

Multivariable Calculus

FOURTH EDITION

HOWARD ANTON
DREXEL UNIVERSITY

with contributions from

Albert Herr, Drexel University



JOHN WILEY & SONS, INC.

New York Chichester Brisbane Toronto Singapore

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Design Supervisor: Madelyn Lesure
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Photo Researcher: Jennifer Atkins
Photo Research Manager: Stella Kupferberg
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Cover and Chapter Opening Art: Hudson River Studio
Text Design: Hudson River Studio
Electronic Illustration: Techsetters, Inc.
Compositor: General Graphic Services, Inc.

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Library of Congress Cataloging-in-Publication Data:

Anton, Howard.
Multivariable calculus / Howard Anton : with contributions from
Albert Herr. -- 4th ed.
p. cm.
Includes index.
ISBN 0-471-58247-6
I. Calculus. I. Herr, Albert. II. Title.
QA303.A54 1992
515--dc20

92-16559
CIP

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Preface

□ ABOUT THIS TEXT

This book consists of the multivariable portion of my text, *Calculus with Analytic Geometry*, 4th Edition (Chapters 14–18), as well as Chapter 19 on second-order differential equations. Also included as appendices are reprints of the chapter on infinite series (Chapter 11) and the section on first-order differential equations (Section 7.7). In addition, there is an appendix that covers the basic concepts of complex numbers.

This book, in combination with the *Brief Edition of Calculus with Analytic Geometry*, 4th Edition, covers all of the material in *Calculus with Analytic Geometry*, 4th edition, plus the added material on complex numbers.

It is assumed that the reader has completed a course on single variable calculus, but for reference a brief review of key concepts from single-variable calculus is given at the front of the text.

Reviewing and Class Testing

This edition is the outgrowth of twelve years of classroom use and input from hundreds of students and instructors who have written to me with constructive comments over the years. In addition, the exposition and exercise sets have been refined and polished in response to input from a team of professors who carefully critiqued every single line of the third edition as they used it in the classroom. It is my hope that this painstaking attention to pedagogical detail has resulted in a book that surpasses the earlier editions for clarity and accuracy.

Rigor


The challenge of writing a good calculus textbook is to strike the right balance of rigor and clarity. It is my goal in this text to present precise mathematics to the fullest extent possible for the freshman audience. The theory is presented in a style tailored for beginners, but in those places where precision conflicts with clarity, the exposition is designed for clarity. However, I believe it to be of fundamental importance for the student to understand the distinction between a careful proof and an informal argument; thus, when the arguments are informal, I make this clear to the reader.

Illustrations

This text is more heavily illustrated than most calculus books because illustrations play a special role in my philosophy of pedagogical exposition. Freshmen have great difficulty in reading mathematics and extracting concepts from mathematical formulas, yet they can often grasp a concept immediately when

the right picture is presented. This is not surprising since mathematics is a language that must be learned. Until the language is mastered one cannot hope to understand the ideas that the language conveys. I use illustrations to help teach the language of mathematics by supporting the theorems and formulas with illustrations that help the reader understand the concepts embodied in the mathematical symbolism. In keeping with current trends, I have used modern four-color typography with a consistency of style and color selection throughout the text.

Exercises

The exercises in this new edition have been extensively modified and expanded. Each exercise set begins with routine drill problems and progresses gradually toward problems of greater difficulty. I have tried to construct well-balanced exercise sets with more variety than is available in most calculus texts. Each chapter ends with a set of supplementary exercises to help the student consolidate his or her mastery of the chapter. In addition, many of the exercise sets now contain problems requiring a calculator as well as so-called “spiral” problems (problems that use concepts from earlier chapters). Answers are given to odd-numbered problems, and at the beginning of each exercise set there is a list of those exercises that require a calculator; these are labeled with the icon .

A NOTE FROM THE AUTHOR

I am gratified that the third edition of this text continued the tradition of its predecessors as the most widely used textbook in calculus and that after twelve years in print users and reviewers have continued to praise the clarity of the exposition.

We are now on the threshold of major changes in the way calculus and, indeed, all of mathematics will be taught. Fueled by rapid advances in technology and a reevaluation of traditional course content, we are entering a period of exploration in the teaching of calculus. This new edition reflects a clear, but cautious, commitment to the newer visions of calculus. My goal in this revision is to provide instructors with all of the *supplemental* resources required to begin experimenting with new ideas and new technology within the framework of the traditional course structure.

