
OPTIMAL RADAR TRACKING SYSTEMS

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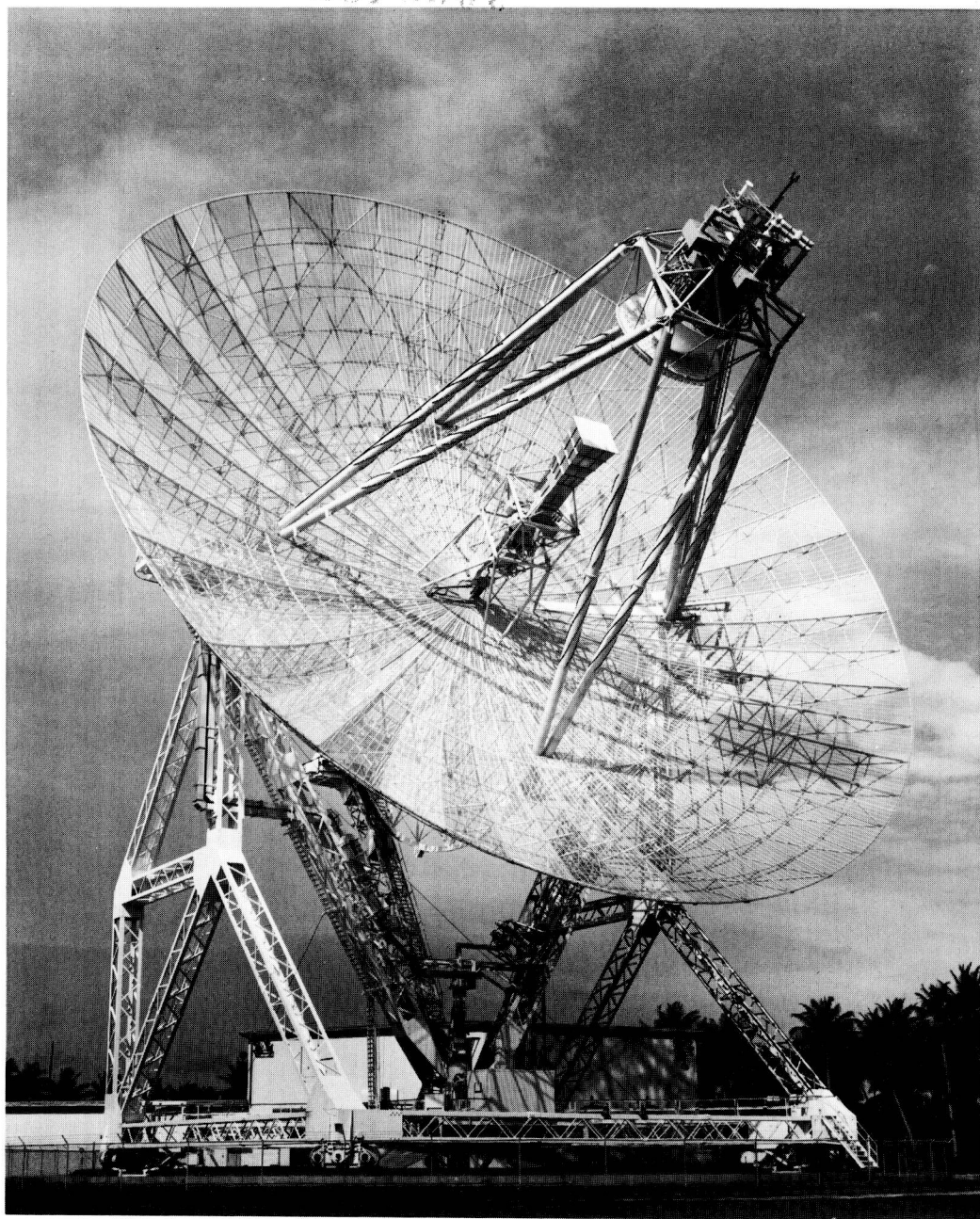
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OPTIMAL RADAR TRACKING SYSTEMS



Altair Radar Antenna on Kwajalein Atoll

Preface

This book is based on the study of a sophisticated radar tracking system, the Altair radar, which uses Kalman filtering to perform optimal long-range tracking of ballistic missile warheads. This engineering example provides a means for explaining Kalman filter theory in a relatively simple manner as well as many other technical issues that are critical to the design of a modern optimal radar tracking system.

