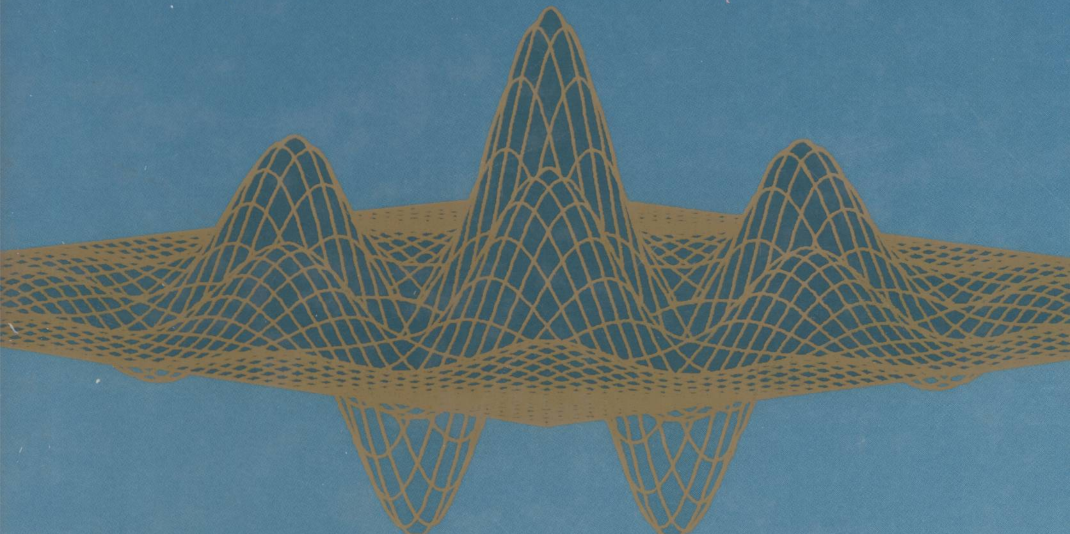
WAVELET ANALYSIS AND ITS APPLICATIONS VOLUME 6

MULTISCALE WAVELET METHODS

For Partial Differential Equations



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Multiscale Wavelet Methods for Partial Differential Equations

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ACADEMIC PRESS
San Diego London Boston New York Sydney Tokyo Toronto

ACADEMIC PRESS
525 B Street, Suite 1900, San Diego, CA 92101-4495, USA
1300 Boylston Street, Chestnut Hill, MA 02167, USA
http://www.apnet.com

ACADEMIC PRESS LIMITED
24–28 Oval Road, London NW1 7DX, UK
http://www.hbuk.co.uk/ap/

Library of Congress Cataloging-in-Publication Data

Multiscale wavelet methods for partial differential equations / edited by Wolfgang Dahmen, Andrew Kurdila, Peter Oswald.
p. cm. — (Wavelet analysis and its applications ; v. 6)
Includes bibliographical references and index.
ISBN 0-12-200675-5 (alk. paper)
1. Differential equations, Partial—Numerical solutions.
2. Wavelets (Mathematics) I. Dahmen, Wolfgang.
II. Kurdila, Andrew. III. Oswald, Peter. IV. Series.
QC20.7.D5M83 1997 97-12672
515'.2433—DC21 CIP

Printed in the United States of America
97 98 99 00 01 IP 9 8 7 6 5 4 3 2 1

Multiscale Wavelet Methods for Partial Differential Equations

Wavelet Analysis and Its Applications

The subject of wavelet analysis has recently drawn a great deal of attention from mathematical scientists in various disciplines. It is creating a common link between mathematicians, physicists, and electrical engineers. This book series will consist of both monographs and edited volumes on the theory and applications of this rapidly developing subject. Its objective is to meet the needs of academic, industrial, and governmental researchers, as well as to provide instructional material for teaching at both the undergraduate and graduate levels.

