

Engineering Modelling and Analysis

**Martin Lambert,
Andrew Metcalfe, David Walker
and Michael Leonard**

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ENGINEERING MODELLING AND ANALYSIS

*David Walker, Michael Leonard,
Andrew Metcalfe & Martin Lambert*



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ENGINEERING MODELLING AND ANALYSIS

This introduction to numerical analysis and computing covers a range of topics suitable for undergraduate engineering students with a particular focus on civil and environmental engineering.

Taking a balanced approach to teaching computing and computer methods at the same time, *Engineering Modelling and Analysis* satisfies the need to be able to use computers using formal languages such as Fortran and other applications such as Matlab and Microsoft Excel. Realistic and relevant problems are provided throughout the book in short accessible chapters which follow the structure of a degree course.

David Walker, **Michael Leonard** and **Martin Lambert** are in the School of Civil, Environmental and Mining Engineering, and **Andrew Metcalfe** is in the School of Mathematical Sciences, all at the University of Adelaide, Australia. They are all active in teaching and research and the content of the book reflects a strong belief that the one should complement the other.

Preface

The application of science and mathematics in engineering has developed to the point where a significant portion of design, analysis and assessment is now computer-based. While this has enhanced the ability of the engineer in his or her work there are some trends developing that should ring alarm bells. One such trend is that towards more and more user-friendly computer software. In many cases now it appears that it is simply not necessary to understand the theory of a particular solution procedure; the requested data are simply fed in to an appropriate program and an answer appears. It almost sounds too good to be true, and in many respects it is. While the experienced engineer, the one who understands the problem and its solution, is able to use such tools in an appropriate fashion, there is the potential for the inexperienced to use these tools incorrectly and without realising this. Such practice should be a serious concern.

This book, then, is aimed at educating engineers-in-training in the basics of computer modelling and analysis, with a clear emphasis on students developing their own code and procedures in a range of modes from a formal computing language (such as Fortran or C) to sophisticated spreadsheets (Microsoft Excel) and mathematical packages (Matlab). It is through such development that a deep understanding of the topics develops leading to an increased ability to apply the methods appropriately and to extend well beyond the confines of the enclosed chapters.

In designing the book a deliberate decision was taken to keep the chapters short and self-contained. In many cases a series of chapters together define an overall topic but there is nothing to suggest that all need to be followed, or that they should be followed in any particular way. Computer programming takes time, and there is likely to be a significant difference in the time required to master a topic from student to student and, indeed, for a particular student from topic to topic. As authors we would like to see a situation where the student spends twice as much time programming as reading and we see a short chapter as our part of the equation.

In many cases the methods covered will be those that were developed first. This is done quite deliberately for two reasons: firstly, the early methods are often the ones that can be used to explain the fundamental principles most simply; secondly, these methods often give a good feel for the processes at work in the solution and it is the *feel* for the methods that is so important in understanding the topics to the depth required.

The coverage in the book is clearly focussed on engineering applications, and in particular civil and environmental applications. We have designed the book to be well aligned with the strengths and interests of engineering undergraduates through the nature of the coverage of the theory of the methods and also through a wide

range of examples of the application of such methods in current engineering practice.

A common topic when discussing numerical methods is the competition between engineers and mathematicians. One way of expressing the difference comes from paraphrasing Patrick Rivett and Samuel Eilon (Operations Research practitioners) (Rivett, 1981; Eilon, 1982):

the mathematician is concerned with exact solutions to approximate problems while the engineer is more likely interested in the provision of approximate solutions to exact problems.

This idea of approximate solutions is central to much of this book and the methods covered. Approximate does not mean incorrect, nor does it mean a rough guess. Approximate means accurate to a known tolerance or level of error.

Another feature of the book is the interconnection of topics. Although it may be possible to learn all about finite difference modelling and the normal distribution independently (for example), that learning can be enhanced considerably by appreciating the links between the two topics and understanding how they are inter-related.

We have taken much care in trying to promote a deep learning environment by highlighting where possible *some* of the links between topics: students will no doubt find many more through their own efforts and reflections.

David Walker
Michael Leonard
Andrew Metcalfe
Martin Lambert

Adelaide, 2008.

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CHAPTER ONE

Introduction (Engineering Modelling and Analysis)

1.1 INTRODUCTION

The word “model” is one that is used commonly in everyday language, and has a range of meanings. Dictionary definitions include “a three-dimensional reproduction of something, usually on a smaller scale”, “a design or style of structure, e.g. this year’s model”, and “to design or plan (a thing) in accordance with a model, e.g. the new method is modelled on the old one”. However, when the term is used in an engineering context it is often quite specific and conceptually different. To an engineer a model can be defined as an “abstraction of reality” (Izquierdo et al., 2004). This concept is illustrated in Figure 1.1 (Dandy et al., 2008) where a real world system (e.g. a bridge, highway, open channel, or dam wall) that needs to be understood sufficiently to make predictions about how it will behave under a range of situations is conceptualised in an idealised form, which is then analysed. This conceptualisation leads to the development of a model and forms an important part of engineering design. Based on the analysis that is then carried out, the properties of the real world system are inferred.

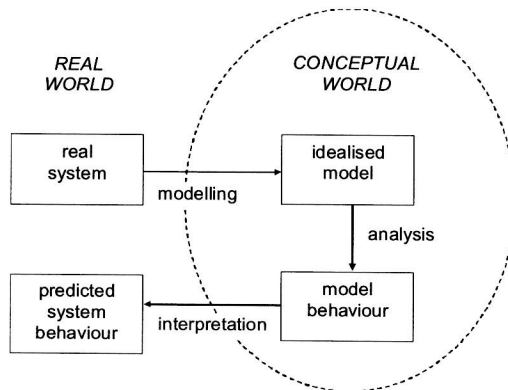


Figure 1.1 Modelling and analysis in a systems approach. From Dandy et al. (2008).

It might be thought that once the model is developed the hard work has been done; however, this is only partly true. Certainly, a good model is a requirement for