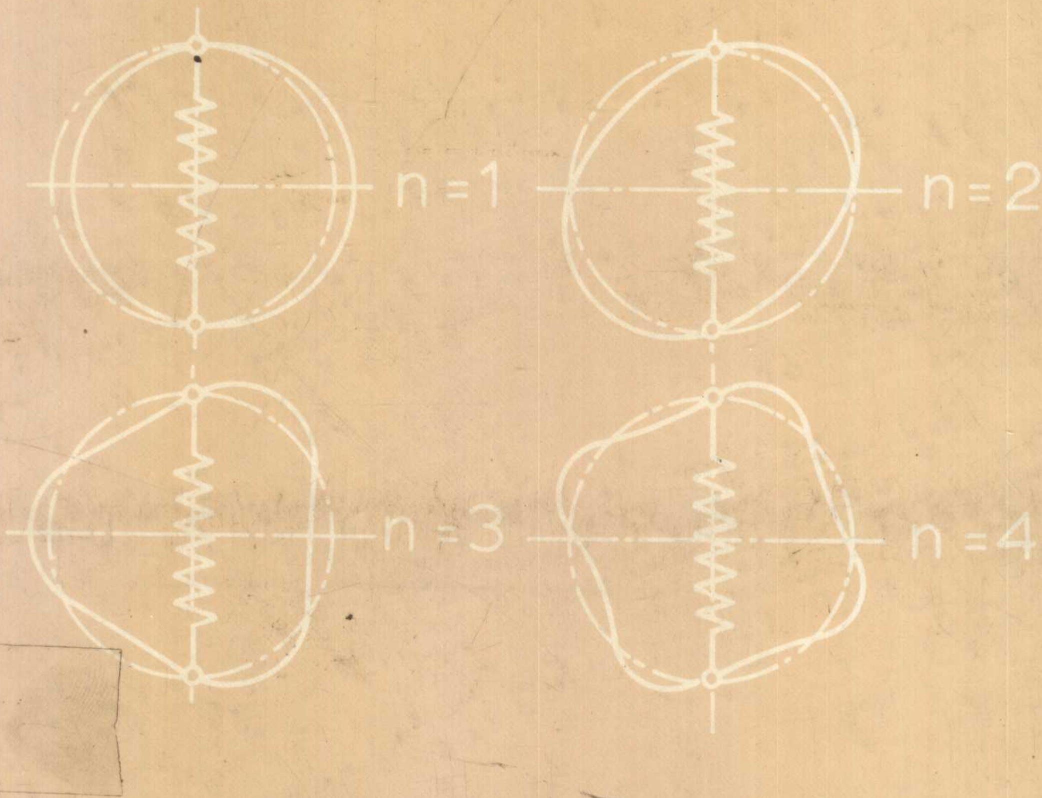
MECHANICAL ENGINEERING/10

VIBRATIONS OF SHELLS AND PLATES



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Vibrations of Shells and Plates

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PREFACE

This book attempts to give engineering graduate students and practicing engineers an introduction to the vibration behavior of shells and plates. It is also hoped that it will prove to be a useful reference to the vibration specialist. It fills a need in the present literature on this subject, since it is the current practice to either discuss shell vibrations in a few chapters at the end of texts on shell statics that may be well written but are too limited in the selection of material, or to ignore shells entirely in favor of plates and membranes, as in some of the better known vibration books. There are a few excellent monographs on very specialized topics, for instance, on natural frequencies and modes of cylindrical and conical shells. But a unified presentation of shell and plate vibration, both free and forced, and with complicating effects as they are encountered in engineering practice, is still missing. This collection attempts to fill the gap.

The state of the art of modern engineering demands that engineers have a good knowledge of the vibration behavior of structures beyond the usual beam and rod vibration examples. Vibrating shell and plate structures are not only encountered by the civil, aeronautical, and astronautical engineer, but also by the mechanical, nuclear, chemical, and industrial engineer. Parts or devices such as engine liners, compressor shells, tanks, heat exchangers, life support ducts, boilers, automotive tires, vehicle bodies, valve reed plates, and saw disks, are all composed of structural elements that cannot be approximated as vibrating beams. Shells especially exhibit certain effects that are not present in beams or even plates and cannot be interpreted by engineers who are only familiar with beam-type vibration theory. Therefore, this book stresses the understanding of basic phenomena in shell and plate vibrations and it is hoped that the material covered will be useful in explaining experimental measurements or the results of the ever-increasing number of finite element programs. While it is the goal of every engineering manager that these programs will eventually be used as black boxes, with input provided and output obtained by relatively untrained technicians, reality shows that the interpretation of results of these programs requires a good background in finite element theory and, in the case of shell and plate vibrations, in vibration theory of greater depth and breadth than usually provided in standard texts.

