MICROSYSTEMS

Nicolae Lobontiu

Dynamics of Microelectromechanical Systems

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Nicolae Lobontiu
University of Alaska Anchorage
USA

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With love to my wife, daughters, mother and father

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PREFACE

Through its objective, scope, and approach, this book offers a systematic view to the dynamics of microelectromechanical systems (MEMS). While providing an in-depth look at the main problems that involve reliable modeling, analysis, and design, the main focus of this book is the mechanical/structural micro domain, which is at the core of most MEMS. Although not designed for a specific course, the book could be used as a text at the upper-undergraduate/graduate level, and, as such, it contains numerous fully solved examples as well as many end-of-the-chapter proposed problems whose comprehensive solutions can be accessed/downloaded from the publisher's website by the qualified instructor. At the same time, it is hoped that this book might be useful to the researchers, professionals, and academics involved with modeling/designing mechanically based MEMS.

This text is a continuation of the book *Mechanics of Microelectromechanical Systems* by Lobontiu and Garcia (Kluwer Academic Publishers, 2004), and therefore it relies on the elements developed in its precursor, such as compliance/stiffness formulations for microcantilevers, microhinges, microbridges, and microsuspensions, as well as on the treatment given to means of actuation and sensing. However, an effort has been made to ensure that this book is self-contained as much as possible.

The material is structured into four parts (conventionally named chapters), which are briefly discussed here. Each chapter contains exposition of the theory that is necessary to developing topics specific to that part. *Chapter 1* studies the bending and torsion resonant responses of microcantilevers and microbridges by employing the distributed-parameter approach and the Rayleigh's quotient approximate method, which provides means for direct derivation of the resonant frequencies. Lumped-parameter modeling, which enables calculation of the above-mentioned resonant frequencies via the equivalent stiffness and inertia properties, is also used. Several microcantilever and microbridge configurations are analyzed, and closed-form equations are provided for the bending and torsional resonant (natural) frequencies by taking into account the number of profiles that longitudinally define the member, the number of layers in a cross-section, and the type of cross-section (either constant or variable). Designs that contain circular perforations are also analyzed together with configurations that contain externally attached matter whose quantity and position alter the main resonant frequencies.

Chapter 2 analyzes the resonant/modal response of more complex micromechanical systems by considering their components are either inertia or spring elements. The lumped-parameter modeling approach is applied to derive the free vibratory response of micromechanical systems that behave as either single degree-of-freedom (DOF) ones, or as multiple DOF systems—in case they undergo more complex vibratory motion and/or are composed of several mass elements. Lagrange's equations are employed in modeling the free response of multiple DOF micromechanical systems. Numerous examples of mass-spring microsystems undergoing linear or/and rotary resonant vibrations are presented.

Chapter 3 addresses the main mechanisms responsible for energy losses in MEMS. Quality factors and corresponding viscous damping coefficients are derived owing to fluid–structure interaction (as in squeeze- and slide-film damping), anchor (connection to substrate) losses, thermoelastic damping (TED), surface/volume losses and phonon-mediated damping.

Chapter 4 discusses MEMS by taking into account the forcing factor and therefore the forced response is analyzed. For harmonic (sinusoidal, cosinusoidal) excitation, the frequency response is modeled by quantifying the amplitude and phase shift over the excitation frequency range. The Laplace transform and the cosinusoidal transfer function approach are employed in analyzing topics such as transmissibility, coupling, mechanical-electrical analogies, as well as applications such as microgyroscopes and tuning forks. For non-harmonic excitation, the time response of MEMS is studied by means of the Laplace transform, the state-space approach and time stepping schemes. Nonlinear problems, such as those generated by large deformations are also discussed, and dedicated modeling/solution methods such as time-stepping schemes or the approximate iteration method are presented. All the solutions for the problems that appear at the ends of the chapters can be accessed at http://www.springer.com/west/home/generic/search/results?-SGWID=4-40109-22-173670220-0 by a qualified instructor.