This is the sixth book of the series. While wavelet techniques and algorithms have proved to be very powerful in several areas of applications such as signal processing and image compression, the wavelet approach to solving complex problems governed by physical models such as partial differential equations (PDEs) has to compete with the well-established and already very effective methods. There is, however, a common link between the multiresolution approximation structure in wavelet analysis and multilevel and multigrid techniques in numerical solutions of PDEs. This volume is aimed at bridging these two most fruitful approaches and is designed to update current developments in PDEs that are somewhat related to the wavelet approach.

The series editor wishes to congratulate the editors of this volume for an outstanding job in selecting the most relevant chapters and in carefully editing the volume. He would also like to thank the authors for their very fine contributions.

Preface

Wavelet methods are by now well-established as a novel and successful mathematical tool with applications in Signal Analysis and Image Processing, Theoretical Physics and Mathematics itself. Meanwhile several books and edited volumes have appeared, in particular, in this series *Wavelet Analysis and Its Applications*, addressing this fast growing field of reasearch. More recently the intriguing aspects of these concepts have stirred considerable interest in applying them also to more traditional areas of numerical analysis and engineering applications. The high-level contributions to the *Special Session on Wavelet Galerkin Methods* at the 31st *Annual Meeting of the Society of Engineering Science* held in College Station, TX, in October 1994 as well as the *Special Session on Meshless and Wavelet Methods* at the 3rd *U.S. National Congress in Computational Mechanics* held in Dallas in June 1995 and the partly very controversial panel discussions on these meetings indicated a strong need of an up-to-date publication on current developments and further prospects of this methodology, specifically in the area of partial differential equations (PDEs).

It is fair to say that wavelet analysis as it stands has by far not yet reached a steady state in this field. As promising as many of the underlying concepts are, it would be naive to expect their immediate practical success in complex, real-life applications governed by PDEs. Current wavelet methods and corresponding software tools are still primarily confined to model problems. While these new ideas may also actually trigger new developments in the context of well-established multilevel techniques, they will ultimately have to compete with the discretization methods and existing software packages for PDEs. This volume aims at contributing to the progress in this direction. We do not claim to reflect an exhaustive account of the state of the art. However, we have made an effort to address several aspects which we feel are important and typical in connection with solving partial differential equations and which bear potential for further progress. Key ideas such as sparsity of wavelet representations of operators, economic representations of functions with localized singularities or strongly scale dependent behavior, the availability of new libraries of flexible localized basis functions (wavelets and wavelet packets) as well as resulting fast multiscale algorithms will be presented in the context of linear and nonlinear operator equations.

To support a first orientation we have grouped the material into six chapters which are, however, interrelated in many respects. The following comments might serve as a brief guide.

The first chapter may be viewed as a bridge to finite element based multilevel preconditioning and multigrid techniques. Recent development has shown that the multiscale space decomposition framework typical for wavelets provides significant insight into the understanding of algorithms like hierarchical bases or BPX schemes. On the other hand, multigrid technology adds further algorithmical variety and fuels intertwining of these concepts. Oswald focusses on frame based multilevel Schwarz preconditioners for elliptic boundary value problems on bounded domains in $\mathbb{R}^d$. The main goal is to preserve as much as possible the algorithmic advantages of scale- and shift-invariant discretizations typical for the classical wavelet setting by introducing local modifications only near the boundary. As for finite element schemes, adaptive nested refinement can be incorporated. Vassilevski and Wang start from the hierachical basis method by Yserentant for linear finite element discretizations (which is asymptotically nonoptimal), and propose an improvement based on approximately computing wavelet-like complement basis functions. They also discuss the multiplicative algorithms corresponding to these space decompositions which provides a link to multigrid V-cycle solvers.

