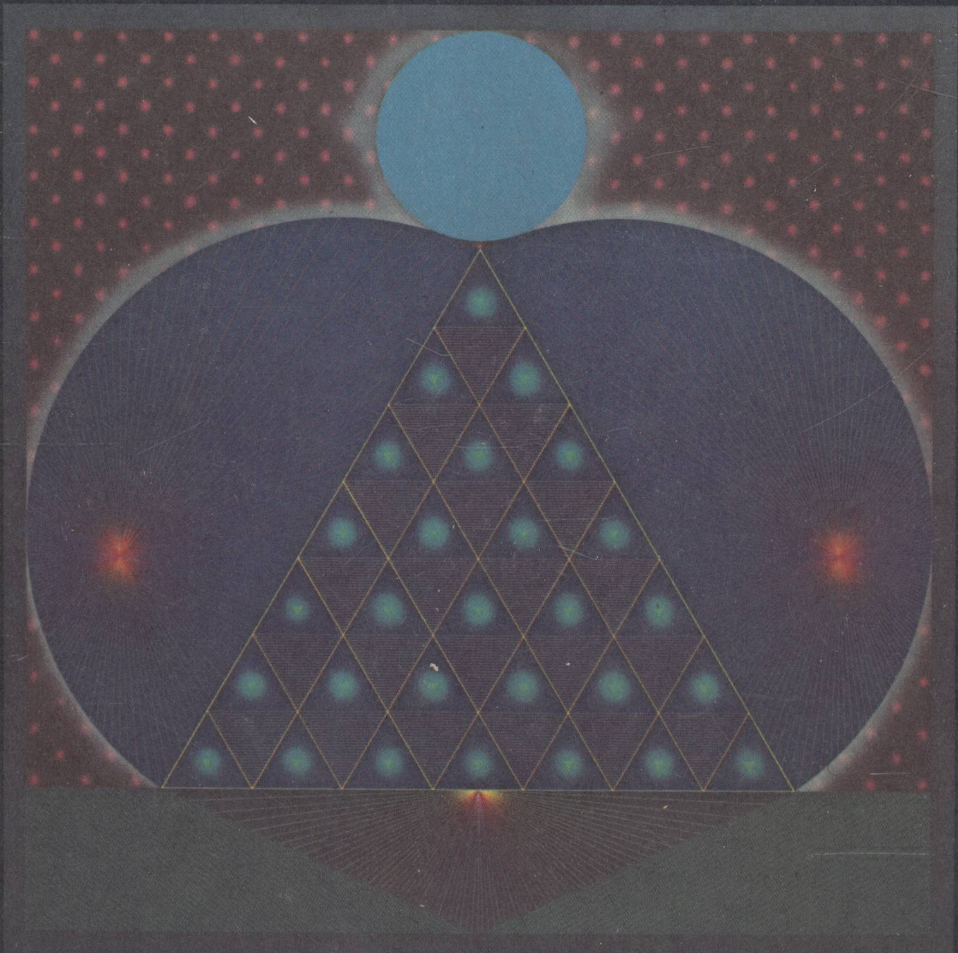
College Algebra

Third Edition



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Edwin F. Beckenbach
University of California, Los Angeles

Irving Drooyan
Los Angeles Pierce College

William Wooton

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To the student

A partially programmed study guide based on the text is available from your local bookstore under the name A Semi-Programmed Study Guide for College Algebra, Third Edition, *by Bernard Feldman.*

