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Heat Transfer in Turbulent Mixed Convection

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Editor

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monograph by Petukhov and Polyakov thus allows a critical digest of the Russian contributions in this field up to 1983. It also provides, so far as I am aware, the first attempt at a comprehensive statement of the field as a whole.

In editing the translation, my aim has been to provide a readable text in correct technical English while not restructuring sentences more than comprehension or idiom demands. As a result, the prose retains a slightly pedantic ring—what some might recognize as a Russian accent. This is, perhaps, no bad thing since an author's style of writing gives insight to his directions of thought and thence to his approach to research. A few editorial incursions by way of footnotes have been made, mainly to provide references to more readily available or more recent literature, the latter being kindly provided by Dr. B. Axcell and Professor J. D. Jackson of the Engineering Department of the University of Manchester and Dr. J. C. Wyngaard of the National Center for Atmospheric Research (NCAR).

My thanks are also due: to the translator Richard Hainsworth for painstakingly clarifying more than one hundred nuances of meaning; to Dagny Simonsen for help in removing the abundance of errors carried over from the Russian edition in the English-language references; and to Mrs. L. J. Ball for correcting French and German citations and for preparing the editorial matter for printing.

B. E. Launder

INTRODUCTION

For a number of physical systems, gravitation is the major factor governing the flow of a fluid and the transfer of heat. A good example is the movement of atmospheric and oceanic masses. These movements are due primarily to density differences in the Earth's gravitational field. However, there are also cases on a much smaller scale—systems typical of a current industrial plant—where the effect of gravitation on fluids with nonuniform densities is the main driving force (free convection). It may also be significant in forced flows, changing the character of the convective transfer of momentum and heat.

The combined action of laminar free and forced convection (laminar mixed convection) has been studied for many years and some important scientific and practical results have been obtained. We would here particularly mention the pioneering work of the Soviet scientist G. A. Ostroumov*. His work focused on laminar flow and heat transfer in vertical channels. Notwithstanding his results, the firm opinion until the 1970's was that gravitational effects were absent in turbulent flows in channels, and this was reflected in heat transfer textbooks of the period.

The advance in power engineering and the development of new technolo-

*Ostroumov, G. A. "Free Convection for Internal Boundary-Value Problems," Moscow (1952).

gies led to the appearance in various flow and heat transfer equipment of significant effects associated with buoyant forces in turbulent flows, and which, in time, led to breakdowns. The items included nuclear reactors (particularly the thermal reliability of the core under conditions of relatively low coolant velocities), nuclear power stations with liquid metal coolants, supercritical steam generators, pipelines for hot or fluidized flows, superconducting cables, thermal gas optic lenses, and ventilation and airconditioning systems.† This situation caused us to revise our opinions concerning the neglect of gravitation in turbulent flows in channels at high pressure and stimulated appropriate scientific investigations. These have been most actively pursued at the USSR Academy of Sciences' High Temperature Institute, at the University of Manchester in the UK, and at the Fluid Mechanics Laboratory of the University of Paris in France.

Meanwhile, a considerable body of research had been carried out on atmospheric and oceanic turbulence brought about by the effect of gravity on turbulent stratified flows on a geophysical scale, and a number of correlations had been established for these processes. These results are reported in the appropriate sections in A. S. Monin and A. M. Yaglom [1.9], N. K. Vinichenko et al. [5.3], and Turner's monograph‡ It was to be expected that some of these results would be applicable to turbulent flows in channels, but they could not be taken without verification given the completely different conditions of channel and environmental flows. Moreover, a variety of specialized problems arose from the peculiarities of turbulence in flows under pressure, the cross sectional form of the channels and their orientation in space, and the use of a diversity of coolants (ranging from liquid metal to oils).

The study—stimulated by the demands of engineering practice—of turbulent mixed convection leads, in fact, to the development of a separate branch of the theory of convective heat transfer.