Users of the earlier editions will be able to ease comfortably into the new edition, but will sense a more contemporary philosophy in the exposition and exercises, with increased attention to numerical computations and estimation, as well as flexibility for varying the order and emphasis of topics. For instructors on the cutting edge of calculus reform, there is an extensive array of supplements that use new technology: symbolic algebra software, calculators, Hyper-card stacks, and even CD-ROM.

Although there are many changes in this new edition, I remain committed to the philosophy that the heart and soul of a fine textbook is the clarity of its exposition; there are very few sections in this new edition that have not been polished and refined. My goal, as in earlier editions, is to *teach* the material in the clearest possible way with a level of rigor that is suitable for the mainstream calculus audience.

Supplements

Graphing Calculator Supplement

The following supplement contains a collection of problems that are intended to be solved on a graphing calculator. The problems are not specific to a particular brand of calculator. The manual also provides an overview of the types of calculators available, general instructions for calculator use, and a discussion of the numerical pitfalls of roundoff and truncation error.

- *Discovering Calculus with Graphing Calculators*
Joan McCarter, *Arizona State University*
ISBN: 0-471-55609-2

Symbolic Algebra Supplements

The following supplements are collections of problems for the student to solve. Each contains a brief set of instructions for using the software as well as an extensive set of problems utilizing the capabilities of the software. The problems range from very basic to those involving real-world applications.

- *Discovering Calculus with DERIVE*
Jerry Johnson and Benny Evans, *Oklahoma State University*
ISBN: 0-471-55155-4
- *Discovering Calculus with MAPLE*
Kent Harris, *Western Illinois University*
ISBN: 0-471-55156-2
- *Discovering Calculus with MATHEMATICA*
Bert Braden, Don Krug, Steve Wilkinson, *Northern Kentucky University*
ISBN: 0-471-53969-4

Macintosh Hypercard Stacks

This supplement is a set of Hypercard 2.0 stacks that are designed primarily for lecture demonstrations. Each stack is self-contained, but all have a common interface. There is an initial set of six stacks (with more to be developed). The six initial stacks are concerned with limits, Newton's Method, the definition of the definite integral, convergence of Taylor polynomials, polar coordinates, and mathematics history.

- *HYPERCALCULUS*
Chris Rorres and Loren Argabright, *Drexel University*
ISBN: 0-471-57052-4

CD-ROM Version of Calculus for IBM Compatible Computers

This supplement is an electronic version of the entire text and the *Student's Solutions Manual* on compact disk for use with IBM compatible computers equipped with a CD-ROM drive. All text material and illustrations are stored on disk with an interconnecting network of hyperlinks that allows the student to access related items that do not appear in proximity in the text. A complete keyword glossary and step-by-step discussions of key concepts are also included.

- *CD-ROM Version of Anton Calculus/4E: An Electronic Study Environment*
Developed by Electric Book Company
ISBN: 0-471-55803-6

Linear Algebra Supplement

The following supplement gives a brief introduction to those aspects of linear algebra that are of immediate concern to the calculus student. The emphasis is on methods rather than proof.

- *Linear Algebra Supplement to Accompany Anton Calculus/4E*
ISBN: 0-471-56893-7

Student Study Resources

The following supplement is a tutorial, review, and study aid for the student.

- *The Calculus Companion to Accompany Anton Calculus/4E, Vols. 1 and 2*
William H. Barker and James E. Ward, *Bowdoin College*
ISBN: 0-471-55139-2 (Volume 1); ISBN: 0-471-55138-4 (Volume 2)

The following supplement contains detailed solutions to all odd-numbered exercises.

- *Student's Solutions Manual to Accompany Anton Calculus/4E*
Albert Herr, *Drexel University*
ISBN: 0-471-55140-6

Resources for the Instructor

There is a resource package for the instructor that includes hard copy and electronic test banks and other materials. These can be obtained by writing on your institutional letterhead to Susan Elbe, Senior Marketing Manager, John Wiley & Sons, Inc., 605 Third Avenue, New York, N.Y., 10158-0012.