Symbols

[1.1]	$\{a, b\}$	the set whose elements (or members) are a and b
	$A = B$	A is equal to, or equals, B
	$A \sim B$	A is equivalent to B
	$A \subset B$	A is a subset of B
	$\emptyset$	the null, or empty, set
	$a \in A$	a is an element of, or is a member of, A
	U	the universal set
	$\not\subset$, $\notin$, etc.	is not a subset of, is not an element of, etc.
	$\{x \mid \ldots\}$	the set of all x such that ...
[1.2]	$A \cup B$	the union of sets A and B
	$A \cap B$	the intersection of sets A and B
	A'	the complement of set A
[1.3]	N	the set of natural numbers
	J	the set of integers
	Q	the set of rational numbers
	H	the set of irrational numbers
	R	the set of real numbers
	I	the set of imaginary numbers
	C	the set of complex numbers
	${}^{-}a$, or $-a$	the additive inverse, or negative, of a
	a^{-1}, or $1/a$	the multiplicative inverse, or reciprocal, of a
	$a - b$	the difference when b is subtracted from a
	$\frac{a}{b}$, or $a \div b$	the quotient when a is divided by b

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[1.5]	R_-	the set of negative real numbers
	R_+	the set of positive real numbers
	$a < b$	a is less than b
	$a > b$	a is greater than b
	$a \leq b$	a is less than or equal to b
	$a \geq b$	a is greater than or equal to b
	$\|a\|$	the absolute value of a
[2.1]	a^n	the nth power of a, or a to the nth power
	$P(x)$, $D(y)$, etc.	P of x, D of y, etc.
[3.1]	a^0	1
	a^{-n}	$1/a^n$
[3.2]	$a^{1/n}$	the real nth root of a for $a \in R$, or the positive one if there are two
	$a^{m/n}$	$(a^{1/n})^m$
[3.3]	$\sqrt[n]{a}$	the real nth root of a for $a \in R$, or the positive one if there are two
[3.5]	$0.33\bar{3}$, etc.	repeating decimal
	$a \approx b$	a is approximately equal to b
[4.3]	$\pm$	plus or minus
[5.1]	(a, b)	the ordered pair of numbers whose first component is a and whose second component is b
	$A \times B$	the Cartesian product of A and B
	$R \times R$, or R^2	the Cartesian product of R and R
	f, g, h, F, etc.	names of functions
	$f(x)$	f of x, or value of f at x
[5.2]	d	the distance between two points
	m	the slope of a line

(*continued inside back cover*)

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[5.4]	f^{-1}	the inverse function of f
[5.5]	$[x]$	the greatest integer not greater than x
[7.2]	$\log_b x$	the logarithm of x to the base b
[7.3]	$\text{antilog}_b\, x$	the antilogarithm of x to the base b
[7.6]	e	an irrational number, approximately equal to 2.7182818
[8.2]	(x, y, z)	the ordered triple of numbers whose first component is x, second component is y, and third component is z
[8.5]	$\mathscr{R}$, $\mathscr{S}$, etc.	a set of points
[9.1]	$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$, etc.	matrix
	$A_{m \times n}$	m by n matrix
	$a_{i,j}$	the element in the ith row and jth column of the matrix A
	A^t	the transpose of the matrix A
	$0_{m \times n}$	the m by n zero matrix
	$-A_{m \times n}$	the negative of $A_{m \times n}$
[9.2]	$I_{n \times n}$	the identity matrix for all n by n matrices
[9.3]	$A \sim B$	A is equivalent to B (for matrices)
[9.4]	$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, etc.	determinant
	M_{ij}	the minor of the element a_{ij}
	A_{ij}	the cofactor of a_{ij}
	$\delta(A)$	the determinant of A
[9.6]	A^{-1}	the inverse of A
[10.1]	z	complex number
	$-z$	the additive inverse, or negative, of z
	z^{-1}	the multiplicative inverse, or reciprocal, of z

