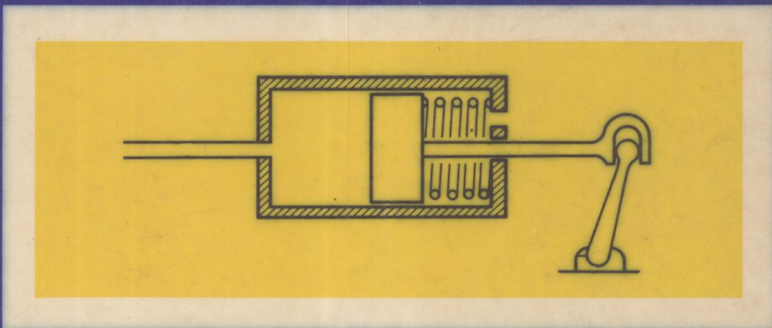
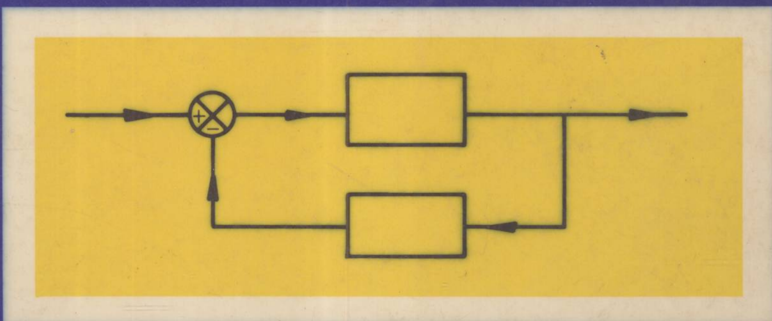
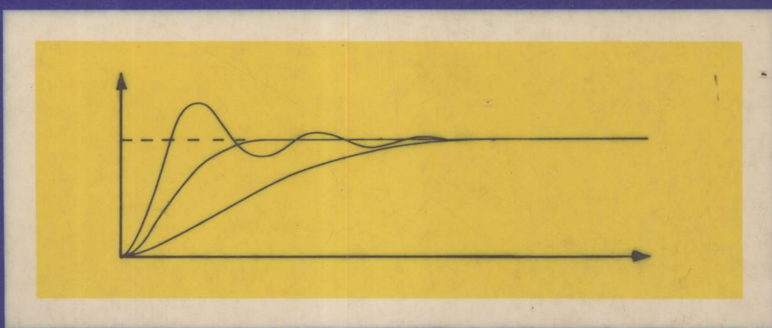
System Modelling and Control

Second Edition

J. Schwarzenbach and K. F. Gill







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Second Edition

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Edward Arnold

First published in Great Britain 1978 by
Edward Arnold (Publishers) Ltd, 41 Bedford Square, London WC1B 3DQ

Edward Arnold, 300 North Charles Street, Baltimore, Maryland 21201, U.S.A.

Edward Arnold (Australia) Pty Ltd, 80 Waverley Road, Caulfield East, Victoria 3145, Australia

Reprinted with corrections 1979
Second edition 1984

ISBN 0 7131 3518 2

Typeset and printed in Great Britain at
The Pitman Press, Bath

Preface to First Edition

Since the early 1940s, the number of practical applications of the principle of feedback has grown rapidly and the range of application has become very wide, with the consequence that an increasing proportion of engineers, scientists, and technologists require a basic appreciation of the fundamentals of automatic control theory. As the requirements for system dynamic performance have become more exacting so also have the demands on the engineer. Many who are new to the subject find feedback control theory difficult to understand, largely because of the rather abstract nature of some of the concepts involved. Of the textbooks available many are so comprehensive in their coverage that they are more suited to the reader who already has some understanding of the subject rather than to the beginner. The sheer volume of theoretical material tends to discourage the latter type of reader, and the detail often obscures the significance of the main principles.

The primary objective in writing this book has therefore been to sift from the large volume of literature on control theory the material believed to be most pertinent to industrial practice, and to present it in such a way that the student or practising engineer can attain a sound physical understanding of the basic principles of control. Familiarity with the material presented in the book will enable the reader to converse with specialists in the field, to design simple control loops adequate for many industrial applications, and with the aid of more advanced texts to design more complex control schemes. The aim throughout has been to present the fundamental theory in such a way that the reader can see the practical relevance of the material and that he can build up a clear mental picture to aid understanding. The mathematical manipulations can readily be mastered with practice; understanding the significance of the procedures and of their results is the real problem.

