

# Calculus Calculus Calculus Calculus

## One and Several Variables

second edition

**Saturnino L. Salas**  
**Einar Hille**

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# one and several variables

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## **Calculus**

# Calculus Calculus

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**Saturnino L. Salas • Einar Hille**

**XEROX**

# Preface

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*Calculus: One and Several Variables* is intended for a three-semester (or five-quarter) course in calculus and analytic geometry:

- Chapters 1–9 for the first two semesters (three quarters);
- Chapters 12–15 for a third semester (two additional quarters);
- Chapters 10 and 11 for either sequence as time permits.

The text is available both as a complete volume and in two parts. In the two-part version, Part 1 deals with functions of one variable, analytic geometry, and infinite series (Chapters 1–11 of the complete volume). Part 2 deals with infinite series, vectors, and functions of several variables (Chapters 10–15 of the complete volume).

The wide acceptance of the first edition has been gratifying and has given us a dividend: a generous flow of reaction from the classroom, a flood of constructive suggestions. As a result, we have added some new material and reworked much of the old. As in the first edition, we have continued to avoid the exotic and tried to stay within the mainstream of calculus—fundamental ideas, basic techniques, standard applications. We have tried to keep the language informal and the presentation uncluttered.

For those of you who are familiar with the first edition, here are some of the changes:

- Chapter 1 A more leisurely discussion of composite functions and one-to-one functions.
- Chapter 2 More explanation on how to find a  $\delta$  that “works.” More illustrations.
- Chapter 3 The differentiability of inverse functions is discussed with more precision; a proof is given in the Appendix. Applications to economics have been added: elasticity, supply and demand curves, isocost lines. More applied max-min problems. More emphasis on



differentials and on the  $df$  notation. A little more on implicit differentiation. The rudiments of partial differentiation.

- Chapter 4 Adoption of the increment notation  $\Delta x_i = x_i - x_{i-1}$ . Emphasis on the  $u$ -substitution.
- Chapter 5 The center stage is now occupied by the notion of exponential growth and decline. The logarithm is motivated by the functional equation  $f(xy) = f(x) + f(y)$ .
- Chapter 6 A discussion of simple harmonic motion has been added. The hyperbolic functions remain, but are now optional.
- Chapters 7, 8 A reordering of topics. First, a chapter on the computation of integrals (with less reliance on formulas and more emphasis on general techniques such as the  $u$ -substitution); then the chapter on analytic geometry.
- Chapter 9 Addition of a variant of Bliss's theorem to help with applications of the integral. New sections on fluid pressure and revenue streams.
- Chapter 10 Expansion of the discussion of power series, giving emphasis to finding the interval of convergence. More attention to numerical estimation.
- Chapter 11 Exploration of some of the common misapplications of L'Hospital's rule.
- Chapter 12 Expansion of the discussion of lines and planes in space. More on vector geometry.
- Chapter 13 Curvature is now optional.
- Chapter 14 Addition of a brief catalog of the quadric surfaces—standard equations, pictures, basic properties.
- Chapter 15 More illustrations; more detailed explanations on how to evaluate multiple integrals. More exercises.
- Appendix Addition of trig tables, log tables, and exponential tables.

*Solutions Manual.* Expanded to include detailed solutions to all nonroutine exercises.

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We owe much to Arthur B. Evans, who directed the project, to Marret McCorkle, who edited the manuscript from beginning to end, and to Dagmar Noll, who assisted throughout.

Special thanks also to Xerox College Publishing for permission to use Tables 1, 4, and 5 in the Appendix and to include exercises from Granville, Smith, and Longley's *Elements of Calculus*.

S.L.S.  
E.H.