Although many applications in this text qualify as *nano* devices, the prefix *micro* has been utilized throughout, with the understanding that both the micro and nano domains are covered by the generic denomination of *microelectromechanical systems*. Particular care has been paid to the accuracy of this text, but it is possible that unwanted errors have slipped in, and I would be extremely grateful for any related signal.

In closing, I would like to address my sincere thanks to Alex Greene, Springer Editorial Director of Engineering, for all the positive interaction, support, and profound comprehension of this project.

Anchorage, Alaska

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Chapter 1

MICROCANTILEVERS AND MICROBRIDGES: BENDING AND TORSION RESONANT FREQUENCIES

1.1 INTRODUCTION

Microcantilevers and microbridges are the simplest mechanical devices that operate as standalone systems in a variety of microelectromechanical systems (MEMS) applications, such as nano-scale reading/writing in topology detection/creation, optical detection, material properties characterization, resonant sensing, mass detection, or micro/nano electronic circuitry components such as switches or filters.

This chapter studies the bending and torsion resonant responses of microcantilevers (fixed-free flexible members) and microbridges (fixed-fixed flexible members) by mainly utilizing the distributed-parameter approach and the related Rayleigh's quotient approximate method, which enable direct derivation of the resonant frequencies. The lumped-parameter modeling, which permits separate calculation of equivalent stiffness and inertia properties en route of obtaining the above-mentioned resonant frequencies, is also used in this chapter for certain configurations.

Structurally, microcantilevers and microbridges can be identical, it is only the boundary conditions that differentiate them, and this is the reason the two members are discussed together in this chapter. The configuration of a particular microcantilever or microbridge is a combination of three features, namely: number of profiles that longitudinally define the member (there can be a single profile [geometric curve], or multiple profiles [case in which there is a series connection between various single-profile segments]), number of layers in a cross-section (there can be single-layered, homogeneous members or multi-layer [sandwich] ones), and the type of cross-section (either constant or variable). These variables are illustrated in Figure 1.1 as a three-dimensional (3D) space. Because each of the three variables can take one of two possible values, eight different configuration classes are possible by combining all possible variants (in Figure 1.1 these categories are represented by the cube's vertices). The origin of the 3D space, which is one specific design category, is defined by the parameters *SL*, *SP*, and *CCS*, and represents the subclass of microcantilevers/microbridges that is made of a single layer (*SL*). Their geometry (width) is defined by a single profile (one geometric curve—*SP*), and are of constant cross-section. This particular combination results in a homogeneous, constant cross-section member, one of the simplest and most used cantilevers/bridges. The other seven subclasses (corresponding to the remaining cube vertices in Figure 1.1) can simply be described in a similar manner.

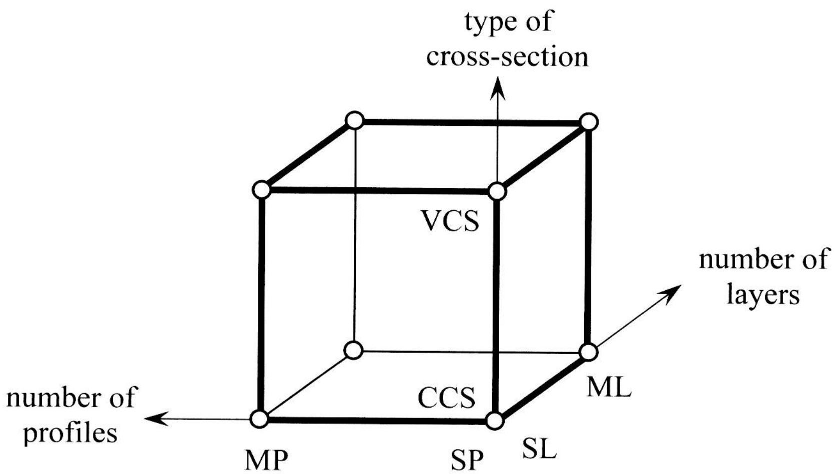

Figure 1.1 Three-dimensional space characterizing the geometric and material parameter categories that define microcantilevers/microbridges (*SL*, single layer; *ML*, multiple layer; *SP*, single profile; *MP*, multiple profile; *CCS*, constant cross-section; *VCS*, variable cross-section)