The first reasonably systematic temperature measurements of heated surfaces showed up a number of unexpected features and demonstrated that the influence of buoyant forces on turbulent heat transfer—depending as it does on the geometrical form and orientation with respect to the vertical of the heat transfer surface—was completely different from the influence of those forces when encountered in laminar flows. The first attempts at theoretical solutions indicated the inadequacy of the available methods and the necessity of investigating the structure of the flow and the turbulent transport of heat and momentum.

†Examples of the unexpected and disastrous appearance of buoyancy forces in specific cases are described in M. A. Styrikovich, O. I. Martynova, and Z. L. Miropol'sky, *The Process of Generating Steam in Power Stations* [in Russian], Energiya, Moscow (1969); H. Nittel, "Spannung. Krafte. Rohre." Energie, W. Germany, *31*, No. 4, pp. 123–125 (1979); O. G. Martynenko, P. M. Kolesnikov, and V. L. Kalpashikov, *Introduction to the Theory of Convective Gas Lenses* [in Russian], Nauka i Tekhnika, Minsk (1972).

‡J. Turner, *The Effects of Buoyancy in Liquids*, Cambridge, 1973.

The academic interest in the subject of turbulent mixed convection in shear flows arises from the production of turbulence by both mean velocity gradients and the action of buoyancy forces. At present, the approach being most actively pursued to calculate complex turbulent flows is to set up models based on balances for the single-point moments of the various quantities (usually termed 'second-moment' or 'second-order' closure.-Ed.). The larger of these systems contain ten to twelve complex differential equations to require many assumptions and empirical constants. The most useful of these models in practice include a transport equation for the turbulent energy together with velocity-velocity and velocity-temperature correlations and an equation for the dissipation of the turbulent energy. A widely-used model is one developed at Imperial College London, which is based on a $k - \epsilon$ (energy-dissipation) model and algebraic correlation relations. It is not possible yet to recommend any single model as being appropriate of even yielding satisfactory results for a relatively wide range of mixed convection problems near walls. We can illustrate this by a single example. The fluid dynamics and heat transfer in gas flows in vertical circular pipes have been investigated at the USSR Academy of Sciences' Institute for High Temperatures, the Moscow Power Engineering Institute, and Imperial College, London. Each investigation used a completely different model of turbulence and in each case the experimental heat transfer data was described quite well. However, when it came to the other parameters (even an integral quantity such as the friction factor) the results that came out were neither quantitatively nor qualitatively in agreement with the experiment.[1]

A consequence of this inability to describe mixed turbulent convection in channels mathematically is that a large volume of reliable experimental data has been accumulated in the field.

It has become apparent recently that the material on heat transfer and the pattern of turbulent flows in channels under significant influence of gravity (material that has been accumulated by solving both academic and practical problems), *should be analyzed as a whole and generalized.* The systematic presentation of these results is what we attempt here.

We had conceived both the plan and layout of this book in 1983, but the serious illness and passing in December 1984 of my mentor, Boris Sergeevich Petukhov, interrupted our joint effort. It then became my duty to complete our work and to abide by our agreements concerning its content and presentation.

The book is structured as follows. Chapter 1 provides a short presentation of the basic equations governing the dynamics of viscous fluids, the approximate equations for turbulent heat and momentum transport, and the boundary

[1]The level of development of second-moment closure is at the present time rather more advanced than the authors' remarks suggest. Mixed turbulent convection provides, however, an extremely complex set of phenomena that poses a most searching test of the turbulence model. Good recent examples of engineering computations for buoyancy-affected flows are provided in references [A7, A15, A4 and A5] listed in the Appendix (Editor's Footnote).

conditions for mean and fluctuating quantities, together with a similarity analysis of mixed convection in channels. This is followed by a description of the properties of turbulent flow and heat transfer for: the flow around a plane surface in purely forced convection; fluid flow through channels of rectangular or circular cross section, again in the absence of body forces; and the purely free convection around a vertical plane surface (Ch. 2). The relationships covered in Ch. 2 provide the limiting cases of turbulent mixed convection. The information on laminar mixed convection given in Ch. 3 can be used for comparison to identify those features in the changes in heat transfer and skin friction in similar situations due to turbulence, i.e., those features determined by the specific behavior of turbulent heat and momentum transport that occur when the influence of gravity is significant. Many of the results in Chapters 2 and 3 are original and interesting in themselves.

