



**Proceedings of the
34th Annual ACM Symposium
on Theory of Computing**

**Montreal, Quebec, Canada
May 19-21, 2002**

Sponsored by

SIGACT

(The ACM Special Interest Group for Algorithms and Computation Theory)

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Foreword

The papers in this volume were presented at the *Thirty-Fourth Annual ACM Symposium on Theory of Computing (STOC2002)*, held in Montreal, Quebec, Canada, May 19-21, 2002. The Symposium was sponsored by the ACM Special Interest Group on Algorithms and Computation Theory (SIGACT).

In response to a call for papers, 287 paper submissions were received. All were submitted electronically. The program committee conducted its deliberations electronically, via an on-line meeting that ran from January 10 to January 19. The committee selected 91 papers from among the submissions. The submissions were not refereed, and many of these papers represented reports of continuing research. It is expected that most of them will appear in a more polished and complete form in scientific journals.

The papers encompassed in wide variety of areas of theoretical computer science. The topics included algorithms and computational complexity bounds for classical problems in algebra, geometry, topology, graph theory, game theory, logic and machine learning, as well as theoretical aspects of security, databases, information retrieval, and networks, the web, computational biology, and alternative models of computation including quantum computation and self-assembly.

The program committee would like to thank all authors who submitted papers for consideration. The committee is very grateful to the SIGACT Electronic Publishing Board for use of their software for the electronic meeting. The committee is especially grateful for the many colleagues listed below who helped us review the submissions.

We would also like to thank Lisa Tolles-Efinger of Sheridan Printing for her excellent aid in the proceedings production.

The Special Joint Session on Complexity Theory: This was held on May 21 jointly with the Seventeenth Annual IEEE Conference on Computational Complexity (COMPLEXITY2002).

The Knuth Prize: Christos Papadimitriou was awarded the Knuth Prize and gave a Plenary Talk on "*The Joy of Theory*."

The Machtey Award: The SIGACT Best Student Paper was awarded to Tim Roughgarden for his submission "*The Price of Anarchy is Independent of the Network Topology*."

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Recognizing String Graphs in NP

Marcus Schaefer*

Eric Sedgwick†

Daniel Štefankovič‡

ABSTRACT

A *string graph* is the intersection graph of a set of curves in the plane. Each curve is represented by a vertex, and an edge between two vertices means that the corresponding curves intersect. We show that string graphs can be recognized in **NP**. The recognition problem was not known to be decidable until very recently, when two independent papers established exponential upper bounds on the number of intersections needed to realize a string graph [18, 20]. These results implied that the recognition problem lies in **NEXP**. In the present paper we improve this by showing that the recognition problem for string graphs is in **NP**, and therefore **NP**-complete, since Kratochvíl [12] showed that the recognition problem is **NP**-hard. The result has consequences for the computational complexity of problems in graph drawing, and topological inference.

1. STRINGS, DRAWINGS, AND DIAGRAMS

A *string graph* is the intersection graph of a set of curves in the plane. A (*Jordan*) *curve*, or *string*, is a set homeomorphic to $[0, 1]$. Given a collection of curves $(C_i)_{i \in I}$ in the plane, the corresponding intersection graph is $(I, \{\{i, j\} : C_i \text{ and } C_j \text{ intersect}\})$. The *size* of a collection of curves is the number of intersection points (we assume that no three curves intersect in the same point). A graph isomorphic to the intersection graph of a collection of curves in the plane is called a *string graph*.

The string graph problem asks how string graphs can be recognized. The problem made its first explicit appearance

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in a 1966 paper by Sinden on circuit layout [21], although a similar question had been suggested earlier by Benzer on genetic structures [1]. The string graph problem was introduced to the combinatorial community by Ron Graham in 1976 [9].

From a combinatorial point of view we are interested in $c_s(G)$, the smallest number of intersections of a set of curves realizing a string graph. For graphs G that are not string graphs, we let $c_s(G)$ be infinity. With that we can define $c_s(n) = \max\{c_s(G) : G \text{ is a string graph on } n \text{ vertices}\}$. A computable upper bound on $c_s(n)$ implies decidability of the string graph problem. In 1991 Kratochvíl and Matoušek [14] showed rather surprisingly, that $c_s(n) \geq 2^{cn}$ for some constant c , and conjectured that $c_s(n) \leq 2^{cn^k}$ for some k . The papers by Pach and Tóth [18], and Schaefer and Štefankovič [20] established upper bounds of this form, implying decidability of the string graph problem in nondeterministic exponential time.