# The Greek Alphabet

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A	$\alpha$	alpha
B	$\beta$	beta
$\Gamma$	$\gamma$	gamma
$\Delta$	$\delta$	delta
E	$\epsilon$	epsilon
Z	$\zeta$	zeta
H	$\eta$	eta
$\Theta$	$\theta$	theta
I	$\iota$	iota
K	$\kappa$	kappa
$\Lambda$	$\lambda$	lambda
M	$\mu$	mu
N	$\nu$	nu
$\Xi$	$\xi$	xi
O	$\omicron$	omicron
$\Pi$	$\pi$	pi
P	$\rho$	rho
$\Sigma$	$\sigma$	sigma
T	$\tau$	tau
$\Upsilon$	$\upsilon$	upsilon
$\Phi$	$\phi$	phi
X	$\chi$	chi
$\Psi$	$\psi$	psi
$\Omega$	$\omega$	omega



# Index

- Abscissa, 7
- Absolute convergence of series, 502
- Absolute maximum value, 136
- Absolute minimum value, 136
- Absolute value, 5
- Acceleration, 155, 627
- Additivity
  - of arc length, 642–643
  - of integration, 185, 779
- Alternating series, 503
- Amplitude, 298
- Angle(s)
  - direction, 580
  - between lines, 8, 587
  - between planes, 594
  - between vectors, 576
- Angular momentum, 632
- Antiderivative, 191
- Apollonius of Perga, 355
- Approximate integration, 343–349
  - left and right endpoint estimates, 346
  - midpoint estimate, 346
  - parabolic estimate (Simpson's rule), 346
  - trapezoidal estimate, 346
- Arc cosecant, 293
- Arc cosine, 292
- Arc cotangent, 292
- Archimedes, 420
  - spiral of, 445, 650
- Arc length, 441
  - formulas, 442, 443, 444
  - parametrization, 643
  - vector form, 637
- Arc secant, 292
- Arc sine, 286
- Arc tangent, 288
  - series for, 517
- Area, 179, 412, 413
  - by double integration, 771, 779
  - in polar coordinates, 434, 437
  - of a surface of revolution, 447, 450
- Asymptote(s)
  - horizontal, 537
  - of a hyperbola, 374
  - vertical, 547
- Attractive force, 630
- Average of a function, 406
  - three variables, 810
  - two variables, 809
- Axes, rotation of, 399
- Bernoulli brothers, 4
- Binomial coefficient, 533
- Binomial series, 533
- Bliss, G. A., 428, A13
- Bliss's theorem, a variant of, 428, A13
- Bolzano, Bernard, A5
- Boundary, 676

- Boundary curves, 278
- Boundedness, 6
  - for functions, 653, A6
  - for sequences, 465
- Box, 798
- Branches of a hyperbola, 374
- Cardioid, 384, 387
- Cartesian space coordinates, 559
- Cauchy, Augustin-Louis (Baron de), 484
  - mean-value theorem, 543
  - sequence, 484
- Center
  - of mass, 818
  - of a quadric surface, 656
- Central force, 630
- Central line of a quadric surface, 656
- Centroid
  - of a plane region, 812
  - in polar coordinates, 819
  - of a solid, 816
- Chain rule(s), 99, 103
  - in Leibniz notation, 101
  - several variables, 703, 705
  - for vector functions, 616
- Circular helix, 627, 628
- Closed curve, 761
- Closed interval, 6
- Closed set, 677
- Collinear vectors, 588
- Comparison theorem
  - for improper integrals, 557
  - for series, 497
- Component(s)
  - of one vector in the direction of another, 578
  - scalar, 573
  - vector, 573
- Composition of functions, 20
- Compound interest, 253
- Concavity, 146
- Cone
  - circular, 658
  - quadric, 657
- Conic section, 355
  - classification by discriminant, 402
- Containment (for sets), A1
- Continuity (one variable), 65
  - of composites, 67
  - $\epsilon, \delta$  definition, 67
  - implied by differentiability, 80
  - on an interval  $[a, b]$ , 71
  - one-sided, 69
  - uniform, A9
  - for vector functions, 610
- Continuity (several variables), 679
  - of composites, 680
  - in a particular variable, 681
- Continuous compounding, 253
- Continuous differentiability, 703
- Convergence
  - absolute (for series), 502
  - for improper integrals, 552, 554, 555
  - for sequences, 472
  - for series, 487
- Coordinates
  - cylindrical, 819
  - polar, 378
  - rectangular, 7, 559
  - spherical, 824
- Coplanar vectors, 592
- Cosecant
  - hyperbolic, 308
  - trigonometric, 9
- Cosine(s)
  - hyperbolic, 302
  - law of, 10
  - trigonometric, 9
- Cotangent
  - hyperbolic, 308
  - trigonometric, 9
- Critical point, 130, 135
- Cross product of two vectors, 598
  - properties of, 599
- Curvature, 645
  - formulas for, 645, 646, 647, 648
  - vector, 644
- Curve
  - closed, 761
  - differentiable, 703
  - given parametrically, 390, 622
  - level, 664
  - piecewise smooth, 754
  - smooth, 752
- Cylinder
  - circular, 660
  - elliptic, 659
  - hyperbolic, 660
  - parabolic, 659

