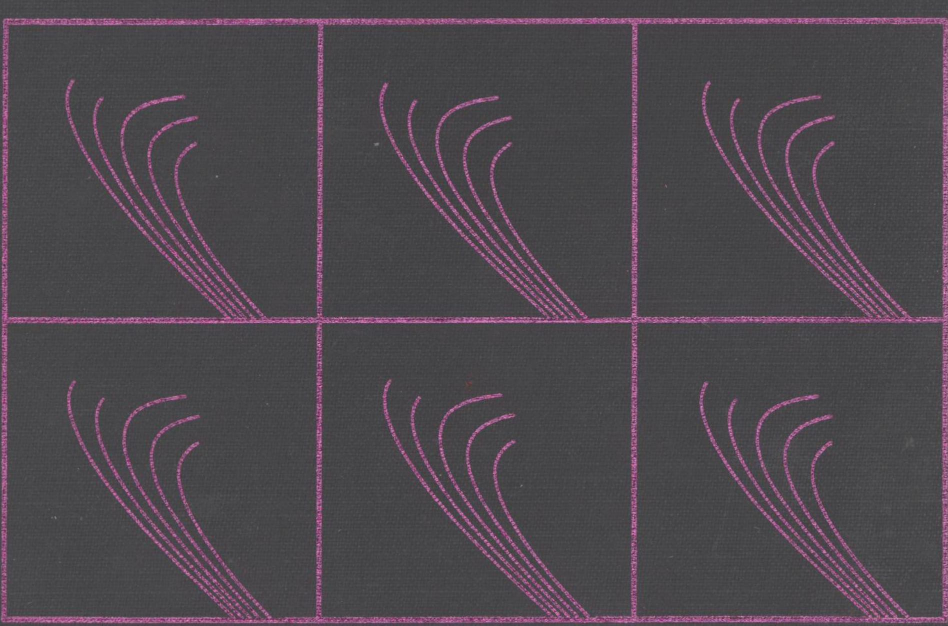
INTRODUCTION TO OPERATIONS RESEARCH



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INTRODUCTION TO OPERATIONS RESEARCH

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Rensselaer Polytechnic Institute

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INTRODUCTION TO OPERATIONS RESEARCH

To Juanita and to
Kelly and Steve

To Gail

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ABOUT THE AUTHORS

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PREFACE

This textbook is intended for use in a two-semester sequence of courses introducing the mathematical methods of operations research. Part I can also be used alone for a one-semester course on linear programming. We have chosen to provide deep and thorough coverage of the most important methods in operations research rather than a superficial treatment of a larger number of topics. The level of exposition is appropriate for juniors and seniors who are majoring in engineering, computer science, mathematics, and quantitative methods in management.

The basic techniques of operations research are simple and straightforward, and only a small amount of advanced mathematics is needed for a technically accurate introduction to the subject. This textbook assumes a knowledge of high school elementary algebra and a familiarity with simple matrix notation such as would be introduced in the first class of an undergraduate linear algebra course. In addition, Chapters 9 and 10 assume a knowledge of elementary differential calculus, and Part III assumes a knowledge of elementary probability and statistics. A concise appendix on matrix notation makes the book accessible to students who have not previously had any linear algebra.

Both the style of exposition and the mathematical notation have been chosen to reflect the simplicity of the subject, and the readability of the text is considered more important than rigor. Examples are used extensively to introduce and motivate the topics. In this way, the presentation reflects the inductive process of scientific discovery rather than imitating the retrospective deduction that is typical in research papers. This approach gives the student the opportunity to rediscover the important results personally instead of merely reading about them in theorems. Proofs are given for some results, but only after the result has been illustrated by example, and only when the proof provides a constructive method for solving problems. Thus the book is not a treatise on mathematical theory.

The simplicity of the methods used in the book means that they can be deeply understood even by beginning students of the subject, and the treatment is mathematically precise even though the results are often stated informally. Thus the discussion is not a cookbook tabulation of trite formulae, and the student should reasonably be expected to understand the mathematical basis for the techniques in addition to being able to apply them.

Answers to selected exercises are given at the end of the book, and a separate answer book is available that contains complete solutions to all of the exercises.

The development of this text benefited greatly from the comments and suggestions made by our colleagues during its use in preliminary form over the past four years. In particular, we express our gratitude to Professors Carlton E. Lemke (Rensselaer Polytechnic Institute), Richard T. Wong (Purdue University), and Thomas M. Liebling (École Polytechnique Fédérale de Lausanne). We also thank the many RPI students who used the preliminary versions, proofread the text, and tested out the exercises. Special thanks are due to Richard Sych, Lori Grieb, Laura Ripans, Carla Bryan, and Robert Bosch.

Joseph G. Ecker
Michael Kupferschmid

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CHAPTER 1

INTRODUCTION

During World War II, the U.S. Army Air Corps suffered many casualties in the course of flying strategic bombing missions over Nazi Germany. An anecdote about that era concerns a study conducted to determine how to reduce those horrible losses. Army B-17 aircraft were examined for holes made by bullets and flak during bombing missions, and the location of each hole was marked on an outline drawing of a B-17. When all of the holes had been marked, it was clear that some parts of a B-17 were much more likely to suffer battle damage than others. The general in charge of the study convened a briefing of military planners to propose that the most heavily damaged areas be provided with additional armor plating. Finding the best places to put the extra armor was very important because only a small amount of weight could be added to the airplanes. At the conclusion of the briefing, after many of those in attendance had agreed to the soundness of the plan, a junior officer in the back of the room timidly raised his hand and was recognized to speak. Clearing his throat nervously, the lieutenant asked in a small voice if it might not be better to armor the parts of the airplane showing the fewest holes rather than the parts with the most severe damage. "After all," he pointed out, "the airplanes that you measured the holes in are the ones that came back."

1.1 OPERATIONS RESEARCH

The preceding story is probably apocryphal, but the question of where to put the armor has many features that are typical of problems in **operations**

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