Notes and Reports
in
Computer Science and Applied Mathematics

Reliability Theory and Models

Stochastic Failure Models, Optimal Maintenance Policies, Life Testing, and Structures

Edited by

MOHAMED S. ABDEL-HAMEED

ERHAN ÇINLAR

JOSEPH QUINN

Reliability Theory and Models

Stochastic Failure Models, Optimal Maintenance Policies, Life Testing, and Structures

Edited by

Mohamed S. Abdel-Hameed
Department of Mathematics and Computer Science
University of North Carolina at Charlotte
Charlotte, North Carolina

Erhan Çınlar
Department of Industrial Engineering
and Management Sciences
Northwestern University
Evanston, Illinois

Joseph Quinn
Department of Mathematics and Computer Science
University of North Carolina at Charlotte
Charlotte, North Carolina

1984

ACADEMIC PRESS, INC.
(Harcourt Brace Jovanovich, Publishers)
Orlando San Diego New York London
Toronto Montreal Sydney Tokyo

Academic Press Rapid Manuscript Reproduction

Proceedings of a Symposium on Stochastic Failure Models, Replacement and Maintenance Policies, and Accelerated Life Testing, Sponsored by the University of North Carolina at Charlotte and the U.S. Army Research Center, Held in Charlotte, North Carolina, June 24–26, 1983

ACADEMIC PRESS, INC.
Orlando, Florida 32887

United Kingdom Edition published by
ACADEMIC PRESS, INC. (LONDON) LTD.
24/28 Oval Road, London NW1 7DX

Library of Congress Cataloging in Publication Data

Main entry under title:

Reliability theory and models.

Includes bibliographical references.
1. Reliability (Engineering)--Congresses.
I. Abdel-Hameed, M. (Mohamed) II. Cinlar, E. (Erhan), date . III. Quinn, J.
TA169.R46 1984 620'.00452 84-45220
ISBN 0-12-041420-1 (alk. paper)

PRINTED IN THE UNITED STATES OF AMERICA

84 85 86 87 9 8 7 6 5 4 3 2 1

Reliability Theory and Models

Stochastic Failure Models, Optimal Maintenance Policies, Life Testing, and Structures

Notes and Reports in Computer Science and Applied Mathematics

Editor
Werner Rheinboldt
University of Pittsburgh

1. JOHN R. BOURNE. Laboratory Minicomputing
2. CARL TROPPER. Local Computer Network Technologies
3. KENDALL PRESTON, JR. AND LEONARD UHR (Editors). Multicomputers and Image Processing: Algorithms and Programs
4. STEPHEN S. LAVENBERG (Editor). Computer Performance Modeling Handbook
5. R. MICHAEL HORD. Digital Image Processing of Remotely Sensed Data
6. SAKTI P. GHOSH, Y. KAMBAYASHI, AND W. LIPSKI (Editors). Data Base File Organization: Theory and Applications of the Consecutive Retrieval Property
7. ULRICH W. KULISCH AND WILLARD L. MIRANKER (Editors). A New Approach to Scientific Computation
8. JACOB BECK, BARBARA HOPE, AND AZRIEL ROSENFELD (Editors). Human and Machine Vision
9. EDGAR W. KAUCHER AND WILLARD L. MIRANKER. Self-Validating Numerics for Function Space Problems: Computation with Guarantees for Differential and Integral Equations
10. M. ABDEL-HAMEED, ERHAN ÇINLAR, AND JOSEPH QUINN (Editors). Reliability Theory and Models: Stochastic Failure Models, Optimal Maintenance Policies, Life Testing, and Structures

Ismail N. Shimi

1935–1982

Ismail N. Shimi, program manager for probability and statistics at the Air Force Office of Scientific Research, died suddenly on the morning of August 31, 1982. He was 47 years old.

He obtained his doctorate from the University of North Carolina in 1964. He taught at the University of California at Riverside, Ain Shams University in Cairo, and Florida State University at Tallahassee before joining the Air Force Office of Scientific Research in 1975.