- Cylindrical coordinates, 819
- Cylindrical wedge, 819
- Damped oscillation, 278
- Decimal fraction, 490
- Decreasing function, 122
- Decreasing sequence, 467
- Definite integral, 179
- Degree (angle measurement), 8, 9
- Density
  - of irrationals, 5
  - of rationals, 5
- Derivative(s), 76
  - of a composite function, 99
  - directional, 693
  - of higher order, 105
  - of an inverse, 108
  - partial, 172, 669, 671
  - of a product, 83
  - of a quotient, 87
  - as a rate of change, 95
  - of a reciprocal, 86
  - of a scalar multiple, 83
  - of a sum, 83
  - of a vector function, 611
- Determinant, 598, 603
- Difference quotient, 77
  - for a vector function, 611
- Differentiability, 76
  - for functions of several variables, 687
  - for vector functions, 611
- Differentiable curve, 703
- Differential, 160
  - several variables, 740
- Differential equations, 261–266, 296–302
- Differentiation
  - implicit, 164
  - logarithmic, 227
  - of power series, 526
- Differentiation formulas, 82–87, 99
  - in the Leibniz notation, 90–91
  - for vector functions, 616
- Directional derivative, 693
  - as a dot product, 694
- Direction angles, 580
- Direction numbers for a line, 584
- Direction (same and opposite, for vectors), 569
- Direction vectors for a line, 584
- Directrix
  - of an ellipse, 372
  - of a hyperbola, 377
  - of a parabola, 356
- Dirichlet function, 46
- Discriminant
  - of a second-degree equation, 402
  - for second-partials test, 726
- Displacement, 562
- Distance
  - as the integral of speed, 180, 641
  - from a point to a line, 354
  - from a point to a plane, 595
  - between two points, 8, 561
- Divergence
  - for improper integrals, 552, 554, 555
  - for sequences, 472
  - for series, 487
- Domain of a function, 15
  - several variables, 651
- Dot product, 575
  - properties of, 578
- Double integral, 770, 778
  - change of variables, 794
  - evaluation by repeated integrals, 780
  - in polar coordinates, 790, 792
- Double sums, 772
- Dummy variable, 179, 486
- $e$ , 218
  - estimates for, 246–248, 510–511
  - irrationality of, 512
  - as  $\lim_{n \rightarrow \infty} (1 + 1/n)^n$ , 478
- Eccentricity (of an ellipse), 371
- Elasticity, 113
- Element of a set, A1
- Elimination of the  $xy$ -term, 399
- Ellipse, 365
  - equations of, 366, 367, 390
  - reflecting property of, 369
- Ellipsoid, 656
- Empty set, A2
- Endpoint maximum, 136
- Endpoint minimum, 136
- Energy, kinetic, 632, 758
- Equality for sets, A2
- Eudoxos of Cnidos, 421
- Euler, Leonard, 218
- Even function, 10