The string graph problem is closely related to a graph drawing problem, a connection we will make use of later. Given a graph $G = (V, E)$ and a set $R \subseteq \binom{E}{2} = \{\{e, f\} : e, f \in E\}$ on E , we call a drawing D of G in the plane a *weak realization* of (G, R) if only pairs of edges which are in R are allowed to intersect in D (they do not have to intersect, however). In this case we call (G, R) *weakly realizable*. We say that D is a *realization* of G if exactly the pairs of edges in R intersect in D .¹ Let us define $c_w(G, R)$ as the smallest number of intersections in a weak realization of (G, R) , $c_w(G) = \max\{c_w(G, R) : (G, R) \text{ has a weak realization}\}$, and $c_w(m) = \max\{c_w(G) : G \text{ has } m \text{ edges}\}$.

The string graph problem can be reduced in polynomial time to the weak realizability problem [16, 12]. The reduction is as follows. Given a graph $G = (V, E)$, let $G' = (V \cup E, \{\{u, e\} : u \in e \in E\})$, and $R = \{\{\{u, e\}, \{v, f\}\} : \{u, v\} \in E\}$. Then G is a string graph if and only if (G', R) is weakly realizable.

In Theorem 4.4 we show that the weak realizability problem is in **NP**. Because of the reduction of the string graph problem to the weak realizability problem, and Kratochvíl's proof of **NP**-hardness of the string graph problem [12] this implies the following corollaries.

COROLLARY 1.1. *The string graph problem is complete for NP.*

COROLLARY 1.2. *The weak realizability problem is complete for NP.*

¹Kratochvíl [13, 11, 12] calls (G, R) an *abstract topological graph*, and uses the word *feasible* for weakly realizable.

The corollaries imply that the weak realizability problem can be reduced to the string graph problem in polynomial time. No natural polynomial time reduction witnessing this relationship is known (although there is an NP-reduction).

The weak realizability problem is a generalization of the concept of *crossing number* of a graph G , which is the smallest number of intersections necessary to draw G in the plane. Garey and Johnson showed that computing the crossing number is NP-complete [7]. Many variants of this problem have been considered in the literature, including the *pairwise crossing number* (or *crossing pairs number*), which is the smallest number of pairs of edges that need to intersect to draw G . Pach and Tóth recently showed that computing the pairwise crossing number is NP-hard [17]. Since there is an NP-reduction from this problem to the weak realizability problem, we have the following corollary.

COROLLARY 1.3. *The pairwise crossing number problem is complete for NP.*

The string graph problem is also related to Euler (or Venn) diagrams, and through these to topological inference. Given a specification of the relationships of concepts, such as “some A is B , some B is C , but no A is C ”, we can ask whether there is a diagram illustrating the relationship of the concepts (regions homeomorphic to the unit disk). In this particular case Figure 1 illustrates the given situation.

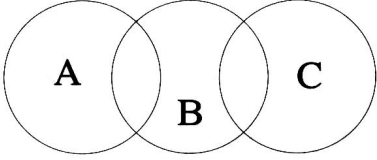


Figure 1: Some A is B , some B is C , but no A is C .

This problem is polynomial-time equivalent to the string graph problem. Topological inference allows a more refined set of predicates to describe relationship between regions, but even in this case a reduction to the string graph problem can be established, giving us the following result.

COROLLARY 1.4. *The existential theory of diagrams and the existential fragment of topological inference are complete for NP.*

Details of this reduction (which is an NP-reduction rather than a polynomial time one) and the definitions involved can be found in the journal version of [20]. Several restricted versions of this problem were shown to be solvable in P and NP earlier, but the general problem was not known to be decidable [10, 2, 22].

For the proof of our main theorem, Theorem 4.4, the same approach as in our earlier paper [20] proves successful: we reinterpret the problem as a problem over words. The necessary background material on words and word equations is covered in Section 2. The topological aspects of the proof are covered in Section 3.