[10.2]	$\bar{z}$	the conjugate of z
[10.3]	i	the imaginary unit, (0, 1)
	$\sqrt{-b}, b>0$	$\sqrt{b}i$
[10.5]	$\mathbf{v}$	vector
	$\lVert\mathbf{v}\rVert$	the norm, or magnitude, of $\mathbf{v}$
	$\mathbf{v}_1 + \mathbf{v}_2$	the sum of $\mathbf{v}_1$ and $\mathbf{v}_2$
	$c\mathbf{v}$	the product of c and $\mathbf{v}$
	$\mathbf{0}$	the zero vector
	$-\mathbf{v}$	the negative of $\mathbf{v}$
	$\mathbf{v}_1 - \mathbf{v}_2$	the difference when $\mathbf{v}_2$ is subtracted from $\mathbf{v}_1$
[12.2]	$s(n)$, or s_n	the nth term of a sequence
[12.3]	S_n	the sum of the first n terms in a sequence
	$\sum$	the sum
	S_∞	the sum of an infinite sequence
[12.4]	L	the limit of a sequence
[12.5]	$n!$	n factorial, or factorial n
	$0!$	0 factorial
	$\binom{n}{r}$	$\frac{n!}{r!(n-r)!}$
[13.1]	$n(A)$	the number of elements in the set A
	$P_{n,n}$	the number of permutations of n things taken n at a time
	$P_{n,r}$	the number of permutations of n things taken r at a time
[13.2]	$\binom{n}{r}$	the number of combinations of n things taken r at a time
[13.3]	E	event
	$P(E)$	the probability of E
[13.5]	$P(E_2 \mid E_1)$	the conditional probability of E_2, given the occurrence of E_1

Preface

This third edition, like its predecessors, has been designed for the freshman college mathematics course that has come to mean many different things in different schools. Its organization allows the instructor to choose from a wide variety of topics that fall under the heading of Algebra, in order to achieve the desired level of mathematical sophistication for his course. Such flexibility seems imperative for a course which is seldom a terminal subject in mathematics but, rather, serves the transitional purpose of equipping the student for more advanced courses. The student is assumed to have had the equivalent of at least one year of high school algebra and one year of high school geometry.

In this edition, we have concentrated on making the material easier for the instructor to teach and, more important, easier for the student to understand. Problem sets have been revised in the light of classroom experience, and many sections have been rewritten to clarify central concepts.

> In Chapter 1 the formal statement-reason format for proofs has been changed to an informal paragraph style.
>
> Chapter 2 has been rewritten to put less emphasis on the structure of polynomials; synthetic division is now in a separate section, and a *new section* on *partial fractions* has been added.
>
> Chapter 3 now includes a separate section on sums and products of radical expressions.
>
> Chapter 4 has been revised to include separate sections on the quadratic formula and on word problems.
>
> Chapter 5, which has been completely rewritten, now includes a *new section* on *inverse relations.*
>
> Chapter 6 has a new treatment of the graphing of quadratic functions, based on the symmetry property of parabolas.
>
> Chapter 8 now includes *new sections* on *convex sets, polygonal regions,* and *linear programming.*
>
> Chapter 9 has been reorganized so that linear systems are solved by means of row-equivalent matrices in Section 9.3, right after the properties of matrices are introduced in the first two sections.

Chapter 10 on complex numbers and vectors has been completely rewritten.

Chapter 11 on the theory of equations has been revised to eliminate some of the formal proofs.

The section on power series has been eliminated in the rewriting and reorganizing of Chapter 12.

Revisions in Chapter 13 center on the sections dealing specifically with probability.

The Appendix now includes, under the heading *Mathematical Structure*, a compact summary of the basic concepts of the text and a brief treatment of mathematical systems as related to the number systems considered in the text.

A *new chapter-review set of problems* has been added to all chapters. Sectional problem sets, as before, include examples with solutions that enable the students to work through the exercises on their own.

The first chapter, which introduces the student to the complete ordered field of real numbers, is followed by three chapters—on polynomials, rational exponents, and open sentences in one variable—that may be optional for the student who shows a mastery of second-year high school algebra. Beginning with Chapter 5, discussions generally center around the function concept. Polynomial and rational functions (and their graphs) are covered in detail; variation is treated from the function standpoint; logarithms are developed from a consideration of exponential functions; determinants are presented as functions of matrices; sequences are treated as functions having sets of positive integers as domain; and probability is discussed from a set-function standpoint.