Acknowledgments

It has been my good fortune to have the advice and guidance of many talented people, whose knowledge and skills have enhanced this book in many ways. For their valuable help I thank:

Reviewers and Contributors to Earlier Editions

Edith Ainsworth, *University of Alabama*
David Armacost, *Amherst College*
Larry Bates, *University of Calgary*
Marilyn Blockus, *San Jose State University*
David Bolen, *Virginia Military Institute*
George W. Booth, *Brooklyn College*
Mark Bridger, *Northeastern University*
John Brothers, *Indiana University*
Robert C. Bueker, *Western Kentucky University*
Robert Bumcrot, *Hofstra University*
Chris Christensen, *Northern Kentucky University*
David Cohen, *University of California, Los Angeles*
Michael Cohen, *Hofstra University*
Robert Conley, *Precision Visuals*
A. L. Deal, *Virginia Military Institute*
Charles Denlinger, *Millersville State College*
Dennis DeTurck, *University of Pennsylvania*
Jacqueline Dewar, *Loyola Marymount University*
Irving Drooyan, *Los Angeles Pierce College*
Hugh B. Easler, *College of William and Mary*
Joseph M. Egar, *Cleveland State University*
Garret J. Etgen, *University of Houston*
James H. Fife, *University of Richmond*
Barbara Flajnik, *Virginia Military Institute*
Nicholas E. Frangos, *Hofstra University*
Katherine Franklin, *Los Angeles Pierce College*
Michael Frantz, *University of La Verne*
William R. Fuller, *Purdue University*
Raymond Greenwell, *Hofstra University*
Gary Grimes, *Mt. Hood Community College*
Jane Grossman, *University of Lowell*
Michael Grossman, *University of Lowell*

Douglas W. Hall, *Michigan State University*
 Nancy A. Harrington, *University of Lowell*
 Albert Herr, *Drexel University*
 Peter Herron, *Suffolk County Community College*
 Robert Higgins, *Quantics Corporation*
 Louis F. Hoelzle, *Bucks County Community College*
 Harvey B. Keynes, *University of Minnesota*
 Paul Kumpel, *SUNY, Stony Brook*
 Leo Lampone, *Quantics Corporation*
 Bruce Landman, *Hofstra University*
 Benjamin Levy, *Lexington H.S., Lexington, Mass.*
 Phil Locke, *University of Maine, Orono*
 Stanley M. Lukawecki, *Clemson University*
 Nicholas Macri, *Temple University*
 Melvin J. Maron, *University of Louisville*
 Thomas McElligott, *University of Lowell*
 Judith McKinney, *California State Polytechnic University, Pomona*
 Joseph Meier, *Millersville State College*
 David Nash, *VP Research, Autofacts, Inc.*
 Mark A. Pinsky, *Northeastern University*
 William H. Richardson, *Wichita State University*
 David Sandell, *U.S. Coast Guard Academy*
 Donald R. Sherbert, *University of Illinois*
 Wolfe Snow, *Brooklyn College*
 Norton Starr, *Amherst College*
 Richard B. Thompson, *The University of Arizona*
 William F. Trench, *Trinity University*
 Walter W. Turner, *Western Michigan University*
 Richard C. Vile, *Eastern Michigan University*
 James Warner, *Precision Visuals*
 Candice A. Weston, *University of Lowell*
 Yihren Wu, *Hofstra University*
 Richard Yuskaitis, *Precision Visuals*

Content Reviewers

The following people critiqued the previous edition and recommended many of the changes that found their way into the new edition:

Ray Boersma, *Front Range Community College*
 Terrance Cremeans, *Oakland Community College*
 Tom Drouet, *East Los Angeles College*
 Ken Dunn, *Dalhousie University*
 Kent Harris, *Western Illinois University*
 Maureen Kelly, *Northern Essex Community College*
 Richard Nowakowski, *Dalhousie University*
 Robert Phillips, *University of South Carolina at Aiken*
 David Randall, *Oakland Community College*
 George Shapiro, *Brooklyn College*
 Ian Spatz, *Brooklyn College*

Accuracy Reviewers of the Fourth Edition

The following people worked with me in reading the manuscript, galley proofs, and page proofs for mathematical accuracy. Their perceptive comments have improved the text immeasurably:

Irl C. Bivens, *Davidson College*
 Hannah Clavner, *Drexel University*
 Garret J. Etgen, *University of Houston*
 Daniel Flath, *University of South Alabama*
 Susan L. Friedman, *Bernard M. Baruch College, CUNY*
 Evelyn Weinstock, *Glassboro State College*

Problem Contributors

The following people contributed numerous new and imaginative problems to the text:

Irl C. Bivens, *Davidson College*
 Daniel Bonar, *Denison University*
 James Caristi, *Valparaiso University*
 G. S. Gill, *Brigham Young University*
 Albert Herr, *Drexel University*
 Herbert Kasube, *Bradley University*
 Phil Kavanaugh, *Illinois Wesleyan University*
 John Lucas, *University of Wisconsin–Oshkosh*
 Ron Moore, *Ryerson Polytechnical Institute*
 Barbara Moses, *Bowling Green State University*
 David Randall, *Oakland Community College*
 Jean Springer, *Mount Royal College*
 Peter Waterman, *Northern Illinois University*

Answers, Solutions, and Index

The following people assisted with the critically important job of preparing the index and obtaining answers for the text and solutions for the *Student's Solutions Manual*:

Harry N. Bixler, *Bernard M. Baruch College, CUNY*
 Hannah Clavner, *Drexel University*
 Michael Dagg
 Stephen L. Davis, *Davidson College*
 Susan L. Friedman, *Bernard M. Baruch College, CUNY*
 Shirley Wakin, *University of New Haven*

Computer Illustrations

Many of the illustrations involving two-dimensional mathematical graphs were generated electronically using software developed by Techsetters, Inc. A number of people contributed to the development of the software:

Edward A. Burke, *Hudson River Studio*—color separation
 John R. DiStefano (student), *Drexel University*—programmer
 Theo Gray, *Wolfram Research*—technical assistance
 Julie Varbalow (student), *Skidmore College*—testing

Some of the three-dimensional surfaces were generated by Mark Bridger of Northeastern University using the SURFS software package that he developed.

The Wiley Staff

There are so many people at Wiley who contributed in special ways to this project that it is impossible to mention them all. However, a word of appreciation is due to the people I worked with very closely: Joan Carrafiello, Lucille Buonocore, Ann Berlin, Steve Kraham, Susan Elbe, and my editor Barbara Holland.

Other Supplementary Materials

The following people provided additional material for tests and other supplements:

Pasquale Condo, *University of Lowell*
Maureen Kelley, *Northern Essex Community College*
Catherine H. Pirri, *Northern Essex Community College*

Special Contributions

I owe an enormous debt of gratitude to Albert Herr of Drexel University, who worked so closely with me on this edition that his name appears on the title page. Professor Herr is a gifted, award-winning teacher of mathematics with years of experience in the calculus classroom. His careful attention to technical detail and his dogged challenges to the clarity of virtually every line of exposition fostered hours of debate over pedagogy and helped bring this edition to a level of quality I could not have achieved alone. Professor Herr is also the source for many of the imaginative new exercises in this edition. I feel fortunate that Al was by my side in preparing this revision.

If I had the power to grant an honorary degree in mathematics, I would give it to my assistant, Mary Parker, who worked on virtually every aspect of this text—from the tedious task of xeroxing tens of thousands of pages to the technical tasks of accuracy checking and coordination. Her warm sense of humor and dedication to the quality of this book were a constant source of inspiration.

H. A.

Introduction

The creation of calculus is attributed to two of the greatest geniuses in the history of mathematics and science—Gottfried Wilhelm Leibniz and Isaac Newton.

□ GOTTFRIED WILHELM LEIBNIZ (1646–1716)

This gifted genius was one of the last people to have mastered most major fields of knowledge—an impossible accomplishment in our own era of specialization. He was an expert in law, religion, philosophy, literature, politics, geology, metaphysics, alchemy, history, and mathematics.

Leibniz was born in Leipzig, Germany. His father, a professor of moral philosophy at the University of Leipzig, died when Leibniz was six years old. The precocious boy then gained access to his father's library and began reading voraciously on a wide range of subjects, a habit that he maintained throughout his life. At age 15 he entered the University of Leipzig as a law student and by the age of 20 received a doctorate from the University of Altdorf. Subsequently, Leibniz followed a career in law and international politics, serving as counsel to kings and princes.