His research work, which appeared in over 20 publications, covers most of applied probability: branching processes, traffic theory, reliability and life testing, stopping rules, maintenance, inventories, and so on. In addition, he was familiar with much of statistics and kept up with the latest developments in the theory of stochastic processes. This research experience, his taste for good mathematics, and his administrative skills were combined to make him an exceptional program manager. Under his leadership, the AFOSR program in probability and statistics became a most innovative, high-quality, modern research program. He had the rare ability to balance the short-term needs of his organization with long-term interests in basic research.

In 1976, together with C. P. Tsokos, he organized a conference on reliability theory and its applications. Its proceedings were published in two volumes by Academic Press in 1977. He was planning a similar venture when fate failed him. The present volume represents a partial fulfillment of his intentions. We dedicate it to his memory.

Contributors

Numbers in parentheses indicate the pages on which the authors' contributions begin.

Gary R. Adams (297), *New Engines Division, ASD/YZE, Wright-Patterson Air Force Base, Ohio 45433*

Harold S. Balaban (65), *ARINC Research Corporation, Annapolis, Maryland 21401*

R. E. Barlow (221), *Operations Research Center, University of California, Berkeley, Berkeley, California 94720*

H. W. Block (231), *Department of Mathematics and Statistics, University of Pittsburgh, Pittsburgh, Pennsylvania 15260*

Philip J. Boland (243), *Department of Mathematics, University College Dublin, Belfield, Dublin 4, Ireland*

Mark Brown (257, 267), *Department of Mathematics, City College, City University of New York, New York, New York 10021*

Erhan Çınlar (3), *Department of Industrial Engineering and Management Sciences, Northwestern University, Evanston, Illinois 60201*

Anthony J. Feduccia (295), *Systems Reliability and Engineering Branch, Rome Air Development Center, Griffiss Air Force Base, New York 13441*

Robert A. Fontenot (83), *Whitman College, Walla Walla, Washington 99362*

Guangping Ge (267), *Mathematics Department, Hebei Teachers' University, Shijiazhuang, Hebei, The People's Republic of China*

Gary Gottlieb (103), *New York University, New York, New York 10003*

Ina Parks S. Howell (199), *Department of Mathematical Sciences, Florida International University, Miami, Florida 33199*

Mei-Ling Ting Lee (273), *Bridgewater State College, Bridgewater, Massachusetts 02324, and Department of Mathematics, Boston University, Boston, Massachusetts 02215*

Benny Levikson (103), *Department of Statistics, Haifa University, Haifa, Israel*

D. T. McNichols (155), *Department of Statistics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061*

Richard J. Meinhold (169), *Department of Operations Research, George Washington University, Washington, D.C. 20052*

Francis J. O'Meara (292), *Operations Analysis Division, Science and Research (NR), HQ SAC, Offutt Air Force Base, Nebraska 68113*

W. J. Padgett (155, 177), *Department of Mathematics and Statistics, University of South Carolina, Columbia, South Carolina 29210*

Frank Proschan (83, 243), *Department of Statistics, Florida State University, Tallahassee, Florida 32306*

Joseph Quinn (115), *Department of Mathematics and Computer Science, University of North Carolina at Charlotte, Charlotte, North Carolina 28223*

T. H. Savits (231), *Department of Mathematics and Statistics, University of Pittsburgh, Pittsburgh, Pennsylvania 15260*

Margaret K. Schaefer (141), *Department of Mathematics, College of William and Mary, Williamsburg, Virginia 23185*

Moshe Shaked (43), *Department of Mathematics, University of Arizona, Tucson, Arizona 85721*

Nozer D. Singpurwalla (65, 169), *Departments of Operations Research and Statistics, George Washington University, Washington, D.C. 20052*

Robert T. Smythe (291), *Department of Statistics/Computer and Information Systems, George Washington University, Washington, D.C. 20052*

Preface

This volume collects most of the papers presented at a conference on reliability held at Charlotte, North Carolina, during June 24–26, 1983. All the papers have been refereed.

The aim of the conference was to bring together reliability theorists, statisticians, and experts on stochastic processes to discuss new and current directions for research on stochastic failure models, maintenance and replacement policies, and statistical and computational aspects of reliability.