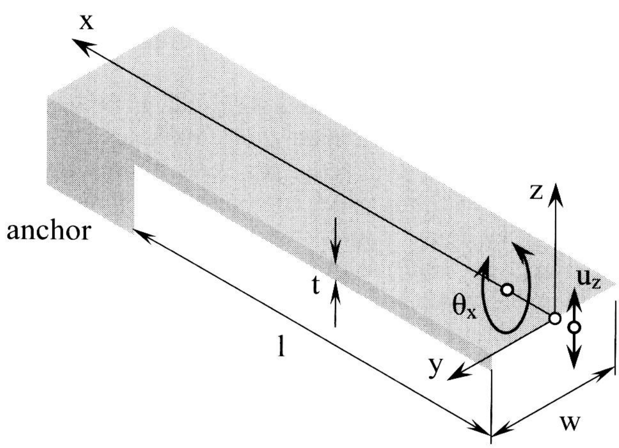

Figure 1.2 Constant cross-section microcantilever: dimensions and degrees of freedom

The assumption will be used in this chapter that variable cross-section (*VCS*) micromembers are of constant thickness and of variable width, assumption which is consistent with the usual microfabrication procedures.

Figures 1.2 and 1.3 show a microcantilever and a microbridge, respectively, both of constant rectangular cross-sections. As shown in Figure 1.2, a microcantilever is a fixed-free member, whose reference frame (which monitors the out-of-the-plane bending about the *z*-axis, and torsion about the *x*-axis) is placed at the free end where both bending and torsion deformations are maximum. A microbridge is a fixed-fixed member, as illustrated in Figure 1.3, and the reference frame can be located either at one fixed end or at its midpoint. The maximum deformations are taking place at the microbridge's midpoint.

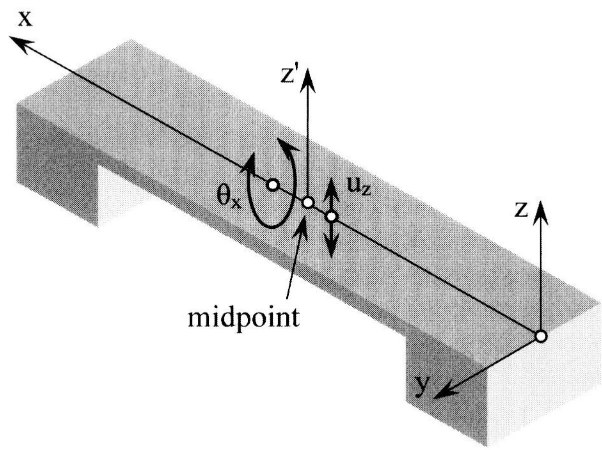

Figure 1.3 Constant cross-section microbridge: dimensions and degrees of freedom

The topic of detecting and evaluating the amount of substance that attaches to MEMS structures by monitoring the shift in the bending and torsion resonances of microcantilevers and microbridges is approached in Section 1.5.

1.2 MODAL ANALYTICAL PROCEDURES

Calculating the modal or resonant response of flexible structures can be performed by means of *analytical* and *numerical methods*. Numerical procedures, of which the *finite element method* (which is not addressed here) is the most popular, are versatile and yield precise solutions for problems that are described by partial differential equations with complex boundary conditions and geometric shapes. Although the method of choice in both academia and industry, for structures with relatively simple geometry and boundary conditions, such as microcantilevers and microbridges, the finite element method can be supplemented by simpler analytical models that are based on closed-form solutions and that offer the advantage of faster processing times.