The principles of feedback control theory are very general; thus the topic is broadly based and is of relevance for a wide range of dynamic systems. The main variation in the potential areas of application lies in the differing characteristics and complexity of the systems to be controlled. Electrical engineers probably have the least difficulty in understanding control theory since many of the concepts are relevant to their other areas of study. They probably also face the fewest problems of application since the systems with which they deal, although often complex, are well defined because of the discrete lumped nature of most components and of the ease of measurement of system variables.

Many existing textbooks are intended primarily for the electrical engineer. The method of approach used in this book should make it particularly useful for mechanical engineers, chemical engineers, and other technologists and scientists (and in part also for life scientists, economists and others with an interest in the dynamic behaviour of systems and in the concept of feedback). The main problem in designing a control loop for non-electrical systems normally arises when attempting to obtain an adequate mathematical model for the system since, in general, components cannot readily be represented by simple discrete ideal elements; often non-linearities are dominant, measurement is difficult, and noise is significant.

In our teaching of control to undergraduates, postgraduates, industrial engineers and non-engineers we have experimented with the method of approach, the topics included, and the order of presentation. The approach that has evolved, and appears to be the most effective, forms the basis for this book. We have found that the 'classical' approach based on the transfer function and associated techniques of analysis is more easily comprehended and related to practice by the beginner than is 'modern control theory' which is based on a state space approach. The emphasis in this book is accordingly on classical linear control theory. Some understanding of the ideas of the state space approach and its relationship to the classical approach is nevertheless highly desirable; hence a chapter is included to introduce the reader to the more advanced theoretical procedures which have been developed over the last decade or so and which are particularly useful for the mathematical analysis of multivariable systems. The material is presented in such a way as to make the transition from the classical to the modern approach as smooth as possible. With regard to the order of presentation of material we have found definite advantage in analysing in some detail the dynamic behaviour of components of systems in both the time and the frequency domain prior to any detailed consideration of a closed loop system. This gives the student a clear awareness of the nature of the dynamic response of a system component and how the response varies with the form of the transfer function and the input excitation. It shows him how the response to any given input can be calculated from a knowledge of the transfer function and conversely how a transfer function can be determined by practical testing of a system component. The latter, the process of system identification, is used for verifying mathematical models derived theoretically and may be the only means, or the easiest means, of obtaining a transfer function representation where theoretical derivation is difficult. When this foundation has been laid the principle of feedback can be introduced and rapid progress made in analysing the dynamic behaviour of closed loop systems and, in particular, how accuracy and stability are affected by components within the loop. This then leads logically and easily to the most important stage, consideration of the design of feedback control systems to meet specific dynamic performance specifications.

The material presented in this book should cover the control engineering content of most undergraduate degree schemes which include the subject of automatic control. The book should be of equal value to the engineer in industry who did not include control in his studies but who is now faced with having to deal with some aspect of control or to communicate with others working in the field. It is suggested that at a first reading (or where a minimum of time is available) Chapters 5 and 7 can be omitted without detriment to the understanding of later material. Where time is not available for these chapters it is nevertheless recommended that the reader tries to gain a general idea of the contents.

We wish to express our thanks to those who have contributed most to the development of this book—the many students of differing backgrounds who by their attempts at learning control theory have highlighted points of particular difficulty in understanding. We are especially indebted to our colleague Mr. J. L. Douglas who has endeavoured to learn the fundamentals of the subject by using this book for self-teaching. In doing so he has made valuable suggestions which have enabled us to remove some of our errors and ambiguities, and to make minor additions where our steps have been rather large. We thank also Mrs M. Fernando for her valuable contribution of a neatly typed text.

Leeds
1978

J. Schwarzenbach
K. F. Gill

Preface to Second Edition

It is encouraging to find that this book appears to a large extent to have attained its primary objectives and that it is therefore helping to meet the need for textbooks which explain simply and clearly the basic fundamentals of control engineering. In the period since it was first published digital computers and microprocessors have come to play a very prominent role both as control system components and as tools for analysis, and the main purpose of this second edition is to introduce supplementary material to reflect this change. Sections have been incorporated describing digital simulation and simulation languages, Section 7.5 has been rewritten to include digital computation of correlation functions and power spectra, and a new chapter has been included to deal with the analysis of discrete data systems. The important topic of design has also been given greater emphasis by expanding the final chapter. Solutions to the problems have been included, and the bibliography has been updated.