- Excluded region, 373
- Exponential function, 230
- Exponential growth, 250
- Exponential series, 508
- Exponential tables, A16, A17
- Extreme point, 721
- Extreme values (one variable), 128–146
  - absolute, 136
  - endpoint, 136
  - local (relative), 129
  - summary of technique for finding them, 137
  - tests, 132, 134
- Extreme values (several variables), 719–739
  - local, 719
  - with side conditions, 731–739
  - tests, 719, 726, 733, 734
- Factorials, 7
- First-derivative test, 132
- First mean-value theorem for integrals, 408
  - extension of, 514
- Fluid pressure, 457
- Focus (foci)
  - of ellipse, 365
  - of hyperbola, 372
  - of parabola, 356
- Foot-pound, 452
- Force of a fluid on a wall, 457–458
- Frequency, 297
- Function(s), 15
  - review, 15–30
  - several variables, 651
  - vector, 609
- Fundamental theorem
  - of integral calculus, 192
  - for line integrals, 760
- Galilei, Galileo, 362
- General quadratic formula, 7
- Geometric series, 488
- Gradient, 687
  - formulas for, 688, 692
  - as the most rapid increase vector, 697
  - as a normal, 711, 712
  - uniqueness of, 691
- Gradient field, 759
- Graph
  - sketching of, 148–152
  - two variables, 661
- Greatest integer function, 61
- Greatest lower bound theorem, 439
- Gregory, James, 517
- Harmonic motion, 297
- Harmonic series, 495
- Helix, circular, 627, 628
- Hooke, Robert, 453
- Hooke's law, 453
- Horizontal asymptote, 537
- L'Hospital's rule
  - $(0/0)$ , 541
  - $(\infty/\infty)$ , 549
- Hyperbola, 372
  - equations of, 373, 390
  - in range finding, 376
- Hyperbolic functions, 302, 308
  - relation to the hyperbola  $x^2 - y^2 = 1$ , 305
- Hyperbolic inverses, 310
- Hyperboloid
  - of one sheet, 656
  - of two sheets, 657
- Implicit differentiation, 164
- Improper integrals, 552–558
- Increasing function, 122
- Increasing sequence, 467
- Increment, 160
  - several variables, 740
- Indefinite integral, 201
- Induction, axiom of, 11, A4
- Infinite limits, 546, 547, 548
- Infinite series, 487
- Inflection point, 146
- Inner product, 575
- Integer, 5
- Integrability theorem, A12
- Integral(s)
  - definite, 179
  - double, 770, 778
  - indefinite, 201
  - line, 753
  - properties of (one variable), 200, 206–210
  - table of, A21–A22
  - triple, 800
  - of a vector function, 612

- Integral sign, 179
- Integral test, 494
- Integration (one variable), 178
  - approximate, 343–349
  - as an averaging process, 405–410
  - by parts, 258
  - of powers and products of the trigonometric functions, 326–337
  - of power series, 528
  - of rational expressions in  $\sin x$  and  $\cos x$ , 337–341
  - of rational functions, 319–326
  - by substitution, 203–206, 341–343
- Integration (several variables)
  - double, 770, 778, 780
  - triple, 800, 802, 803
- Intercept form
  - of equation of a line, 8
  - of equation of a plane, 597
- Intercept of a surface, 655
- Interior, 675
- Intermediate-value theorem, 71, A5
- Intersection of sets, A2
- Interval, 6
  - of convergence, 521
- Inverse function, 25
  - continuity of, A7
  - differentiability of, A8
- Inverse hyperbolic functions, 310–311
- Inverse trigonometric functions, 286–295
- Irrational number, 5
- Irrational powers, 241
- Irrational powers of  $e$ , 230
  
- Kepler's second law, 633
- Kinetic energy, 632, 758
  
- Lagrange, Joseph Louis, 76, 507
  - form of the remainder, 507
  - multiplier, 734
- Law of sines, 10, 607
- Least upper bound axiom, 438
- Left-hand limit, 60
- Leibniz, Gottfried Wilhem, 4, 90, 518
  - notation for derivative, 90–91
- Lemniscate, 381
- Length
  - of a circular arc, 9
  - of a curve, 441, 637
  - of a vector, 570
- Level curve, 664
- Level surface, 666
- Limit
  - of a function of one variable, 38
  - of a function of several variables, 679
  - infinite, 546–549
  - one-sided, 60
  - of a sequence, 471
  - of a vector function, 610
  - as  $x \rightarrow \pm\infty$ , 537–540
- Line integral(s), 753
  - fundamental theorem for, 760
- Linearity of integration, 200, 779
- Lines
  - elementary notions, 8
  - vector form, 584
- Local extreme value, 129, 719
- Logarithm(s)
  - base  $e$ , 215
  - base  $p$ , 245
  - general notion, 213
  - graph of, 219
  - series for, 516
  - table of, A15
- Logarithmic differentiation, 227
- Lower bound, 6, 439
- Lower sum, 178, 768, 800
  