Analytical procedures dedicated to evaluating the resonant response of elastic members comprise *distributed-parameter methods* and *lumped-parameter methods*. In a modal analysis, the distributed-parameter approach studies vibrating elastic structures by considering the time response of all points of the structure, and therefore by assuming the system's properties are distributed over the entire structure. Lumped-parameter approaches, on the other hand, consider that the system's properties are concentrated (lumped) at convenient locations and focus on the dynamic behavior at those selected locations. Small deformations of the elastic members will be assumed in this chapter, which will result in linear models.

1.2.1 Rayleigh's Quotient Method

Rayleigh's quotient method (Timoshenko [1], Thomson [2], Rao [3]) is a distributed-parameter procedure enabling calculation of various resonant frequencies of freely vibrating elastic structures. In the case of conservative systems, the method starts from the equality between the maximum kinetic energy and maximum potential energy:

$$T_{max} = U_{max} \tag{1.1}$$

The next assumption is the one considering the harmonic motion of a vibrating component, according to which the deformation at a given point of the structure is a product between a spatial function and a time-dependent one:

$$u(x,t) = u(x)\sin(\omega t) \tag{1.2}$$

where the deformation can be produced through bending, axial, or torsional free vibrations. The next step into Rayleigh's quotient approach is assuming a certain distribution of the elastic deformation *u (x)*. By combining all these steps yields the resonant frequency of interest. This method will be discussed in the following sections with reference to bending and torsional vibrations only.

1.2.1.1 Bending

In out-of-the-plane bending of single-component MEMS, such as the microcantilever and the microbridge illustrated in Figures 1.2 and 1.3, the kinetic energy is:

$$T = \frac{1}{2}\int_V \left[\frac{\partial u_z(x,t)}{\partial t}\right]^2 dm = \frac{\rho}{2}\int_l A(x)\left[\frac{\partial u_z(x,t)}{\partial t}\right]^2 dx \tag{1.3}$$

where $u_z\ (x,t)$ is the deflection at an arbitrary point x on the microcantilever (microbridge) and time moment t. The member's length is l, its cross-sectional area (potentially variable) is denoted by $A(x)$, and the mass density is ρ.

The elastic potential energy stored in a bent member is:

$$U = \frac{1}{2}\int_l EI_y(x)\left[\frac{\partial^2 u_z(x,t)}{\partial x^2}\right]^2 dx \tag{1.4}$$

where E is the elasticity (Young's) modulus and $I_y(x)$ is the cross-sectional moment of area with respect to the y-axis (see Figures 1.2 and 1.3). By considering the assumption:

$$u_z(x,t) = u_z(x)\sin(\omega t) \tag{1.5}$$

the maximum kinetic energy and maximum elastic potential energy that result from Equations (1.3) and (1.4)—corresponding to values of 1 (one) for the involved sine and cosine factors—are substituted into Equation (1.1), which yields the square of the bending resonant frequency:

$$\omega_b^2 = \frac{\int_l EI_y(x)\left[\frac{\partial^2 u_z(x)}{\partial x^2}\right]^2 dx}{\int_l \rho A(x) u_z(x)^2\, dx} \tag{1.6}$$

The deflection $u_z\ (x)$, which is measured at an arbitrary point along the beam and is positioned at a distance x from the origin (as already mentioned, the origin is the free end for a cantilever and either one fixed end or the midpoint for a bridge), is related to the maximum deflection u_z by means of a bending *distribution function* as:

$$u_z(x) = u_z f_b(x) \tag{1.7}$$

By combining Equations (1.6) and (1.7), the bending resonant frequency can be reformulated as:

$$\omega_b^2 = \frac{\int_l EI_y(x)\left[\frac{d^2 f_b(x)}{dx^2}\right]^2 dx}{\int_l \rho A(x) f(x)^2\, dx} \tag{1.8}$$

Equations (1.6) and (1.8) are two forms of Rayleigh's quotient corresponding to bending under the assumption the cross-section is variable.