Leeds
1984

J. Schwarzenbach
K. F. Gill

List of Principal Symbols

Symbols which appear in one part of the book only, and whose meaning is clear from the accompanying text or figures, have been omitted from this list.

Variables which are functions of time are normally represented by lower case letters, and the Laplace transforms of the variables are normally represented by the corresponding capital letters e.g. $F(s)$ is the Laplace transform of $f(t)$, i.e. $F(s) = \mathscr{L}[f(t)]$.

Subscripts are used where more than one of a given variable is used, e.g. $f_1(t), f_2(t), \ldots, F_1(s), F_2(s), \ldots$

Variables are assumed to be relative to appropriate datum or design values (system components are normally assumed to be linear).

Starred symbols are used to indicate time functions that are in sampled form e.g. $f^*(t)$, the sampled version of $f(t)$, is the series of values $f(0), f(T), f(2T), f(3T) \ldots$; it has Laplace transform $F^*(s)$ and z transform $F(z) = \mathscr{Z}[f(t)]$.

$u(t)$ $U(s)$	Input signal to system or system component
$r(t)$ $R(s)$	Reference input to (or set point of) feedback system
$c(t)$ $C(s)$, $y(t)$ $Y(s)$	Output signal from system or system component
$e(t)$ $E(s)$	Error signal
$n(t)$ $N(s)$	Noise signal
$m(t)$ $M(s)$	Manipulated variable, output from controller
$s = \sigma + j\omega$	Laplace operator, (real part σ, imaginary part ω)
$G(s)$	Transfer function of component of a system
$H(s)$	Transfer function of component in a feedback path
$\delta(t)$	Unit impulse function
$g(t)$, or $w(t)$	Unit impulse response, or weighting function, $\mathscr{L}^{-1}[G(s)]$
$G(j\omega)$	Transfer function with $s = j\omega$; gives harmonic characteristics of system, i.e. $\lvert G(j\omega)\rvert$ = magnitude, $\angle G(j\omega)$ = phase angle φ of output relative to input for input frequency ω
$p_1, p_2, p_3, \ldots$	Poles of a transfer function (factors of denominator) i.e. roots of the characteristic equation
$z_1, z_2, z_3, \ldots$	Zeros of a transfer function (factors of numerator)
K	Gain constant
τ	Time constant of a first order component (in Chapter 7, τ represents time shift of a signal)
ζ and ω_n	Damping factor and undamped natural frequency, respec-

	tively, for a second order system component (or associated with a pair of complex conjugate roots)
ω_s	Sampling frequency
T	Sampling interval, $T = 2\pi/\omega_s$
z	Alternative to Laplace operator used with sampled signals, $z = e^{sT}$
$G(z)$	Pulse transfer function
K_p, K_v, K_a	Positional, velocity, and acceleration error coefficients respectively
M_p and ω_p	Peak magnification of a closed loop system, and the frequency at which it occurs
M	Closed loop magnification
k_1, k_2, k_3; k_c, T_i, T_d	Coefficients of P + I + D controller
$\{\boldsymbol{x}(t)\}$	State vector, comprising the n state variables $x_1(t)$, $x_2(t)$, $x_3(t) \ldots x_n(t)$
$\{\boldsymbol{y}(t)\}$	Response or output vector
$\{\boldsymbol{u}(t)\}$	Control or input vector
$\boldsymbol{A}$	Coefficient matrix
$\boldsymbol{B}$	Driving matrix
$\boldsymbol{C}$	Output matrix
$\boldsymbol{D}$	Transmission matrix
$\boldsymbol{I}$	Unit matrix
$\boldsymbol{\varphi}(t)$	Solution matrix or transition matrix
$\varphi_{xx}(\tau)$	Autocorrelation function of a variable $x(t)$
$\varphi_{xy}(\tau)$, $\varphi_{yx}(\tau)$	Cross correlation function of a pair of variables $x(t)$, $y(t)$
$\Phi_{xx}(\omega)$	Power spectral density of a variable $x(t)$, i.e. the Fourier transform of $\varphi_{xx}(\tau)$
$\Phi_{xy}(\omega)$	Cross spectral density, Fourier transform of $\varphi_{xy}(\tau)$

Contents

1 Introduction

As technological processes increase in complexity, and the required performance specifications become more severe, analytical design procedures assume great importance. It has become essential for engineers to have an understanding of the nature of the dynamic behaviour of systems, and of the methods available for analysing and improving dynamic performance.