During his numerous foreign missions, Leibniz came in contact with outstanding mathematicians and scientists who stimulated his interest in mathematics—most notably, the physicist Christian Huygens. In mathematics Leibniz was self-taught, learning the subject by reading papers and journals. As a result of this fragmented mathematical education, Leibniz often duplicated the results of others, and this ultimately led to a raging conflict as to whether he or Isaac Newton should be regarded as the inventor of calculus. The argument over this question engulfed the scientific circles of England and Europe, with most scientists on the continent supporting Leibniz and those in England supporting Newton. The conflict was unfortunate, and both sides suffered in the end. The continent lost the benefit of Newton's discoveries in astronomy and physics for more than 50 years, and for a long period England became a second-rate country mathematically because its mathematicians were hampered by Newton's inferior calculus notation. It is of interest to note that Newton and Leibniz never went to the lengths of vituperation of their advocates—both were sincere admirers of each other's work. The fact is that both men invented calculus independently; Leibniz invented it 10 years after Newton, in 1685, but he published his results 20 years before Newton published his own work on the subject.

Leibniz never married. He was moderate in his habits, quick-tempered, but easily appeased, and charitable in his judgment of other people's work. In spite

of his great achievements, Leibniz never received the honors showered on Newton, and he spent his final years as a lonely embittered man. At his funeral there was one mourner, his secretary. An eyewitness stated, “He was buried more like a robber than what he really was—an ornament of his country.”

ISAAC NEWTON
(1642–1727)

Newton was born in the village of Woolsthorpe, England. His father died before he was born and his mother raised him on the family farm. As a youth he showed little evidence of his later brilliance, except for an unusual talent with mechanical devices—he apparently built a working water clock and a toy flour mill powered by a mouse. In 1661 he entered Trinity College in Cambridge with a deficiency in geometry. Fortunately, Newton caught the eye of Isaac Barrow, a gifted mathematician and teacher. Under Barrow’s guidance Newton immersed himself in mathematics and science, but he graduated without any special distinction. Because the Plague was spreading rapidly through London, Newton returned to his home in Woolsthorpe and stayed there during the years of 1665 and 1666. In those two momentous years the entire framework of modern science was miraculously created in Newton’s mind—he discovered calculus, recognized the underlying principles of planetary motion and gravity, and determined that “white” sunlight was composed of all colors, red to violet. For some reason he kept his discoveries to himself. In 1667 he returned to Cambridge to obtain his Master’s degree and upon graduation became a teacher at Trinity. Then in 1669 Newton succeeded his teacher, Isaac Barrow, to the Lucasian chair of mathematics at Trinity, one of the most honored chairs of mathematics in the world. Thereafter, brilliant discoveries flowed from Newton steadily. He formulated the law of gravitation and used it to explain the motion of the moon, the planets, and the tides; he formulated basic theories of light, thermodynamics, and hydrodynamics; and he devised and constructed the first modern reflecting telescope.



Gottfried Leibniz
(Culver Pictures)



Isaac Newton
(Culver Pictures)