1.2.1.2 Torsion

Rayleigh's quotient method can also be applied to torsion problems involving microcantilevers and microbridges. The kinetic energy of a variable rectangular cross-section rod, for instance, is expressed as:

$$T = \frac{1}{2}\int_l \frac{\rho w(x)t(x)\left[w(x)^2 + t(x)^2\right]}{12}\left[\frac{\partial\theta_x(x,t)}{\partial t}\right]^2 dx \tag{1.9}$$

where the cross-section's width w and thickness t are assumed variable across the member's length. The torsion angle $\theta_x(x,t)$ is measured at an arbitrary abscissa x and time moment t.

The elastic potential energy is:

$$U = \frac{1}{2}\int_l GI_t(x)\left[\frac{\partial\theta_x(x,t)}{\partial x}\right]^2 dx \tag{1.10}$$

where $I_t(x)$ is the torsion moment of area of the member's cross-section. By considering the torsional angle is defined as:

$$\theta_x(x,t) = \theta_x(x)\sin(\omega t) \tag{1.11}$$

and by also equalizing the maximum kinetic energy to the maximum potential energy the torsion resonant frequency is calculated as:

$$\omega_t^2 = \frac{12\int_l GI_t(x)\left[\frac{d\theta_x(x)}{dx}\right]^2 dx}{\int_l \rho w(x)t(x)\left[w(x)^2 + t(x)^2\right]\theta_x(x)^2 dx} \tag{1.12}$$

The following relationship is considered relating the torsion angle at an arbitrary abscissa, $\theta_x(x)$ and the maximum (reference) torsion angle θ_x:

$$\theta_x(x) = \theta_x f_t(x) \tag{1.13}$$

where $f_t(x)$ is the torsion distribution function. By substituting Equation (1.13) into Equation (1.12), the torsion resonant frequency becomes:

$$\omega_t^2 = \frac{12\int_l GI_t(x)\left[\frac{df_t(x)}{dx}\right]^2 dx}{\int_l \rho w(x)t(x)\left[w(x)^2 + t(x)^2\right]f_t(x)^2 dx} \tag{1.14}$$

Again, Equations (1.12) and (1.14) express Rayleigh's quotients for torsion.

It should be mentioned that Rayleigh's quotient equations for bending and torsion give the respective resonant frequency of a non-homogeneous, variable cross-section member, irrespective of boundary conditions. The boundary conditions decide the form of the bending and torsion distribution functions, $f_b(x)$ and $f_t(x)$ over the member's length. For microcantilevers and microbridges the boundary conditions are different, and therefore the distribution functions are different as well. The distribution functions are also dependent on the abscissa origin in the case of microbridges.

1.2.2 Lumped-Parameter Method

Rayleigh's quotient method, as seen in the previous section, directly yields the resonant frequency of interest, which is sufficient when this type of response is solely needed. However, there are situations where the static or quasi-static behavior of an elastic member is also of interest, and in such cases the stiffness of that member at a specific location is necessary to use it as a connector between the applied loads and resulting deformations.

An alternative to Rayleigh's quotient distributed-parameter method to evaluating the resonant frequencies of flexible members is the lumped-parameter method, which transforms the real, distributed-parameter properties—elastic (stiffness) and inertial (mass or moment of inertia)—into equivalent, lumped-parameter ones—k_e (equivalent stiffness), m_e (equivalent mass), or J_e (equivalent mechanical moment of inertia)—which are computed separately. In doing so, one can use just the stiffness (for static applications) or both the stiffness and inertia fractions (for modal calculations), because the resonant frequency of interest is expressed as:

$$\omega_e^2 = \frac{k_e}{m_e} \tag{1.15}$$

In Equation (1.15), which is written for an elastic body whose equivalent counterpart undergoes translational motion, m_e is the mass of that body. In case the equivalent lumped body undergoes rotation, the mass of Equation (1.15) is replaced by a mechanical moment of inertia J_e. The specifics of determining the resonant frequencies corresponding to bending and torsion of microcantilevers/microbridges by the lumped-parameter approach will be discussed next.