The next three chapters (4–6) are on heat transfer and the pattern of turbulent mixed convection in boundary layers and channels when the physical properties of the heat carrier changes little. The basic results of the Monin–Obukhov theory of turbulence in the surface layer of a temperature stratified atmosphere are presented in the first part of Ch. 4. We discuss approaches for analyzing data on turbulent wall flows and heat transfer in the presence of significant buoyancy forces in flows of various fluids. The results of a study of heat transfer and flow patterns in horizontal channels are presented in Ch. 5 and in vertical channels in Ch. 6. In most cases, the data are generalized by assuming an isotropic diffusion coefficient and a single equation of turbulence energy to describe the turbulence. This simple model is not only able to explain the basic features of the heat transfer and fluid mechanics of mixed convection near walls; it also leads to generalized relationships for several limiting cases. More complicated models of turbulence are also employed in order to provide a mathematical analysis of the structural features of turbulent heat and momentum transport (Sect. 6.2). The approaches developed for situations when the physical properties remain practically constant are extended to the case of heat transfer in a single phase heat carrier near its critical point, where very large variations in density occur. The mechanism by which local deteriorations in heat transfer occur is clarified.

We have tried to formulate the problems, present the solutions, conduct a physical analysis, and give mathematical generalizations. All the generalizations have been reduced to numerical formulae that can be applied immediately to practical problems. Other approaches to the investigation of turbulent mixed convection, such as numerical simulation, are not considered. Obviously, both these and other aspects have advanced during our studies of this field, but it would have significantly expanded the volume of this book had we considered them too. The properties of the flows were measured using thermal and laser anemometry, the theory and application of which are each independent topics and are not covered here. At the same time, we should note that we have used experimental data whose reliability has been established in other appropriate publications. The literature on numerical methods is even greater, although, as we have said, their application to turbulent mixed convection has not yet led to results that are either reliable in comparison to experiment or add anything new.

This monograph is based on work carried out by ourselves and our colleagues in the Department of Heat Transfer at the Institute for High Temperatures of the USSR Academy of Sciences. I should like to take this opportunity to convey my gratitude to these colleagues, many of whom took part in discussions on the manuscript, while others helped in the preparation of the book for publication.

A. F. Polyakov

NOMENCLATURE

Dimensional values

$A = dT_f/dx$	bulk temperature gradient of fluid, K/m; $A = 4q_w/\lambda Pe$ for a circular pipe and q_w = const; $A = (2q_1 + q_2)\lambda Pe$ for a plane channel and constant q_1 and q_2.
$a = \lambda/\rho c_p$	thermal diffusivity, m^2/s
b	width of a rectangular cross section channel, m
c_p	specific heat at constant pressure, $kJ/(kg \cdot K)$
$d = 2r_0$	diameter of a round channel or internal diameter of a circular pipe, m
$d_e = 4F/S$	hydraulic equivalent diameter, m
$e = \frac{1}{2}(\langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle)$	turbulent energy (this is the particular form for Cartesian coordinates; in general, the