2. WORD EQUATIONS

Let Σ be an alphabet of symbols and Θ be an alphabet of variables. The alphabets Σ and Θ are disjoint. A *word*

equation $u = v$ is a pair of words $(u, v) \in (\Sigma \cup \Theta)^* \times (\Sigma \cup \Theta)^*$. The *size* of the equation $u = v$ is $|u| + |v|$. A *solution* of the word equation $u = v$ is a morphism $h : (\Sigma \cup \Theta)^* \rightarrow \Sigma^*$ such that $h(a) = a$ for all $a \in \Sigma$ and $h(u) = h(v)$ (h being a morphism means that $h(wz) = h(w)h(z)$ for any $w, z \in (\Sigma \cup \Theta)^*$). The *length* of the solution h is $\sum_{x \in \Theta} |h(x)|$.

A *word equation with specified lengths* is a word equation $u = v$ and a function $f : \Theta \rightarrow \mathbb{N}$. The solution h has to respect the lengths, i.e. we require $|h(x)| = f(x)$ for all $x \in \Theta$.

Let w be a word in Σ^* . We can write $w = c_1 f_1 c_2 \dots c_k f_k$ where the c_i are characters in Σ , and the f_i are subwords of w . More precisely, $c_1 = w[1]$ and f_i is the longest prefix of $f_i c_{i+1} \dots f_k$ which occurs in $c_1 f_1 \dots f_{i-1} c_i$. The *Lempel-Ziv (LZ) encoding* of w is $LZ(w) = c_1 [a_1, b_1] c_2 \dots c_k [a_k, b_k]$ where $f_i = w[a_i \dots b_i]$. The *size* of the encoding is $|LZ(w)| = k(\log |w| + \log |\Sigma| + 1)$. Note that some words can be compressed exponentially.

Let $h : (\Sigma \cup \Theta)^* \rightarrow \Sigma^*$ be a solution of an equation $u = v$. The LZ encoding of h is the sequence of LZ encodings of $h(x)$ for all $x \in \Theta$. The size of the encoding is $|LZ(h)| = \sum_{x \in \Theta} |LZ(h(x))|$. The usefulness of LZ encoding for word equations is demonstrated by following two results.

THEOREM 2.1 ([8]). *Let $u = v$ be a word equation. For an LZ encoding of a morphism h we can check whether h is a solution of the equation in time polynomial in $|LZ(h)|$.*

THEOREM 2.2 ([19]). *Let $u = v$ be a word equation with lengths specified by a function f . Assume that $u = v$ has a solution respecting the lengths given by f . Then there is a solution h respecting the lengths such that $|LZ(h)|$ is polynomial in the size of the binary encoding of f and the size of the equation. Moreover, the lexicographically least such solution can be found in polynomial time.*

Given an equation with specified lengths there might be solutions which can not be LZ compressed. However Theorem 2.2 says that there is a solution which can be LZ compressed. In particular if the equation has a unique solution then that solution can be LZ compressed. Note that it is easy to encode several equations into one equation [15, Proposition 12.1.8], hence Theorems 2.1 and 2.2 hold for systems of equations as well.

We will need the following two results which easily follow from [8].

LEMMA 2.3. *For an LZ-encoding $LZ(w)$ of w we can test whether w is a palindrome in time polynomial in $|LZ(w)|$.*

LEMMA 2.4. *Given an LZ encoding $LZ(w)$ of w and $a \in \Sigma$, we can compute the number of occurrences of a in w in time polynomial in $|LZ(w)|$.*

3. COMPUTATIONAL TOPOLOGY

In the following let M be a compact orientable surface with boundary. A simple arc γ such that both its endpoints $\gamma(0), \gamma(1)$ are on the boundary ∂M and the internal points $\gamma(x), 0 < x < 1$ are in the interior \dot{M} is called a *properly embedded arc*. Two properly embedded arcs γ_1, γ_2 are *isotopic rel. boundary* ($\gamma_1 \sim \gamma_2$) if there is a continuous deformation of γ_1 to γ_2 which does not move the endpoints. The *isotopy class* of γ is the set of properly embedded arcs isotopic to