To give the background needed to understand the study of Altair, the book presents simplified explanations of the following:

- Feedback control.

- Modulation and demodulation of signals.

- Digital sampled-data systems.

- Digital computer simulation.

- Statistical analysis of random signals.

- Detection and tracking processes in a radar system.

These discussions start at an elementary level and develop the concepts in a concrete manner.

The book is directed to all who are interested in target tracking, including mathematicians. The reader is assumed to have a basic knowledge of mathematics (including the Laplace transform), physics, and AC circuit analysis.

For security reasons, certain details of the Altair radar system are replaced with a comparable radar model. The calculations applied to the model are the same as those that have been used to analyze the performance of the Altair radar, and so the reader is presented with the equivalent of a direct study of Altair. The Kalman filter tracking equations that are given are implemented in Altair, but the radar parameters and some of the tracking coordinate transformation equations are different.

The Altair radar antenna, shown in the frontispiece, is physically very impressive. It has a huge steerable parabolic dish, 150 ft in diameter, which

operates simultaneously at VHF and UHF. This radar is located in the Kwajalein Atoll of the Marshall Islands, 2100 miles west and south of Hawaii. It performs long-range tracking of warheads from ballistic missiles launched at Vandenburg Air Force Base in California and dropped into the lagoon of the atoll.

Although this book deals directly with radar tracking technology, the basic principles apply to any target-tracking system. Hence, the book should also be useful to individuals involved in other tracking applications that use optical signals, sonar signals, RF telemetry signals, etc.

This study of Altair includes a considerable amount of detail concerning the operation of a complex electronic system and thereby presents a study that is unusual in the unclassified literature. It gives a practical introduction to the system analysis of modern sophisticated electronic equipment by illustrating the multitude of analytical issues involved in its design.

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The cooperation of the U.S. Army in reviewing and approving this material for publication is greatly appreciated. I am indebted to Mr. George Fangmann of the GTE Altair Program Office for his assistance in achieving this approval.

I want to thank Analog Devices, Inc. for graciously providing the illustrations for describing noise signals given in Figs 6.2-7 and 6.2-8.

GEORGE BIERNSON

*Concord, Massachusetts
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Chapter 1

Introduction

Target tracking in modern radar systems is often very sophisticated, involving several engineering disciplines. An effective system engineering approach to this field requires a broad understanding of its many aspects. This should include Kalman filter theory [8.1, 8.2], which has become an important element in the design of tracking systems. However, the esoteric manner in which this theory has been presented has placed it beyond the understanding of many engineers engaged in target tracking.

One of the best ways to learn engineering is to study its application in a practical setting. This book presents a design study of a sophisticated radar tracking system, the Altair radar, which uses Kalman filtering to perform optimal long-range tracking of ballistic missile warheads. A study of this system provides an effective means of explaining Kalman filtering, along with many other engineering issues of a complex radar tracking system.

Although this book deals directly with radar tracking technology, the basic principles apply to any target-tracking system. Hence, the book should also be useful to individuals involved in other tracking applications that use optical signals, sonar signals, radio-frequency (RF) telemetry signals, etc.

1.1 OVERVIEW OF BOOK

The Altair radar, shown in the frontispiece, has a huge steerable parabolic antenna, 150 ft in diameter, which operates simultaneously at very high frequency (VHF) and ultrahigh frequency (UHF). This radar is located in the Kwajalein Atoll of the Marshall Islands, 2100 miles west and south of Hawaii. It tracks warheads from ballistic missiles launched at Vandenburg Air Force Base in California and dropped into the lagoon of the atoll.

The study of Altair, which is presented in Chapter 8, gives a practical explanation of the many engineering aspects of its tracking system. This includes Kalman filter theory, feedback control, digital sampled-data theory,

and radar signal processing. It shows how these different principles are integrated into a complex modern radar tracking system.