The book is designed for a semester course of three, four, or five units, or for two three-unit quarter courses. Chapters 7 through 13 are sufficiently independent of each other that any may be omitted for a short course.

A section at the end of the book provides answers for odd-numbered problems, along with graphs.

A semi-programmed study guide covering topics in the text is available for student use.

As in the second edition, a second color is used functionally to highlight key procedures in routine manipulations and to focus attention on key elements of figures. Marginal annotations direct the reader's attention to important ideas.

We sincerely thank David Wend of the mathematics department of Montana State University for his very careful reading of this new edition and his many good suggestions for improving it. Our special thanks go also to Edwin S. Beckenbach and Charles C. Carico for their assistance in the preparation of this edition and its ancillary materials and to our editor, Don Dellen, for his suggestions and encouragement.

Edwin F. Beckenbach
Irving Drooyan
William Wooton

Supplemental Index for Topics of Analytic Geometry

Contents

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1 Properties of Real Numbers

1.1 Definitions and Symbols

A **set** is simply a collection of some kind. It may be a collection of people, colors, numbers, or anything else. In algebra, we are interested in sets of numbers of various sorts and in their relations to sets of points or lines in a plane or in space. Any one of the collection of things in a set is called a **member** or **element** of the set, and is said to be **contained in** or **included in** (or, sometimes, just **in**) the set. For example, the counting numbers 1, 2, 3, . . . (where the dots indicate that the sequence continues indefinitely) are the elements of the set we call the set of **natural numbers**.

Set notation

Sets are usually designated by means of capital letters, A, B, C, etc. They are identified by means of **braces**, { }, with the members either listed or decribed. For example, the elements might be listed as in {1, 2, 3}, or described as in {first three natural numbers}. The expression "{1, 2, 3}" is read "the set whose elements are one, two, and three"; "{first three natural numbers}" is read "the set whose elements are the first three natural numbers."

Using the undefined notion of set membership, we can be more specific about some other terms we shall be using.

Definition 1.1 *Two sets A and B are* ***equal****, $A = B$, if and only if they have the same members—that is, if and only if every member of each is a member of the other.*

Thus, if A denotes {1, 2, 3}, B denotes {3, 2, 1}, C denotes {2, 3, 4}, and D denotes {natural numbers between 1 and 5}, then $A = B$ and $C = D$. The phrase "if and only if" used in this definition is simply the mathematician's way of making two statements at once. Definition 1.1 means: "Two sets are equal if they have the same members. Two sets are equal only if they have the same members." The second of these statements is logically equivalent to: "Two sets have the same members if they are equal."

Definition 1.2 *If the elements of a set A can be paired with the elements of a set B in such fashion that each element of A is paired with one and only one element of B, and conversely, then such a pairing is called a* **one-to-one correspondence** *between A and B.*

For example, if $A = \{a, b, c\}$ and $B = \{1, 2, 3\}$, then the sets A and B can be put into one-to-one correspondence in six different ways, two of which are shown here:

$$\begin{matrix} \{a, & b, & c\} \\ \updownarrow & \updownarrow & \updownarrow \\ \{1, & 2, & 3\} \end{matrix} \qquad \begin{matrix} \{a, & b, & c\} \\ \updownarrow & \updownarrow & \updownarrow \\ \{2, & 1, & 3\} \end{matrix}$$

Definition 1.3 *Two sets are* **equivalent** *if and only if a one-to-one correspondence exists between them.*

Equivalence symbol

The symbol $\sim$ is used to denote equivalence. Thus, $A \sim B$ is read "A is equivalent to B." Intuitively, equivalent sets are sets that contain the same number of members. Clearly, if two sets are equal, then they are equivalent, but the converse is not necessarily true—equality of sets requires that the members be identical, not merely that the sets be in one-to-one correspondence.