It is hoped that the book will be of interest to both the stress analyst whose task it is to prevent failure and to the acoustician whose task it is to control noise. The treatment is fairly complete as far as the needs of the stress analysts go. For acousticians, this collection stresses those applications in which boundary conditions cannot be ignored.

The note collection begins with a historical discussion of vibration analysis and culminates in the development of Love's equations of shells. These equations are derived in Chapter 2 in curvilinear coordinates. Curvilinear coordinates are used throughout as much as possible, because of the loss of generality that occurs when specific geometries are singled out. For instance, the effect of the second curvature cannot be recovered from a specialized treatment of cylindrical shells. Chapter 3 shows the derivation by reduction of the equations of some standard shell geometries that have a tendency to occur in standard engineering practice, like the circular cylindrical shell, the spherical shell, the conical shell, and so on. In Chapter 4 the equations of motion of plates, arches, rings, beams, and rods are obtained. Beams and rings are sometimes used as supplementary examples in order to tie in the knowledge of beams that the reader may have with the approaches and results of shell and plate analysis.

Chapter 5 discusses natural frequencies and modes. It starts with the transversely vibrating beam, followed by the ring and plate. Finally, the exact solution of the simply supported circular cylindrical shell is derived. The examples are chosen in such a way that the essential behavior of these structures is unfolded with the help of each previous example; the intent is not to exhaust the number of possible analytical solutions. For instance, in order to explain why there are three natural frequencies for any mode number combination of the cylindrical shell, the previously given case of the vibrating ring is used to illustrate modes in which either transverse or circumferential motions dominate.

In the same chapter, the important property of orthogonality of natural modes is derived and discussed. It is pointed out that when two or more different modes occur at the same natural frequency, a superposition mode may be created that may not be orthogonal, yet is measured by the experimenter as the governing mode shape. Ways of dealing with this phenomenon are also pointed out.

For some important applications, it is possible to simplify the equations of motion. Rayleigh's simplification, in which either the bending stiffness or the membrane stiffness is ignored, is presented. However, the main thrust of Chapter 6 is the derivation and use of the Donnell-Mushtari-Vlasov equations.

While the emphasis of Chapter 5 was on so-called exact solutions (series solutions are considered exact solutions), Chapter 7 presents some of the more common approximate techniques to obtain solutions for geometrical shapes and boundary condition combinations that do not lend themselves to exact analytical treatment. First, the variational techniques known as the Rayleigh-Ritz technique and Galerkin's method and variational method are presented. Next, the purely mathematical technique of finite differences is outlined, with examples. The finite element method follows. Southwell's and Dunkerley's principles conclude the chapter.

The forced behavior of shells and plates is presented in Chapters 8, 9, and 10. In Chapter 8, the modal analysis approach is used to arrive at the general solution for distributed dynamic loads in transverse and two orthogonal in-plane directions. The Dirac delta function is then used to obtain the solutions for point and line loads. Chapter 9 discusses the dynamic Green's function approach and applies it to traveling load problems. An interesting resonance condition that occurs when a load travels along the great circles of closed shells of revolution is shown. Chapter 10 extends the types of possible loading to the technically significant set of dynamic moment loading, and illustrates it by investigating the action of a rotating point moment as it may occur when rotating unbalanced machinery is acting on a shell structure.

The influence of large initial stress fields on the response of shells and plates is discussed in Chapter 11. First, Love's equations are extended to take this effect into account. It is then demonstrated that the equations of motion of pure membranes and strings are a subset of these extended equations. The effect of initial stress fields on the natural frequencies of structures is then illustrated by examples.

In the original derivation of Love's equations, transverse shear strains, and therefore shear deflections, were neglected. This becomes less and less permissible as the average distance between node lines associated with the highest frequency of interest approaches the thickness of the structure. In Chapter 12, the shear deformations are included in the shell equations. It is shown that these equations reduce in the case of a rectangular plate and the case of a uniform beam to equations that are well known in the vibration literature. Sample cases are solved to illustrate the effect shear deformation has on natural frequencies.

Rarely are practical engineering structures simple geometric shapes. In most cases the shapes are so complicated that finite element or difference methods have to be used for accurate numerical results. However, there is a category of cases in which the engineering structures can be interpreted as being assembled of two or more classic shapes or parts. In Chapter 13, the method of receptance is presented and used to obtain, for instance, very general design rules for stiffening panels by ring- or beam-type stiffeners. It is also shown that the receptance method gives elegant and easily interpretable results for cases in which springs or masses are added to the basic structure.