- Machin, John, 518
- Magnitude of a vector, 570
- Major axis of an ellipse, 366
- Mass
  - center of, 818
  - in cylindrical coordinates, 821
  - in spherical coordinates, 827
  - as a triple integral, 801
- Maximum-minimum values (one variable), 128–146
  - absolute, 136
  - endpoint, 136
  - local (relative), 129
  - summary of technique for finding them, 137
  - tests, 132, 134
  - theorem, 72, A7
- Maximum-minimum values (several variables), 719–739

## Index

- local, 719
- with side conditions, 731–739
- tests, 719, 726, 733, 734
- Mean-value theorem
  - Cauchy, 543
  - for integrals, 408
  - one variable, 117
  - several variables, 701
- Member of a set, A1
- Minimax, 659
- Minor axis of an ellipse, 366
- Mixed partials, 683, 685
  - equality of, 684
- Momentum, 629
  - angular, 632
- Monotonic sequence, 467
- Multiplier, Lagrange, 734
- Natural logarithm, 215
- Neighborhood, 675
- Newton, Sir Isaac, 4
- Newton's second law of motion, 629, 816
- Nominal interest rate, 253
- Nondecreasing sequence, 467
- Nonincreasing sequence, 467
- Norm, 569
  - properties of, 570
- Normal line, 711
- Normal vector, 590
- Numerical estimates
  - by differentials, 159–164, 740–745
  - for  $e$ , 248, 511
  - for integrals, 343–349, 530
  - for  $\pi$ , 518
  - by Taylor polynomials and series, 509–511, 530, 534, 535, 536
- Odd function, 10
- $o(h)$ , 159
- $o(h)$ , 686
- One-to-one function, 17
- Open interval, 6
- Open set, 676
- Order properties of real numbers, 5, 438
- Ordinate, 7
- Pappus of Alexandria, 814
  - theorem of, 814
- Parabola, 356
  - equations of, 357, 390
- Parabolic reflector, 360
- Parabolic trajectory, 362
- Paraboloid
  - elliptic, 658
  - hyperbolic, 658
- Parallel lines, 587
- Parallel planes, 594
- Parallel vectors, 569
- Parametrizations for curves
  - arc length, 643
  - scalar, 390
  - vector, 622
- Partial derivative, 172, 669, 671
- Partial fraction, 319
- Partial sum of a series, 487
- Partition
  - of a box, 799
  - of an interval, 178
  - of a rectangle, 766
- Period (in harmonic motion), 297
- Perpendicular lines, 8
- Perpendicular vectors, 576
- Pi ( $\pi$ ), computation of, 518
- Piecewise smooth curve, 754
- Pinching theorem
  - for functions, 56
  - for sequences, 477
- Plane, equations for, 591, 592
- Point
  - boundary, 676
  - critical, 130, 135, 137
  - extreme, 721
  - inflection, 146
  - interior, 675
  - saddle, 659, 721
  - stationary, 721
- Polar coordinates, 378
  - arc length, 444
  - area, 434, 437
  - centroid, 819
  - double integral, 792
- Polar rectangle, 790
- Polynomial, 51
  - Taylor, 506
- Position
  - as integral of velocity, 197, 641
  - vector, 583
- Power, 632