These requirements are making the use of mathematical modelling techniques an essential part of design. The nature of the model and the methods employed in obtaining it are dependent on the depth of understanding needed at a particular stage of the design study, and on the use to which the model will be put.

It is hoped that this book will give to both the student and the practising engineer a clear insight into the main facets of system modelling, linear control theory, and control system design, and that it will form a sound foundation for practical application or more advanced study. The level of mathematical knowledge assumed is a familiarity with simple differential equations and with complex numbers.

1.1 What is a system?

It is desirable first to define what is meant by a *system*, a word which is frequently used in conversation. Broadly, a system can be thought of as a collection of interacting components, although sometimes interest might lie just in one single component. These components will often be discrete physical elements of hardware, but can equally well be functional parts of such physical components. The system of interest might be a power station, a steam turbine in the power station, or a control valve on the turbine; it might be an aeroplane, its air conditioning, an engine, or part of an engine; a process plant for the production of a chemical, or a large or small part of the plant; a human being, or some part of the body such as the muscle control mechanism for a limb; or it might be the economic system of a country, or any other from a wide range of fields.

The system would normally be considered conceptually as being that part of the universe in which interest lay. There would be interaction between the system and certain parts of the surroundings known as the environment. The two would be separated by an imaginary boundary. In defining the system and its environment it is necessary to decide where this boundary should be placed; this decision depends both on the physical entities involved and on the purpose of the investigation.

In studying a power station, interest might lie primarily in the relationship between the power station and the community, in which case the system and its environment might be envisaged as in Fig. 1.1. There might, however, be a more specific interest in the speed control system of the turbogenerator, in which case the system could be as in Fig. 1.2.

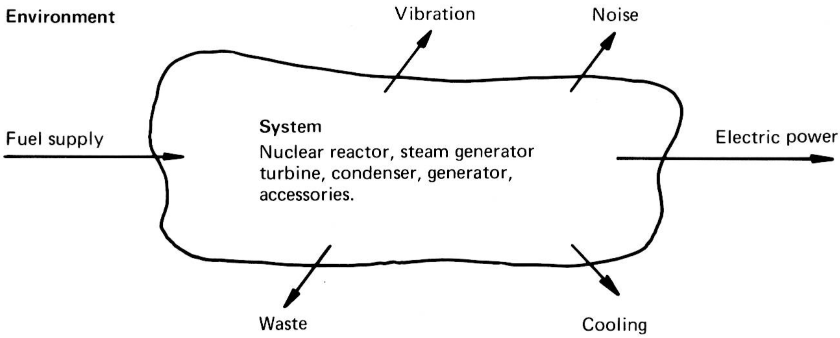

Fig. 1.1 Power station system and its relationship to the environment

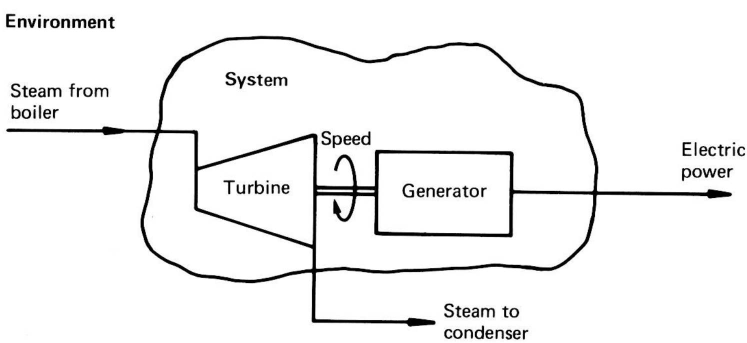

Fig. 1.2 Turbogenerator speed control system

In abstracting from the whole the system of interest, it is necessary to consider carefully where the boundary shall be placed, and closely allied is the need to decide what relevant signals cross the system boundary. In addition, there will be signals of interest within the system boundary, variables which help to describe and define the detailed system behaviour. Some of these signals will be measurable, some not or only indirectly; some will be useful from the viewpoint of analysis, and some not.