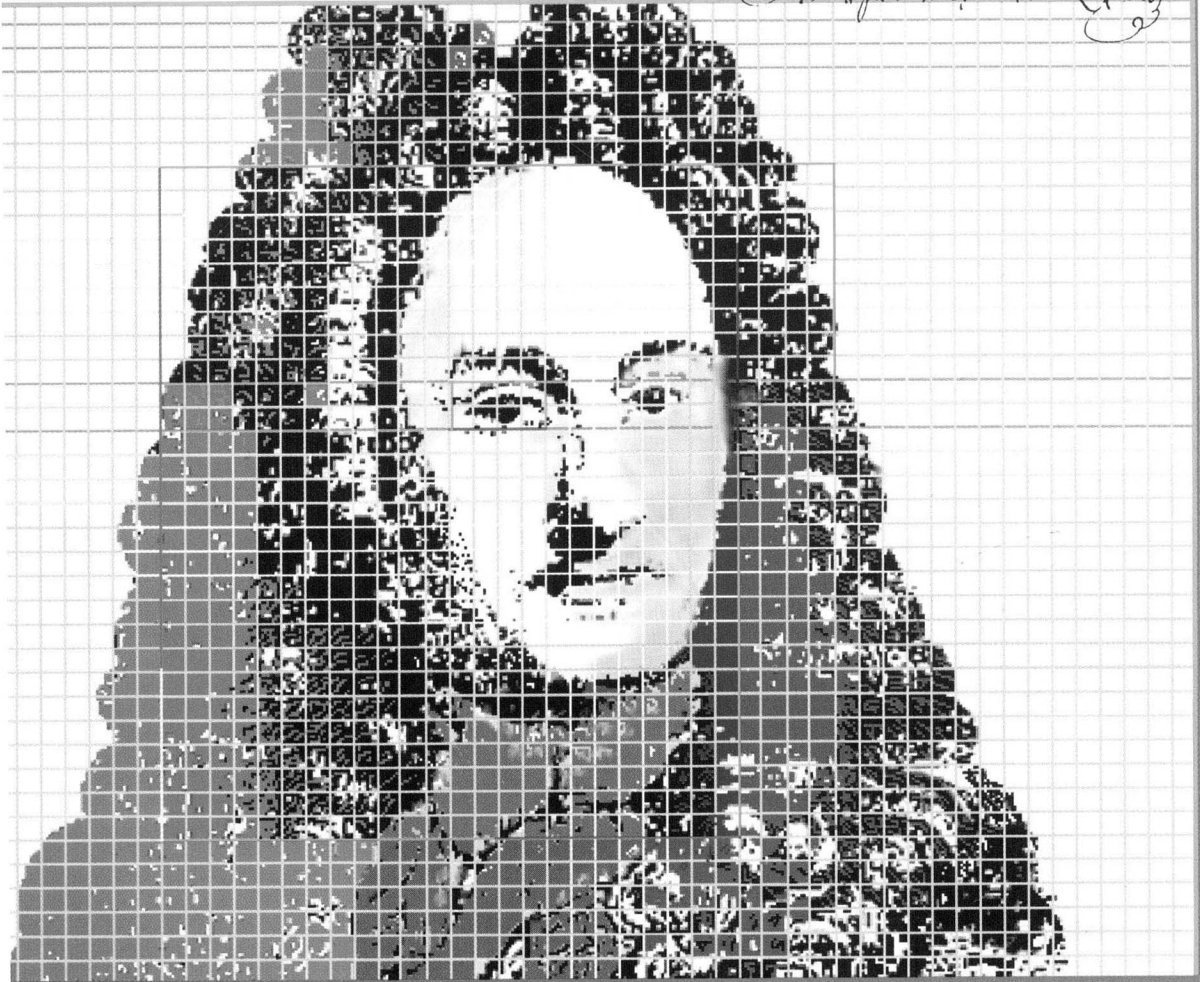
Throughout his life Newton was hesitant to publish his major discoveries, revealing them only to a select circle of friends, perhaps because of a fear of criticism or controversy. In 1687, only after intense coaxing by the astronomer, Edmond Halley (Halley's comet), did Newton publish his masterpiece, *Philosophiae Naturalis Principia Mathematica* (The Mathematical Principles of Natural Philosophy). This work is generally considered to be the most important and influential scientific book ever written. In it Newton explained the workings of the solar system and formulated the basic laws of motion which to this day are fundamental in engineering and physics. However, not even the pleas of his friends could convince Newton to publish his discovery of calculus. Only after Leibniz published his results did Newton relent and publish his own work on calculus.

After 35 years as a professor, Newton suffered depression and a nervous breakdown. He gave up research in 1695 to accept a position as warden and later master of the London mint. During the 25 years that he worked at the mint, he did virtually no scientific or mathematical work. He was knighted in 1705 and on his death was buried in Westminster Abbey with all the honors his country could bestow. It is interesting to note that Newton was a learned theologian who viewed the primary value of his work to be its support of the existence of God. Throughout his life he worked passionately to date biblical events by relating them to astronomical phenomena. He was so consumed with this passion that he spent years searching the Book of Daniel for clues to the end of the world and the geography of hell.

Newton described his brilliant accomplishments as follows: "I seem to have been only like a boy playing on the seashore and diverting myself in now and then finding a smoother pebble or prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."