- irrational, 241
- Power series, 519
- Present value, 461
- Pressure, 457
- Product
  - cross (vector), 598
  - dot (inner, scalar), 575
- $p$ -series, 496
- Quadratic equation, 7
- Quadric surface, 655
- Radian, 8, 9
- Radius of convergence, 521
- Range finding, 376
- Range of a function, 15
  - several variables, 653
- Rational function, 66
- Rational number, 5
- Ratio test, 499
- Reflecting property
  - of ellipse, 369
  - of parabola, 360
- Repeated integrals, 780, 802, 803
- Revenue stream, 460
- Riemann, G. F. B., A8
- Right-hand limit, 60
- Rolle's theorem, 119
- Root test, 497
- Rotation of axes, 399
- Saddle point, 659, 721
- Scalar, 18, 566
- Scalar equations for a line in space, 585
- Scalar multiple of a function, 18
- Scalar product, 575
- Schwarz's inequality, 578
- Secant
  - hyperbolic, 308
  - trigonometric, 9
- Second-derivative test, 134
- Second-order partials, 683
- Second-partials test, 726
- Section of a surface, 656
- Sequence(s) of real numbers, 465
  - convergent, divergent, 472
  - limit of, 471
  - limit theorems, 472–481
- Series (infinite), 487
  - absolutely convergent, 502
  - alternating, 503
  - arc tangent, 517
  - basic comparison theorem, 497
  - binomial, 533
  - convergent, 487
  - cosine, sine, 509
  - divergent, 487
  - exponential, 508
  - geometric, 488
  - harmonic, 495
  - integral test, 494
  - logarithm, 516
  - $p$ -, 496
  - partial sum of, 487
  - power, 519
  - ratio test, 499
  - root test, 497
  - sum of, 487
  - Taylor, 508, 513
  - term-by-term differentiation, 526
  - term-by-term integration, 528
- Set
  - closed, 677
  - empty, A2
  - open, 676
- Shell method, 429
- Sigma ( $\Sigma$ ) notation for sums, 486, 772
- Simple harmonic motion, 297
- Simpson's rule, 346
- Sine(s)
  - hyperbolic, 302
  - law of, 10, 607
  - trigonometric, 9
- Slope
  - of a graph, 77
  - of a line, 8
- Smooth curve, 752
- Smoothness, piecewise, 754
- Speed, 155, 628, 641
- Spherical coordinates, 824
- Spherical wedge, 824
- Stationary point, 721
- Subset, A1
- Substitution, integration by, 203–206, 341–343
- Sum
  - double, 772
  - lower and upper, 178, 767, 768, 800



## Index

- partial, 487
- of a series, 487
- Surface
  - area, 447
  - level, 666
- Symmetric equation for a line, 586
- Table(s)
  - exponential, A16–A17
  - of integrals, A21–A22
  - logarithm, A15
  - trigonometric, A18–A20
- Tangent
  - hyperbolic, 308
  - trigonometric, 9
- Tangent line, 77, 393, 711
  - vector form, 623
  - vertical, 394
- Tangent plane, 714
- Tangent vector, 622
- Taylor, Brook, 506
  - polynomials, 506, 513
  - series, 508, 513
  - theorem, 506
- Term
  - of a sequence, 466
  - of a series, 487
- Torque, 632
- Total differential, 740
- Traces of a surface, 655
- Translation of a coordinate system, 353
- Transverse axis of a hyperbola, 373
- Trapezoidal rule, 346
- Triangle inequalities, 6, 570
- Trigonometric functions
  - differentiation of, 268–281
  - integration of, 281–286
- Trigonometric tables, A18–A20
- Triple integral, 800
  - evaluation by repeated integrals, 802, 803
- Triple scalar product, 602
- Twisted cubic, 624
- Uniform continuity, A9
- Union of sets, A3
- Unit coordinate vectors, 572
- Unit normals, 593
- Upper bound, 6, 438
- Upper sum, 178, 767, 800
- $u$ -substitution, 203
- Vacuous set, A2
- Vector, 566
- Vector function, 609
- Vector product, 598
- Velocity, 155, 627
- Vertex of a parabola, 356
- Vertices of a hyperbola, 373
- Void set, A2
- Volume
  - in cylindrical coordinates, 821
  - by double integration, 771, 778
  - by parallel cross sections, 417
  - by shell method, 429
  - of solids of revolution, 419
  - in spherical coordinates, 824, 827
  - by triple integration, 801
- Wedge
  - cylindrical, 819
  - spherical, 824
- Weierstrass, Karl, A6
- Work
  - emptying out a tank, 454
  - as an integral, 453
  - as a line integral, 753
  - vector notion, 751

# Contents

---

## Introduction **1** 1

1.1	What is Calculus?	1
1.2	Notions and Formulas from Elementary Mathematics	5
1.3	Inequalities; Absolute Value;	11
1.4	Some Comments on Functions	15
1.5	Composition of Functions	19
1.6	Inverse Functions	25

## Limits and Continuity **2** 31

2.1	The Idea of Limit	31
2.2	Definition of Limit	37
2.3	Some Limit Theorems	48
2.4	More on Limits	56
2.5	One-Sided Limits	59
2.6	Continuity	65