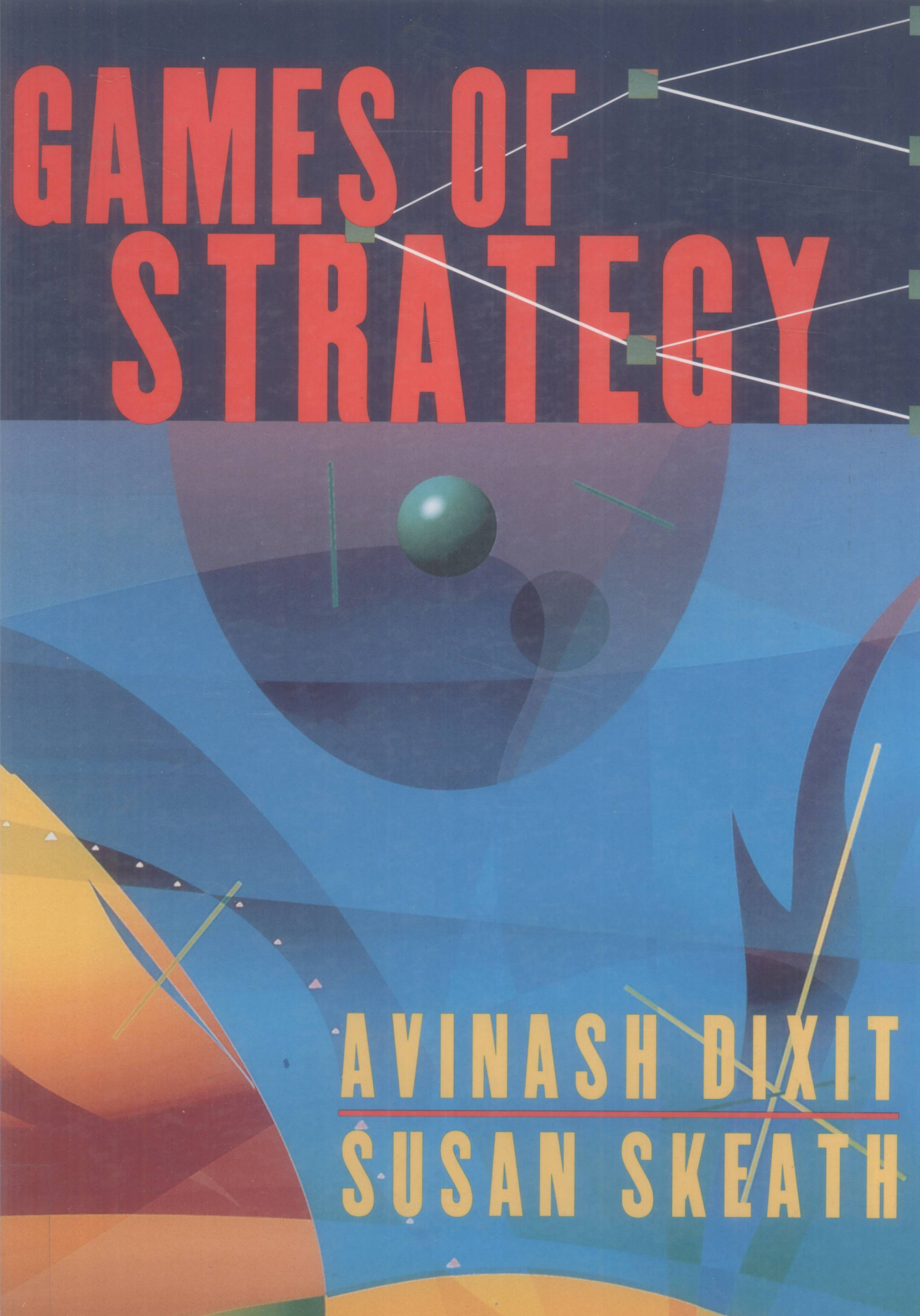


GAMES OF STRATEGY

The background is a complex abstract composition of overlapping geometric shapes. A large, dark blue semi-circle dominates the upper half, containing a green sphere and several thin green lines. Below this, the background transitions into various shades of blue and orange. A yellow line with small white triangles runs diagonally across the lower left. A red line with small white triangles runs diagonally across the lower right. The overall style is modern and geometric.