The signals which pass to the system from the environment will be termed the *system inputs*, while those passing out across the boundary will be the *system outputs*. Often there will be only one system input that is varying and one system output which is affected. The systems to be considered in this book will be predominantly single-input–single-output systems, the type which occurs most frequently in practice.

1.2 System control

The aim of studying dynamic system behaviour is generally one of gaining an understanding of the system, with a view to controlling it to give specific values of certain important variables, to satisfy a required specification.

For the purpose of controlling the system it is necessary to adjust the values of one or more of the inputs to the system. Only certain of the inputs will be available for adjustment and these are referred to as the controlled inputs, whereas others will be disturbance inputs over which no control exists. In the heating of a room, for example, the heat input from the heating device can be altered as required, but the heat flow to or from the environment cannot be controlled in the same way. The variable chosen to be a measure of the desired system output may or may not give a true indication that the control is satisfactory. In the room heating control, the temperature of interest is probably the average temperature or the temperature in the part of the room where people sit, whereas the temperature measured is that at one specific point, the location of the thermostat. This may not even be positioned in the same room, so that appropriate allowances must be made in the design and utilization of the heating system.

Sometimes the incentive for studying a system will be purely one of seeking an understanding of the way in which it functions. In this category come some physiological systems, for which possibilities for designing control loops or improving system behaviour are rather limited.

Two broad classes of control system are available, *open loop control* and *closed loop control* and these are depicted schematically in Fig. 1.3.

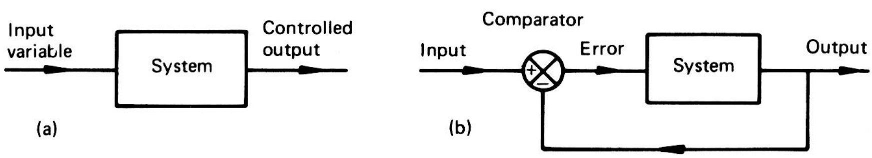

Fig. 1.3 Open loop and closed loop control (a) open loop (b) closed loop.

(a) *Open loop or scheduling control.* On the basis of knowledge about the system and of past experience, a prediction is made of what the input should be to give the desired output; the input is adjusted accordingly. Familiar examples are automatic toasters, programmable washing machines, and interest rate variations as they are used to affect economic systems. Such control is frequently unsatisfactory because any unexpected disturbances to the system can cause a deviation in the output from the desired value. The quality of the toast will vary with the type of bread and the initial temperature of the toaster, the cleanliness of the clothes will depend on correct assessment of amount of soap powder and length of washing cycle required, the effectiveness of an interest rate change will depend on a host of other factors affecting the economy.

(b) *Closed loop or feedback control.* The system output is measured and compared with the desired value; the system continually attempts to reduce the error between the two. Familiar examples are thermostatic controls on domestic and industrial ovens and other heating systems, level controls on water cisterns, and speed regulation by means of engine governors. Frequently the loop is closed through a human being; this is the case with road vehicles, as when a car is driven along an undulating road at a steady speed, or when the car is positioned in its garage at the end of the run.

1.3 The need for analysis

There are many examples of early control systems such as a device of Hero of Alexandria which opened a set of temple doors when a ceremonial fire was lit, and closed them again when the fire died down, or the much later centrifugal governor developed by James Watt for the speed control of steam engines. These systems were produced almost entirely by a trial and observation design process and without the assistance of any theoretical analysis. Simple control loops can still often be made to operate satisfactorily in this way because the specifications have a wide tolerance. As performance requirements become more demanding it becomes necessary to resort to a more analytical approach, since without this the cost in terms of time, manpower, and unnecessary complexity of equipment is not justifiable.

Closing the loop can make the system more accurate by giving a much smaller or a zero steady state error, but it can make the system very oscillatory or even unstable. Basically, problems arise when delays occur within the system; this causes corrective action to be applied too late, leading to alternating overcorrection and undercorrection. It is necessary to achieve a satisfactory compromise between the conflicting requirements of accuracy and stability.