	value of turbulent energy is defined by $\rho\langle v_i v_i\rangle/2$), m^2/s^2
$e_i = \langle v_i^2\rangle$	variances of fluctuating velocity components, m^2/s^2
$e_t = \langle t^2\rangle$	variance of fluctuating temperature, K^2
g	acceleration due to gravity, m^2/s^2, $g = 9.81\ m/s^2$
F	cross sectional area of a channel, m^2
H	distance between plane channel walls or height of a channel with rectangular cross section, m
h	enthalpy, kJ/kg
l	heated section length, m
l'	length of an upstream isothermal section, m
L	turbulence scale, m
p	pressure, N/m^2, Pa
q	heat flux (heat flow per unit area), W/m^2
r	radial coordinate, m
r_0	pipe radius, m
S	wetted perimeter, m
T	mean temperature, K
t	fluctuating temperature component, K
$v_* = \sqrt{\sigma_w/\rho}$	friction velocity, m/s
v_x, v_y, v_z; v_x, v_r, v_φ	fluctuating velocity components, m/s
w	velocity scale, m/s
w_i	mean velocity components, m/s
w_x, w_y, w_z; w_x, w_r, w_φ	mean velocity components (in Cartesian or cylindrical coordinate system), m/s
w_0	axial value of w_x, m/s
w_∞	free-stream value of w_x, i.e., value of outside boundary layer, m/s
$\bar{w}$	cross section average value of w_x
x	Cartesian coordinate in the

	mean flow direction (axial coordinate), m
y	Cartesian coordinate normal to the wall, m
z	Cartesian coordinate tangential to the wall, m (The origin is, as a rule, at the surface in the flow at the point where the heating/cooling starts. When the flat channel is horizontal, the origin is at the lower surface.)
x, r, φ	cylindrical coordinates: axial, m, radial, m, and azimuthal (measured from the lower surface), degree, respectively
x'	distance from the channel entry or from the leading edge of the plate, m
α	heat transfer coefficient, W/(K · m^2)
$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t}\right)_p$	coefficient of thermal expansion, 1/K
δ	length scale, m
δ_0	boundary layer thickness, m
$\delta_{MO} = v_*^3/\kappa g \beta (q_w/\rho c_p)$	Monin-Obukhov scale, m
ϵ	eddy kinematic viscosity (turbulent viscosity), m^2/s
ϵ_q	eddy thermal diffusivity (turbulent conductivity), m^2/s
ϑ	temperature scale, K
$\vartheta_w = (T_w - T_{in})$ or $(T_w - T_\infty)$	temperature scale corresponding to the first kind of boundary condition (given T_w), K
$\vartheta_* = q_w/\rho c_p v_*$, $\vartheta_\lambda = q_w d/\lambda$	temperature scales corresponding to the second kind of boundary condition (given q_w), K
λ	molecular thermal conductivity, W/(m · K)
μ	dynamic viscosity, N · s/m^2
$\nu = \mu/\rho$	kinematic viscosity, m^2/s

ρ	density, kg/m^3
σ	shear stress, N/m^2
τ	time, s
φ	angle measured from the upper generatrix of the channel, rad
ψ	angle between the vertical and the x-axis, rad

Dimensionless values

$c, c_1, \ldots, c_n, c_t$	constants
$A, A_1, \ldots,$ and $B, B_1 \ldots$	constant in the correlations for the velocity and temperature distributions
$E = e/w_x^2 = \frac{1}{2}\sum_{i=1}^{3} E_i,$ $e^+ = e/v_*^2 = 1/2(e_x^+ + e_y^+ + e_z^+)$	turbulent energy in wall coordinates
$E_i = e_i/w_x^2 = \langle v_i^2\rangle/w_x^2,$ $e_i^+ = e_i/v_*^2 = \langle v_i\rangle/v_*^2$	variances of the fluctuating velocity components
$E_* = \mathrm{Gq}/\mathrm{Pr}\,\mathrm{Re}_*^4,\ E = \mathrm{Gq}/\mathrm{Pr}\,\mathrm{Re}^4$	parameters of thermogravity influence
$\mathrm{Gr} = g\beta\delta^3\vartheta/\nu^2$	Grashof number (general definition)
$\mathrm{Gt} = g\beta\delta^3(T_w - T_\infty)/\nu^2,$ $\mathrm{Gq} = g\beta\delta^4 q_w/\nu^2\lambda,$ $\mathrm{GA} = g\beta\delta^4 \mathrm{A}/\nu^2$	Grashof number based on the wall surface temperature, on the wall heat flux, or on the longitudinal gradient of bulk temperature.