AVINASH DIXIT

SUSAN SKEATH

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GAMES OF STRATEGY

Avinash Dixit



Susan Skeath



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GAMES OF STRATEGY





To the memories of our fathers,

Kamalakar Ramachandra Dixit

and

James Edward Skeath



Preface

Game theory is a relative newcomer to the family of academic fields of thought. Its first substantial text, *Theory of Games and Economic Behavior* by John von Neumann and Oskar Morgenstern (Princeton, N.J.: Princeton University Press, 1943), was published less than 60 years ago. But it has rapidly become an important subject. Its basic theory developed through the 1950s and 1960s, with important contributions from John Nash, Thomas Schelling, and others. Then the pace accelerated as the theory began to find applications to issues in such diverse fields as international relations in the 1960s; economics, business, and evolutionary biology in the 1970s and 1980s; and political science in the 1980s and 1990s. Now we are at a point where terms from game theory, such as “zero-sum games” and “the prisoners’ dilemma,” have become a part of the language. As Paul Samuelson says, “To be literate in the modern age, you need to have a general understanding of game theory.”

Colleges and universities should attempt to impart such understanding to all of their students. Courses and textbooks on game theory are indeed proliferating rapidly. However, most of them suffer from two severe limitations—too narrow an orientation and too many prerequisites.

Most game theory courses take their approach from some particular discipline, whether it be economics, politics, business, or biology. These courses are taught with the assumption that students already know the concepts and jargon of their particular science. Many courses also assume a lot of mathematical background; they assume substantial knowledge of, and routinely use, calculus and probability theory. Such disciplinary and mathematical prerequisites re-

strict these courses to the more quantitatively oriented juniors and seniors majoring in a few subjects.

Game-theoretic concepts are actually much more basic, and can be conveyed with benefit to much larger groups of freshmen and sophomores without any prerequisites in any of the fields of application, and with only high school mathematics. Both of us have taught courses of this kind for several years. We have found that students take to the concepts of game theory with ease. Indeed, in economics the game-theoretic view of competition—strategic interactions between firms as they try to outdo each other in their attempts to attract customers—is much more natural and appealing than the standard textbook story of balancing supply and demand in impersonal markets. There are similar examples from other fields.

We believe that there is a strong case for reversing the usual order whereby general introductory courses in each subject are followed by advanced subject-specific courses in game theory. In the more natural progression, all students interested in the social and biological sciences would complete a freshman course in elementary game theory before going on to more detailed study of one of the specialized fields. Students intending to specialize in the natural sciences would also find game theory a more interesting and useful way to satisfy their distribution requirements than many introductory courses in particular social sciences. This book is the product of our experiences teaching game theory at such an elementary level and with such aims, and we hope it will enable others to develop and teach similar courses for beginners.

We emphasize that our book assumes no prior knowledge of economics, political science, or biology. We explain from first principles those concepts we use from these disciplines. Similarly, we do not require any knowledge of college mathematics or statistics. Of course the subject is inherently quantitative; totally nonnumerate students will not flourish in it. But the most basic high school algebra—for example, the ability to solve two linear equations in two unknowns—suffices for almost everything we do. We explain from first principles, and using familiar examples, the few simple rules for manipulating probabilities. In a few places where the use of elementary calculus (taking derivatives of simple functions) actually makes the analysis simpler, we do offer it, but we also offer a non-calculus alternative.

We also recognize that a few students with the best quantitative background and ability will find their intellectual curiosity whetted and will want more. We have therefore gathered some of the more technical and advanced material into one chapter—Chapter 7—for such readers; it can be omitted by others without loss of continuity.