The formulation and use of equivalent viscous damping was advocated in the forced vibration chapters. For steady-state harmonic response problems a complex modulus is often used. In Chapter 14, this type of structural damping, also called hystereses damping, is presented and tied in with the viscous damping formulation.

Because of the increasing importance of composite material structures, the equations of motion of laminated shells are presented and discussed in Chapter 15, along with some simple examples.

This book evolved over a period of almost ten years from lecture notes on the vibration of shells and plates. To present the subject in a unified fashion made it necessary to do some original work in areas where the available literature did not provide complete information. Some of it was done with the help of graduate students attending my lectures, for instance R. G. Jacquot, U. R. Kristiansen, J. D. Wilken, M. Dhar, U. Bolleter, and D. P. Powder. Especially talented in detecting errors were M. G. Prasad, F. D. Wilken, M. Dhar, S. Azimi, and D. P. Egolf. Realizing that I have probably forgotten some significant contributions, I would like to single out in addition O. B. Dale, J. A. Adams, D. D. Reynolds, M. Moaveni, R. Shashaani, R. Singh, J. R. Friley, J. DeEskinazi, F. Laville, E. T. Buehlmann, N. Kaemmer, C. Hunckler, and J. Thompson, and extend my appreciation to all my former students.

I would also like to thank my colleagues on the Purdue University faculty for their direct or indirect advice.

If this book is used for an advanced course in structural vibrations of about forty-five lectures, it is recommended that Chapters 2 through 8 be treated in depth. If there is time remaining, highlights of the other chapters can be presented. Recommended prerequisites are a first course in mechanical vibrations and knowledge of boundary value problem mathematics.

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CONTENTS

1

HISTORICAL DEVELOPMENT OF VIBRATION ANALYSIS OF CONTINUOUS STRUCTURAL ELEMENTS

Vibration analysis has its beginnings with Galileo Galilei (1564-1642), who solved by geometrical means the dependence of the natural frequency of a simple pendulum on the pendulum length [1.1]. He proceeded to make experimental observations on the vibration behavior of strings and plates, but could not offer any analytical treatment. He was partially anticipated in his observations of strings by his contemporary Marin Mersenne (1588-1648), a French priest. Mersenne recognized that the frequency of vibration is inversely proportional to the length of the string and directly proportional to the square root of the cross-sectional area [1.2]. This line of approach found its culmination in Joseph Sauveur (1653-1716), who coined the terminology "nodes" for zero displacement points on a string vibrating at its natural frequency and also actually calculated an approximate value for the fundamental frequency as a function of the measured sag at its center, similar to the way the natural frequency of a single degree of freedom spring-mass system can be calculated from its static deflection [1.3].

The foundation for a more precise treatment of the vibration of continuous systems was laid by Robert Hook (1635-1703) when he

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it is almost a rule in the history of science that people that are credited with an achievement do not completely deserve it. Progress moves in small steps and it is often the individual who publishes at the right developmental step and at the right time who gets the public acclaim.

The longitudinal vibration of rods was investigated experimentally by Chladni [1.9] and Biot [1.10]. However, not until 1824 do we find the published analytical equation and solutions, done by Navier. This is interesting since the analogous problem of the longitudinal vibration of air columns was already done in 1727 by Euler [1.11].

The equation for the transverse vibration of flexible thin beams was derived in 1735 by Daniel Bernoulli [1.12] and the first solutions for simply supported ends, clamped ends, and free ends where found by Euler [1.13] and published in 1744.

The first torsional vibration solution, but not in a continuous sense, was given in 1784 by Coulomb [1.14]. But not until 1827 do we find an attempt to derive the continuous torsional equation [1.15]. This was done by Cauchy (1789-1857) in an approximate fashion. Poisson (1781-1840) is generally credited for having derived the one dimensional torsional wave equation in 1827 [1.16]. The credit for deriving the complete torsional wave equation and giving some rigorous results belongs to Saint-Venant (1797-1886), who published this in 1849 [1.17].

In membrane vibrations, Euler in 1766 published equations for a rectangular membrane that were incorrect for the general case but will reduce to the correct equation for the uniform tension case [1.18]. It is interesting to note that the first membrane vibration case investigated analytically was not the circular membrane, even while the latter, in form of the drum head, would have been the more obvious shape. The reason is that Euler was able to picture the rectangular membrane as a superposition of a number of crossing strings. In 1828 Poisson read a paper to the French Academy of Science on the special case of uniform tension and showed the circular membrane equation and solved it for the special case of