$$\mathrm{Gq} = \frac{g\beta d^4 q_w}{\nu^2\lambda} = \mathrm{Nu}\,\mathrm{Gt} \ \text{ or } \ \mathrm{Gq} = \frac{g\beta(2H)^4 q_w}{\nu^2\lambda} = \mathrm{Nu}\,\mathrm{Gt}$$

$$\mathrm{GA} = \frac{g\beta d^4 \mathrm{A}}{\nu^2} = \frac{4\,\mathrm{Gq}}{\mathrm{Pe}};\ \mathrm{GA}_r = \frac{g\beta r_0^4 \mathrm{A}}{\nu^2} = \frac{g\beta r_0^4 \mathrm{A}}{\nu^2} = \frac{\mathrm{Gq}}{4\,\mathrm{Pe}}$$

$$\mathrm{GA} = \frac{g\beta \mathrm{A}(2H)^4}{\nu^2} = \frac{g\beta(2H)^4}{\nu^2}\,\frac{2(q_1+q_2)}{\lambda\,\mathrm{Pe}}\bigg|_{q_1=q_2} = \frac{g\beta(2H)^4 q_w}{\nu^2\lambda}\,\frac{4}{\mathrm{Pe}}$$

$$= \frac{4\,\mathrm{Gq}}{\mathrm{Pe}}$$

$$\mathrm{GA}_h = \frac{g\beta \mathrm{A}H^4}{\nu^2}\bigg|_{q_1=q_2} = \frac{g\beta H^4 q_w}{\nu^2\lambda}\,\frac{4}{\mathrm{Pe}};\ \mathrm{Gt}_x = \frac{g\beta x^3(T_c - T_\infty)}{\nu^2}$$

$\hat{L} = \frac{L}{d}, L^+ = \frac{Lv_*}{\nu}$	dimensionless turbulent length scale
$M = w/a^*$	Mach number
$\mathrm{Nu} = q_w d/(T_w - T_f)\lambda$ or $\mathrm{Nu} = q_w 2H/(T_w - T_f)\lambda$, $\mathrm{Nu}_x = q_w x/(T_w - T_\infty)\lambda$	Nusselt number
$\mathrm{Pe} = \mathrm{Pr}\cdot \mathrm{Re}$	Peclet number
$\mathrm{Pr} = \nu/a$	(molecular) Prandtl number
$\mathrm{Pr_t} = \epsilon/\epsilon_q$	turbulent Prandtl number
$Q = q/q_w$	relative heat flow
$R = \frac{r}{r_0} = \frac{2r}{d}$	dimensionless radius
$c_f = 2\sigma_w/\rho\bar{w}^2, 2\sigma_w/\rho w_\infty^2$	skin friction coefficient
$\mathrm{RA} = \mathrm{PrGr}, \mathrm{Rt} = \mathrm{Pr\ Gt}, \mathrm{Rq} = \mathrm{PrGq}$	Rayleigh number, RA = PrGA
$\mathrm{Re} = w\delta/\nu$, $\mathrm{Re} = wd/\nu$, $\mathrm{Re} = \bar{w}2H/\nu$, $\mathrm{Re}_x = w_\infty x/\nu$, $\mathrm{Re}_* = v_* d/\nu$	Reynolds number

$$\mathrm{Rf} = \frac{g\beta\langle tv_y\rangle}{\langle v_x v_y\rangle \frac{\partial w_x}{\partial y}} = \frac{\mathrm{Gr}}{\mathrm{Re}^2}\frac{\langle t V_y\rangle}{\langle V_x V_y\rangle \frac{\partial W_x}{\partial Y}} = \frac{\mathrm{Gr}}{\mathrm{Re}^2 \mathrm{Pr_T}}\frac{dT/dY}{(dW_x/dY)^2}$$