Chapter 2 offers information on principal features of wavelet based discretizations centering upon sparse representations of operators and functions as well as resulting adaptivity concepts. Bertoluzza highlights the algorithmic benefits of interpolatory Deslaurier-Dubuc wavelets for adaptive collocation methods. The method is illustrated on several linear problems including dominating convection, but might be attractive for nonlinear problems as well. Beylkin and Keiser present a systematic algorithmic study of a class of periodic nonlinear evolution equations covering, for instance, the viscous Burgers equation and Korteweg-de-Vries equation. Their main objective is to produce a scheme which solves such problems for a given tolerance at a cost which remains proportional to the number of significant wavelet coefficients of the solution. Essential ingredients are sparse operator representations in the so-called nonstandard form and the fast evaluation of nonlinear terms. Joly, Maday, and Perrier apply wavelet packets and compression techniques to the adaptive treatment of nonlinear evolution equations. In particular, a best basis concept based on cardinal entropy is introduced and its practical implementation for time-dependent PDEs is discussed. The numerical examples are concerned again with the Burgers equation with small viscosity, and simple convection-diffusion problems. Dahlke, Dahmen, and DeVore address the issue of adaptivity from a primary analytical viewpoint. The goal is to interrelate the concepts of nonlinear approximation, Besov regularity, and wavelet based adaptive techniques for stationary elliptic problems covering integral as well as differential operators. In particular adaptive refinement strategies are shown to converge without additional a priori assumptions on the solution.

In Chapter 3, two papers on integral equations closely related to the material of the previous chapters are included. Von Petersdorff and Schwab propose a wavelet based, fully discrete Galerkin scheme for a zero-order elliptic boundary integral equation. They combine the wavelet compression of the intergral operator with a carefully designed adaptive quadrature scheme which ensures the same asymptotical complexity in the computation of the compressed matrix as the solver. This is a central step towards efficient practical implementations. The paper by Rieder is devoted to additive and multiplicative wavelet solvers of Tikhonov regularized ill-posed problems. The techniques are similar in spirit to those in Chapter 1.

The paper by Barsch, Kunoth and Urban in Chapter 4 surveys a software toolbox under development which aims at providing an experimental platform for wavelet discretizations of PDEs and integral equations. Some emphasis is put on the use of C++ for treating multidimensional multiscale data structures and algorithms. Ko, Kurdila and Oswald present a comparative study of several multilevel preconditioners for a second order model problem on simple domains. In particular, schemes based on finite elements, Daubechies and AFIF scaling functions and wavelets are compared.

Chapter 5 is different in nature. Rather than the efficiency of algorithms, the main concern here is to employ wavelets as an adequate tool for analyzing and simulating multiscale interaction/separation in flows. In Elezgaray, Berkooz, Dankowicz, Holmes and Myers, local models for the study of coherent structures of solutions of the Kuramoto-Sivashinsky equation on large intervals are investigated. The use of periodic wavelets is proposed and compared with the traditional Fourier approximations. In the paper by Wickerhauser, Farge and Goirand, the complexity of fully developed turbulent flows is investigated with the aid of concepts like theoretical dimension, best bases and wavelet packets. Numerical experiments for the Burgers equation and two-dimensional Navier-Stokes flow are presented which suggest a strong interrelation of the theoretical dimension and the number of coherent structures in two-dimensional viscous turbulent flows.

Chapter 6 is devoted to the use of wavelets primarily as a tool for analysing differential operators. Angeletti, Mazet and Tchamitchian study second order differential operators in divergence and non-divergence forms with diffusion tensors of rather weak regularity on $\mathbb{R}^d$. One of the main results is concerned with the boundedness of Galerkin projection operators in L_p-Sobolev norms. Holschneider presents a wavelet based microlocal analysis of local regularity spaces. As an application, the regularity of elliptic differential operators on domains with cusps is treated.

This volume is comprised of both invited and contributed chapters. All contributions, whether of survey character or containing primarily original material, were refereed according to their respective goal. We wish to thank all authors and reviewers for their most valuable contributions, in particular, for their patience and cooperation. We are indebted to Margaret and Charles Chui for their diligent and kind assistance during the editorial process.

Aachen, Germany
College Station, Texas
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June, 1997

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