Definition 1.4 *If every member of a set A is a member of a set B, then A is a* **subset** *of B. If, in addition, B contains at least one member not in A, then A is a* **proper subset** *of B.*

Subset symbol

The symbol $\subset$ (read "is a subset of" or "is contained in") will be used to denote both the subset relationship and the proper-subset relationship. Thus

$$\{1, 2, 3\} \subset \{1, 2, 3, 4\} \quad \text{and} \quad \{1, 2, 3\} \subset \{1, 2, 3\}.$$

Notice that, by definition, every set is a subset of itself.

The set that contains no elements is called the **empty set**, or **null set**, and is denoted by the symbol $\emptyset$ (read "the empty set" or "the null set"); $\emptyset$ is a subset of every set, and it is a proper subset of every set except itself. If a set S is the null set or is equivalent to $\{1, 2, 3, \ldots, n\}$ for some fixed natural number n, then S is said to be **finite**. A set that is not finite is said to be **infinite**. For example, the set of *all* natural numbers, $\{1, 2, 3, \ldots\}$, is an infinite set.

Definition 1.5 *Two sets A and B are* **disjoint** *if and only if A and B contain no member in common.*

For example, if $A = \{1, 2, 3\}$ and $B = \{5, 6, 7\}$, then A and B are disjoint.

Set-membership notation

The symbol $\in$ (read "is a member of" or "is an element of") is used to denote membership in a set. Thus,

$$2 \in \{1, 2, 3\}.$$

Note that we write

$$\{2\} \subset \{1, 2, 3\} \quad \text{and} \quad 2 \in \{1, 2, 3\},$$

since {2} is a *subset*, whereas 2 is an *element*, of {1, 2, 3}.

When discussing an individual but unspecified element of a set containing more than one member, we usually denote the element by a lowercase italic letter (for example, a, d, s, x), or sometimes by a letter from the Greek alphabet: α (alpha), β (beta), γ (gamma), and so on. Symbols used in this way are called **variables**.

Definition 1.6 *A* ***variable*** *is a symbol representing an unspecified element of a given set containing more than one element.*

The given set is called the **replacement set**, or **domain**, of the variable. If the domain is a set of numbers, then the variable represents a number. Thus

$$x \in A$$

means that the variable x represents an (unspecified) element of the set A. The members of the replacement set are called the **values** of the variable. A symbol with just one value is called a **constant**.

When discussing sets, it is often helpful to have in mind some general set from which the elements of all of the sets under consideration are drawn. For example, if we wish to talk about sets of college students, we may want to consider all college students in this country, or all students in general; or, taking a larger view, we may want to consider students as a special kind of human being—say, all those human beings who are consciously striving to increase their knowledge. Thus, we can draw sets of college students from any one of a number of different general sets. Such a general set is called the **universe of discourse**, or the **universal set**, and we shall usually denote it by the capital letter U. It follows that any set in a particular discussion is a subset of U for that discussion.

Negation symbol

The slant bar, /, drawn through certain symbols of relation, is used to indicate negation. Thus $\neq$ is read "is not equal to," $\not\subset$ is read "is not a subset of," and $\notin$ is read "is not an element of." For example,

$$\{1, 2\} \neq \{1, 2, 3\}, \quad \{1, 2, 3\} \not\subset \{1, 2\}, \quad \text{and} \quad 3 \notin \{1, 2\}.$$

Set-builder notation

Another symbolism useful in discussing sets is illustrated by

$$\{x \mid x \in A \text{ and } x \notin B\}$$

(read "the set of all x such that x is a member of A and is not a member of B"). This symbolism, called **set-builder notation**, is used extensively in this book. What it does is specify a variable (in this case, x) and, at the same time, state a condition on the variable (in this case, that x is contained in the set A and not in B).

Exercise 1.1

Designate each of the following sets by using braces and listing the members.

Example {natural numbers between 8 and 12}

Solution {9, 10, 11}

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