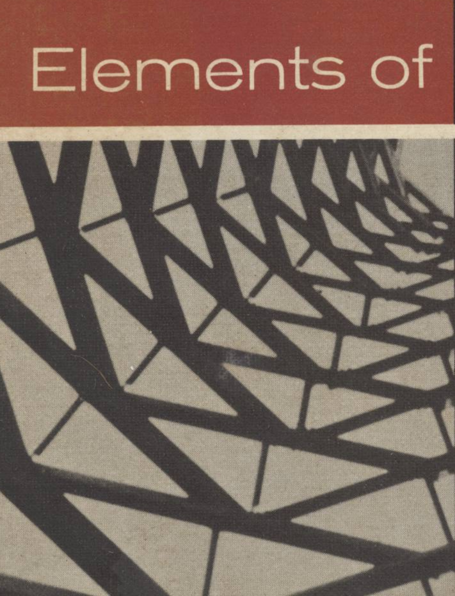
Elements of Trigonometry



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ELEMENTS OF TRIGONOMETRY

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LOWER CASE GREEK ALPHABET

α	ALPHA
β	BETA
γ	GAMMA
δ	DELTA
ϵ	EPSILON
ζ	ZETA
η	ETA
θ	THETA
ι	IOTA
κ	KAPPA
λ	LAMBDA
μ	MU
ν	NU
ξ	XI
o	OMICRON
π	PI
ρ	RHO
σ	SIGMA
τ	TAU
υ	UPSILON
ϕ	PHI
χ	CHI
ψ	PSI
ω	OMEGA

ELEMENTS OF TRIGONOMETRY

Preface

ELEMENTS OF TRIGONOMETRY is suitable for any beginning course designed either for terminal students or for those who will continue to study mathematics.

We begin with a brief review of basic topics, including set theory, relations, and functions. Because these concepts are used throughout the remainder of the text, students who are unfamiliar with them should study these topics carefully before proceeding.

In Chapters 2 through 5 we present the trigonometric functions, identities, graphs of trigonometric functions, and inverse trigonometric relations and functions. Graphs of the trigonometric functions, their corresponding inverse relations and functions, along with the properties of each, are covered thoroughly. In these chapters we also introduce the student to such concepts as circular functions, simple harmonic motion, and polar coordinates.

In Chapters 6, 8, 9, and 10 we present applications of the trigonometric functions to triangles, equations, vectors, and complex numbers. While we treat solutions of triangles, including right triangles, we do not emphasize this subject. The many methods for solving trigonometric equations are explained by means of illustrations. Next, we show the application of the polar coordinate system to systems of trigonometric equations, along with an introduction to vectors and their applications. Utilizing this work with vectors, we study complex variables. To simplify computations in these chapters, we have used three-place tables.

Chapter 7 is a concise, yet self-contained, presentation of exponential and logarithmic functions, including applications of logarithms. Here we use the conventional four-place tables in calculations.

Chapter 11, a Compendium of Exercises, gives the student an opportunity to perfect his skills by solving a wide variety of problems varying in degree of sophistication. Because these problems are an integral part of the presentation, each class should solve a number of them. Students may attempt some of the problems after mastering the material on identities. Many of the problems are rather simple. For those that are more difficult, we have provided hints.

Appendix I is an introduction to finite mathematical induction, a rather thorough study for this level. Several illustrations are developed in detail, and a problem set with answers is included.

A course based on this book may be presented in at least two ways, depending on the preparation of the students. Some classes may review Chapters 1 and 7, study Chapters 2 through 11, and if time permits Appendix 1. Classes that are not well prepared should study the first six chapters and Chapter 8, as the core of the course.

The book includes a sufficient number of problems for either approach. Answers to nearly half the problems appear in the book itself; the remaining answers are available in a Solutions Manual.

As with any textbook, the ideas underlying this one were drawn from many sources and refined by many suggestions. We wish, therefore, to acknowledge our debt to our former teachers, our colleagues, and our students. In particular, we wish to thank the professors and students who used this text in a preliminary edition during the 1966–67 academic year. Special thanks are due Mrs. Lucille Moore, who not only typed the preliminary edition, but also typed and helped to edit the revised manuscript and the Solutions Manual.