$\mathrm{Ri} = \mathrm{Pr_t Rf}$	gradient Richardson number
$\mathrm{St} = q_w/\rho c_p w\Delta T = \mathrm{Nu/Pe}$	Stanton number
$T^+ = (T_w - T)/\vartheta_* = (T_w - T)\rho c_p v_*/q_w$	temperature in wall coordinates
$W_x = w_x/w$, $u = w_x/w_0$, $u = w_x/w_\infty$, $U = w_x/\bar{w}$, $u^+ = w_x/v_*$	velocity
$\langle V_i V_j\rangle = \langle v_i v_j\rangle/w^2$, $\langle v_i v_j\rangle^+ = \langle v_i v_j\rangle/v_*^2$	turbulent stress tensor
$X = x/\mathrm{Pe}\delta$, $X = x/\mathrm{Pe}d$	reduced length
$Y = y/r_0 = 2y/d = 1 - R$ or $Y = 2y/H$, $Y = y/\delta_0$	distance from the surface
$\Gamma = \sigma/\sigma_w$	relative shear stress
$\eta = yv_*/\nu$	distance from the wall (universal coordinate)
$\eta^+ = y/\delta_{MO} = \kappa\chi$	coordinate used for temperature stratified flows, or the stability parameter

$\Theta = (T - T_{in})/\vartheta$ or $\Theta = (T - T_\infty)/\vartheta$ or $\Theta = (T_w - T)/(T_w - T_{ax})$	temperature
κ	Von Karman constant
κ_i	unit vector components
$\delta_{ik} = \begin{matrix} 0 \text{ for } i \neq k \\ 1 \text{ for } i = k \end{matrix}$	Kronecker delta
ϵ_{ijk}	alternating tensor
$\xi = \dfrac{d}{l}\dfrac{2\Delta p}{\rho \bar{w}^2}$	hydraulic drag ($\xi = 4c_f$ when the flow in the channel is fully developed, in which case it is termed the *friction factor*)
$\eta = (q_w/\Delta T_w \lambda)y = \text{Nu}\,(y/\delta)$	distance from the wall

Subscripts

IN, OUT	inlet or outlet (section), respectively
u, 1	upper or lower (surfaces of the flat horizontal channel)
f	bulk mean value of fluid
d	end of the entry region, i.e., the channel cross section with a developed velocity profile in isothermal flow
cr	critical value
L	laminar flow in the absence of body forces
m	corresponds to the maximum velocity
th	threshold value
w	value at the wall
fc	free convection value
T	turbulent flow in the absence of body forces
ax	axial value
i, j, k; 1, 2, 3; x, y, z	coordinate directions
x	means that a dimensionless number is based on the axial coordinate
r_0, H, δ	means that a dimensionless number (for example, Gr, Nu, Re) is based on the radius r_0 of

	a pipe, or on the height (H) or half-height (0.5 H) of the plane channel, or on the boundary layer thickness δ_0; if a dimensionless number has no subscript it is based on the hydraulic equivalent diameter, i.e., $\delta = d$ for a circular pipe and $\delta = 2\,H$ for a plane channel
1, 2	correspond to the plane channel surfaces
0, $\pi/2$, π	values of φ
0	scale, reference, characteristic, initial or axial value
∞	value taken far from the surface in the flow (outside the boundary layer) or far from the start of heating when in a channel

Superscripts

$-$	averaged over a cross section or over a perimeter; also a bulk value
$=$	averaged both over the perimeter and length of a channel
Λ	dimensionless value
$\langle\ \rangle$	time-averaged value
$\sim$	instantaneous value (a mean value plus a fluctuating one)
$'$	fluctuating value
$+$	dimensionless value (employing wall variables)