The Discoverers of Calculus

Gottfried Wilhelm Leibniz



Gottfried Wilhelm Leibniz (1646-1716)

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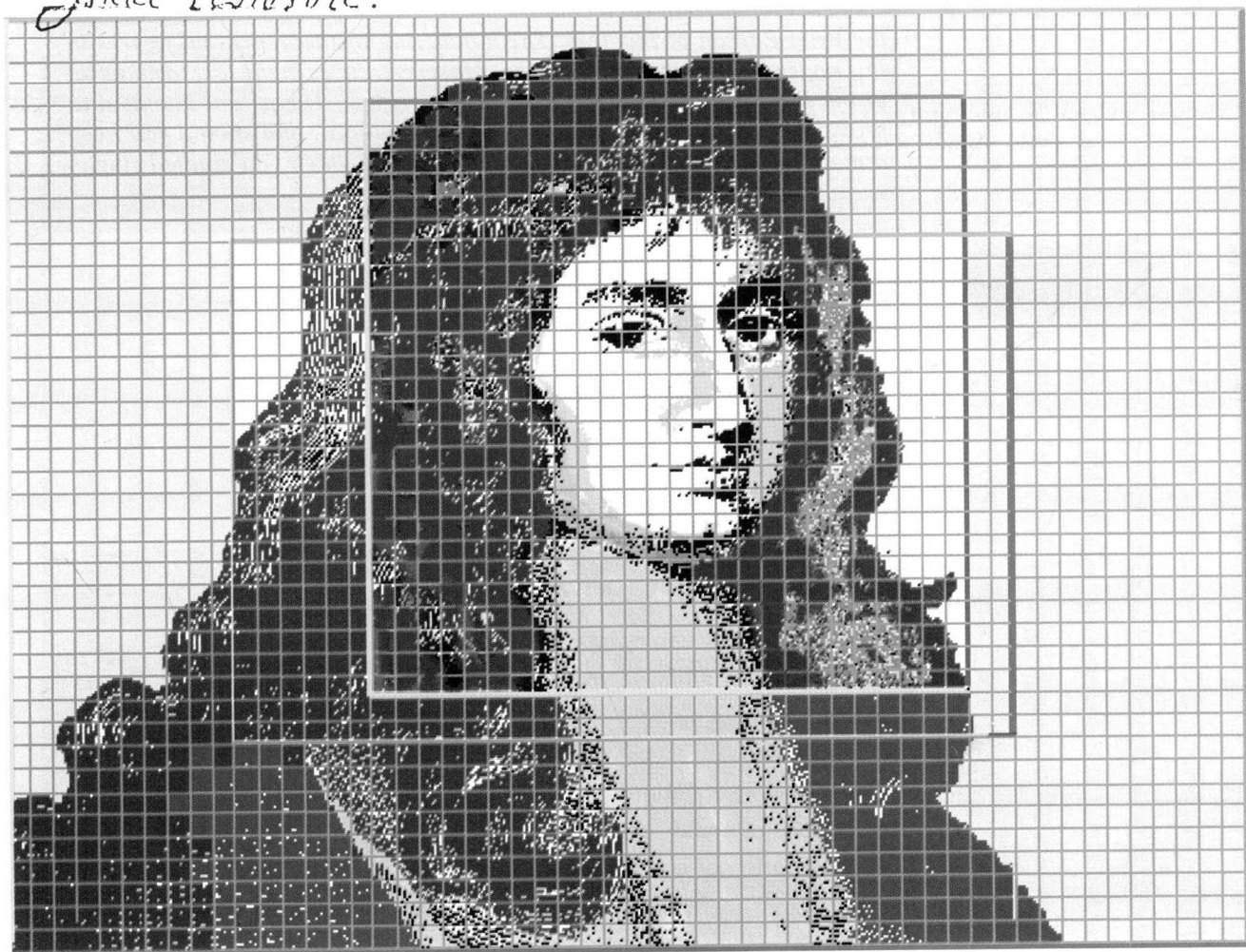
FIRST-ORDER DIFFERENTIAL EQUATIONS AND APPLICATIONS

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Isaac Newton.



Sir Isaac Newton (1642-1727)