The book covers a lot of material, and teachers can select from it to suit the length and special emphases of their courses. The ideas introduced in Chapters 1 and 2, and the theory and techniques developed in Chapters 3 to 5, are basic to

any course. Chapter 6 extends and combines the techniques of Chapters 3 through 5; it can be covered in a more superficial or selective manner. Chapter 7 is technical and optional as we said earlier.

The next five chapters examine specific classes of games, and therefore offer more scope for choice. Chapter 8 on the prisoners' dilemma is important to any treatment of game theory. Chapters 9 on strategic moves, 10 on evolutionary games, 11 on collective-action games, and 12 on uncertainty and asymmetric information are largely independent of one another. Some of these can be studied in depth while others are only skimmed.

The final group of chapters is on applications—Chapters 13 on brinkmanship, 14 on voting, 15 on auctions, 16 on bargaining, and 17 on markets—and allows the greatest freedom of choice in course design. In most of these chapters, we begin with one or two simple numerical examples and then develop the ideas in greater generality. Teachers may choose to cover some of the topics in depth and convey only a flavor of the rest using the examples.

In our own courses we first used Avinash Dixit and Barry Nalebuff's *Thinking Strategically* (Norton, 1991) as the main text. But that book was directed at more of a general "trade" readership, and so lacked the kind of pedagogic material a course textbook needs: repetition and summaries for reinforcement, precise definitions and highlighted statements of key concepts, problems, and questions to allow students to test their understanding, and so on. Therefore we had to supplement the book with lecture notes, problem sets, and other handouts or course packets. In the process of creating these tools, we gradually improved and replaced the examples and cases used to convey the ideas in *Thinking Strategically*, thus effectively replacing almost all of that book with new material. Here we have assembled and organized all of these things into one package, and written a proper textbook at the Dixit–Nalebuff level. However, we have retained a very important feature of that book, namely its reliance on examples and cases. Our ultimate aim is to convey the concepts of game theory, but through the medium of examples and cases instead of setting out the theory by itself in an abstract mathematical way.

We have tried out successive drafts of this manuscript on our own classes for the past two years. Several teachers at other universities and colleges used the manuscript in their courses; some even developed new courses based on the manuscript. This list includes Amanda Bayer (Swarthmore College), Larry Evans (The College of William and Mary), Kimberly Katz (Mount Holyoke College), Greg Trandel (University of Georgia), and Randall Waldron (University of South Dakota). They, along with their students and our own students, gave us extremely valuable feedback. The result is not only the correction of many errors—typographical as well as substantive—but also major improvements in exposition, the introduction of new material, and the deletion of material found to be too difficult or unsuitable. We are grateful to all of these teachers and students—for

their willingness to venture into a new type of course using a draft manuscript; for the encouragement they gave us based on their experience; and for their friendly, perceptive, and constructive criticisms of the manuscript drafts.

In addition, many people read the whole manuscript: Vincent Crawford (University of California, San Diego), Hiroyuki Kawakatsu (University of California, Irvine), Barry Nalebuff (Yale), Ed Parsons (our editor at Norton), and two anonymous readers commissioned by him. Others read individual chapters: Dilip Abreu and Pierpaolo Battigalli (Princeton), Frank Milne (Queen's University, Canada), and Sylvia Nasar (*New York Times*). All of these people gave us many helpful comments concerning the substance as well as the writing, and we are happy to acknowledge our debt to them all. We are also grateful to Kate Barry (our project editor at Norton) for expertly guiding the book through the production process and for being tolerant of our requests for many last-minute changes.

Susan Skeath would also like to thank her husband, Paul van Mulbregt, for his support and encouragement, as well as for reading and commenting on early drafts of various chapters. In addition, her children deserve praise for their patience and tolerance, if not for learning how to push the computer's reset button.

Finally, we were fortunate to have an outstanding copy editor, Susan Middleton. She not only improved our writing at numerous points, but also understood the substance, and caught an embarrassingly large number of slips and errors. If the book is judged to be well written and relatively error-free, she deserves a great deal of the credit. Of course, the two authors retain joint responsibility for any errors and shortcomings that remain, in the sense that each of us blames the other.

A. D.

S. S.

April 1999

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Introduction and General Principles



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