1.4 Methods of system representation

It has been shown that the first step in the study of a system is the important one of defining clearly what constitutes the system of interest, and in what ways the system interacts with the surrounding environment. Having drawn a conceptual boundary round the system it is necessary to represent the system in a convenient pictorial and mathematical way.

A useful and very frequently used pictorial representation of a system is the *block diagram* where individual blocks are used to represent separate functional parts of the system. Fig. 1.3a is a simple block diagram representing a system with a single input and a single output, the lines indicating the signal flow paths, with direction of signal transmission given by the arrows. Where signals are added to or subtracted from one another, summing points are indicated, as shown in Fig. 1.3b. Although any single-input–single-output system could be represented by a single block as in Fig. 1.3a, if the system comprises a number of interacting components it is more useful if it is represented by several blocks interconnected by the appropriate signal flow paths.

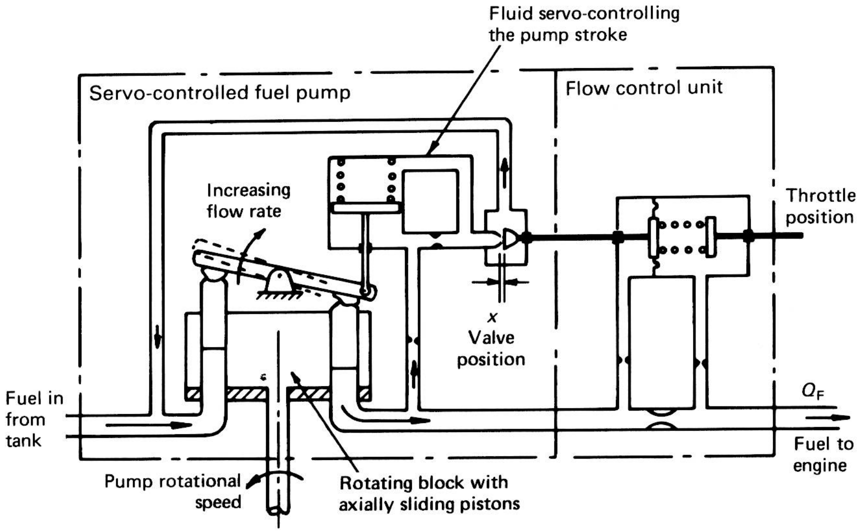

Fig. 1.4 Schematic diagram of pump and flow control on a gas turbine speed control system (with symbols for pressures, flows, springrates, diaphragm areas, flow restrictions, etc. omitted)

In arriving at a block diagram representation, an intermediate schematic diagram in which the functional parts are clearly shown would often be utilized. Consider as an illustration that the system of interest is an aircraft gas turbine speed control system. The engineering drawings, although showing the physical arrangement, are too congested with detail and would not show the type of information required for a dynamic study. A schematic diagram of the form shown in Fig. 1.4 would however show how the system components function and form the basis for an analytical study, and enable the production of a block diagram of the form shown in Fig. 1.5.

Block diagrams show only the interrelationships between the different parts of the system, and for analysis must be supplemented by a quantitative

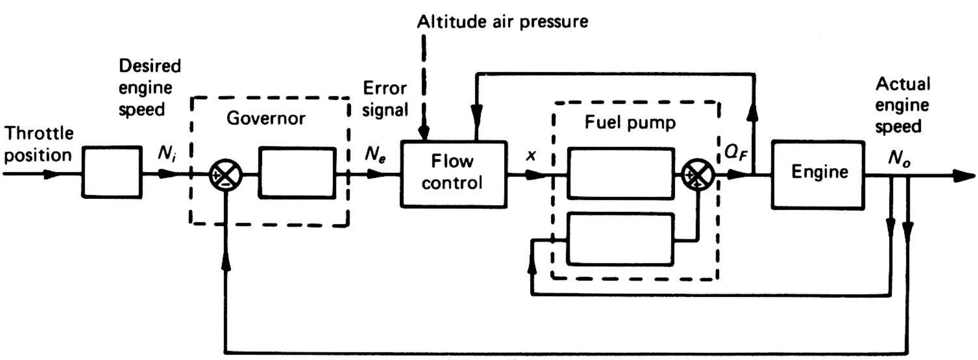

Fig. 1.5 Block diagram of engine and speed control system