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JOHN B. WELLS, JR., *formerly a professor of the Mathematics Department at the University of Kentucky and a co-author of the first draft of the manuscript, regrettably did not live to see the published book.*

1 PRELIMINARY CONCEPTS

2 THE TRIGONOMETRIC FUNCTIONS AND RELATED CONCEPTS

3 TRIGONOMETRIC IDENTITIES AND THEIR APPLICATIONS

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10 COMPLEX NUMBERS

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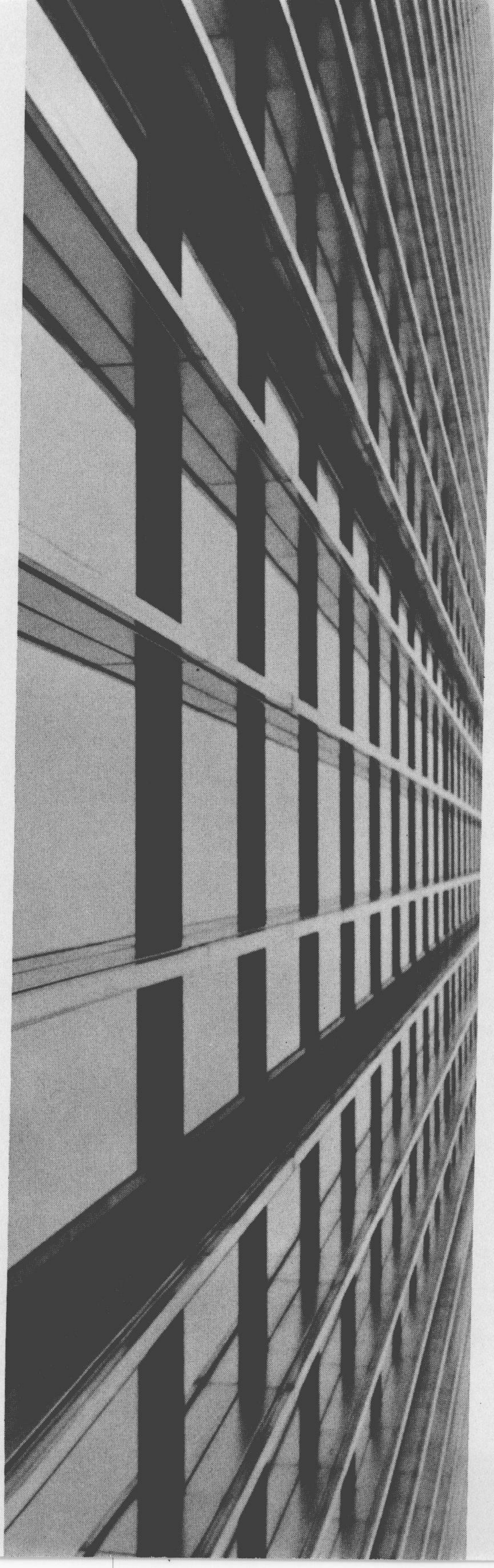

1

Preliminary Concepts

1-1 SETS

The theory of sets is a useful mathematical tool for many studies, including the physical and biological sciences, engineering, and economics. Although the concept of a set is of fundamental importance in mathematics, the word "set" will not be defined. Here, only an intuitive understanding of this concept is required. A similar demand was made when the reader encountered the words "point" and "straight line" in geometry. To illustrate this, consider your own trigonometry class. The members of this set (called a class) are those and only those students who officially enrolled for this class.

In general, upper case letters will be used to name sets, while lower case letters will denote *elements* of sets. If A is a set, the notation $a \in A$ indicates that a is an element (or member) of set A. When the element b is not an element of set A, this is indicated as $b \notin A$. The symbol $\in$ is used only between a symbol that represents an element of a set and a symbol that represents the set; it is not used between the symbols representing either two sets or two elements.

In general, there are two ways of describing a set—the *roster method* and the *rule method.* The set notation $\{. . .\}$ is used in both descriptions. It is important that a set be described precisely so that no confusion exists regarding what is in the set and what is not.

The roster method requires that the elements of the set be listed. For example, if Mary, Bill, and Jane are the members of a set, say A, then the description is given as $A = \{\text{Mary, Bill, Jane}\}$. This collection of symbols means and is read as "A is the set consisting of Mary, Bill, and Jane," or "A is the set whose elements are Mary, Bill, and Jane."

The rule method of describing a set A requires that a rule be given such that a is a member of A, provided a satisfies the rule. Furthermore, only those elements that satisfy the given rule will be members of the set. In this case, the mathematical statement is $\{x \mid$ *the rule given here is satisfied*$\}$ and means "the set of all x such that the rule given here is satisfied." If $A = \{1, 2, 3, 4\}$, then $A = \{x | x \text{ is a positive integer, and } x < 5\}$. A second example of the rule method is to let $A = \{2, 4, 6, 8, 10\}$; then, $A = \{2x | x \text{ is a positive integer, and } x < 6\}$ or $A = \{x | x \text{ is an even positive integer, and } x < 12\}$. Since there are many ways to state a rule, there are many ways to describe a particular set by the rule method.

We shall use the letter I^+ to denote the set of positive integers; that is, $N = \{1, 2, 3, . . .\}$, where . . . means that the established pattern is to be continued. In the case of the rule method, a description of I^+ could be $I^+ = \{x | x \text{ is a positive integer}\}$.