In order that the reader can comprehend this study of the Altair system, background material is presented in Chapters 2–7. The book is directed to all who are interested in target tracking, including the mathematician. The discussions in Chapters 2–7 start at an elementary level and develop the concepts in a concrete manner. The reader is assumed to have a basic knowledge of mathematics (including the Laplace transform), physics, and AC circuit analysis.

In Chapters 2–7, the reader is led by the hand through the following material needed to comprehend the complex issues of a sophisticated optimal radar-tracking system:

- a. Chapter 2 summarizes the principles of feedback control, and Chapter 3 extends this with techniques for calculating the error of a tracking radar due to target motion.
- b. Chapter 4 discusses modulation, carrier-frequency filtering, and demodulation of signals.
- c. Chapter 5 presents the theory of digital sampled-data systems. This includes the design of algorithms for digital filtering and digital computer simulation. A simulation method is presented that allows a very complex dynamic system to be modeled using BASIC computer language on a personal computer. The state variable, state equation, and transition matrix are explained in a simple manner.
- d. Chapter 6 describes the statistical analysis of system response to random inputs, and applies this to receiver noise and wind gusts. Basic noise theory is explained, including noise bandwidth, the Gaussian distribution, and correlation functions.
- e. Chapter 7 analyzes the detection and tracking processes of a tracking radar system. Radar performance is calculated for acquiring a target signal, and for tracking the target in range and angle.

For security reasons, certain details of the Altair radar system are replaced with a comparable radar model. The calculations applied to the model are equivalent to those that have been used to analyze the performance of the Altair radar. Hence, the reader is presented with the equivalent of a direct study of Altair. The Kalman filter tracking equations that are given are the same as those actually implemented in Altair, but the radar parameters and some of the tracking coordinate transformation equations are different.

The following is a summary of the discussion of Altair given in Chapter 8:

- a. In Section 8.1, a radar model is presented, which differs quantitatively from Altair, but explains the radar aspects of the Altair tracking system. Radar parameters are examined along with the equations from which

- they are derived. This model shows how Altair senses radar signal strength to determine the measurement-noise covariance matrix used in the Kalman filter equations.
- b. Section 8.2 describes the basic structure of the target-tracking equations. Radar tracking data are converted to rectangular coordinates, and Kalman filtering is performed in terms of these converted signals. Computations are included in the filters to account for predicted accelerations of the target, caused by gravity, coriolis effects, and atmospheric drag.
 - c. In Sections 8.3 and 8.4, the principle of the Kalman filter is explained in physical terms. From the Kalman-filter matrix equations, algorithms are derived that optimally adjust the smoothing parameters of the Altair Kalman-filtering tracking system. This Altair example yields a simple yet in-depth understanding of Kalman filter theory.
 - d. In Section 8.5, the tracking-filter equations of Altair are analyzed and reduced to signal-flow diagram form by applying the sampled-data and feedback-control principles developed earlier. This derivation shows that the Kalman filter configuration is equivalent to a conventional tracking system that filters the target information in terms of the coordinates of the target. It is the equations for calculating the optimal smoothing coefficients of the tracking filters that are unique to Kalman filter theory.
 - e. In Section 8.6, Altair Kalman-filter responses are simulated, yielding plots that show how this optimal tracking system performs while tracking a ballistic missile warhead. The plots characterize the acquisition phase, the steady-state conditions during exoatmospheric flight, and the strong transient that occurs when the warhead reenters the atmosphere.

Section 1.2 of this Introduction summarizes the background material presented in Chapters 2–7. Section 1.3 summarizes the discussion of the Altair system given in Chapter 8.

1.2 BACKGROUND MATERIAL FOR UNDERSTANDING THE ALTAIR DISCUSSION

1.2.1 Feedback Control Theory

Tracking is a feedback control process, and the Altair tracking system incorporates many feedback control loops. Feedback control is employed in the servos that position the radar antenna, in the range-tracking circuitry that keeps the range gate centered over the target pulse, and in the tracking-filter equations implemented in the tracking computer, which smooth the radar data and determine the trajectory of the target. Thus, feedback is the foundation of target tracking, and so the book begins in Chapter 2 with a summary of the principles of feedback control.