One of the highlights of the meeting was a session titled "U.S. Air Force Perspectives on Failure Models and Reliability/Availability/Maintainability." The session consisted of presentations by representatives of three applied research groups in the Air Force. The presentations are included in this volume and are especially noteworthy for the glimpse they provide into the concerns of those dealing with the reliability and maintenance of very large complex systems.

The conference was supported by the University of North Carolina at Charlotte and the U.S. Army Research Center. We thank them for their support. We also thank M. Kazemi and Z. W. Ras for their time and effort with local arrangements. We owe much gratitude to Marty Grogg for her invaluable assistance both during the conference and throughout the preparation of these proceedings. Finally, we are especially grateful to the participants for a marvelous time and much fine mathematics.

Contents

Life Testing

Reliability Structures, Computability, and Approximations

Special Session: U.S. Air Force Perspectives on Failure Models and Reliability/Availability/Maintainability

Stochastic Failure Models

MARKOV AND SEMIMARKOV MODELS OF DETERIORATION

Erhan Cinlar[1]

IE/MS Department
Northwestern University
Evanston, Illinois

Lifetime distribution of a device is obtained assuming quite general laws for deterioration. Markov and semimarkov models of deterioration are discussed in the light of recent results on representation and characterization of Markov processes. Fairly general models can be obtained from a standard Poisson random measure by appropriate choices of several deterministic functions. The essential notion is a correspondence between the actual time (in actual use under field conditions) and an intrinsic time (in controlled laboratory conditions).

[1]Research supported by Air Force Office of Scientific Research through their grant AFOSR-82-0189.

ISBN 0-12-041420-1

1. INTRODUCTION

The simplest notion in reliability theory is the lifetime of a device, and the main problem connected with it is that of finding the distribution of the lifetime. In the statistical approach, that distribution is obtained from the experimental data on the lifetimes of a number of identical copies of the device. With high-reliability devices, the time and expense needed for such tests are prohibitive, and techniques like accelerated life testing and using censored data introduce further difficulties.

In an effort to circumvent such difficulties, the concept of a deterioration process is introduced, and the lifetime is defined to be the hitting time of a threshold value by the deterioration process. Then, one must deal with three separate problems: choosing an appropriate model for the deterioration process, solving for the hitting time distribution mathematically, and identifying the parameters of the model by statistical means. This paper is concerned with the first of these problems.

Our aim is to discuss the stochastic structure of some general models for deterioration processes and solve for the lifetime distributions in general. We move from the specific toward the general: we discuss deterioration processes that can be modeled by

a) continuous Markov

b) continuous semimarkov

c) right continuous Markov

d) Markov additive, and

e) semimarkov

processes. The last two are very close in structure, and together, they cover all processes proposed in the past literature as models for deterioration processes.

Recent representation and characterization theorems obtained in ÇINLAR [5] and in ÇINLAR and JACOD [6] for Markov processes enable us to represent the processes involved in terms of several deterministic functions. These functions have certain physical meanings and can be determined by laboratory experiments under controlled situations (as opposed to actual use under field conditions). The essential notion is a correspondence between the actual time (in actual use under field conditions) and an intrinsic clock time constructed so that the deterioration process appears smoother and simpler in clock time.

Section 2 is on continuous Markov and semimarkov models of deterioration; this is a simplified version of the results in [5] and provides insight into the techniques to follow. Section 3 describes the structure of increasing Markov processes, obtained in ÇINLAR and JACOD [6], in the setting of deterioration processes. Section 4 constructs a Markov additive process, which is a two-dimensional process (Y,A), Y being the deterioration process in intrinsic clock time and A being the actual time as a function of clock time. Finally, Section 5 constructs the semimarkov process modeling the deterioration process in actual time, and shows that the process is almost the most general possible. The paper ends with Section 6 computing the distribution of the lifetime under a theshold mechanism for failure.

This is an expository paper, written in a non-technical style with minimal notation, concentrating on modeling issues and on meanings to be attached to various parameters and operations. To keep it simple, we limited the exposition to the deterioration of a single component, but multi-component versions are almost as easy and can be constructed by mimicking the procedure here and consulting the sources referenced.