

**WAYNE NELSON**

**APPLIED LIFE  
DATA ANALYSIS**

**WILEY SERIES IN PROBABILITY  
AND MATHEMATICAL STATISTICS**



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# APPLIED LIFE DATA ANALYSIS

**WAYNE NELSON**

*General Electric Co.*

*Corporate Research and Development*

*Schenectady, New York*



E8562524



**JOHN WILEY & SONS**

New York • Chichester • Brisbane • Toronto • Singapore

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***Library of Congress Cataloging in Publication Data:***

Nelson, Wayne, 1936-  
Applied life data analysis.

(Wiley series in probability and mathematical statistics. Applied probability and statistics section, ISSN 0271-6356) (A Wiley publication in applied statistics)

Includes bibliographical references and index.

- I. Reliability (Engineering) I. Title.
- II. Series. III. Series: Wiley publication in applied statistics.

TS173.N44 620'.00452 81-14779  
ISBN 0-471-09458-7 AACR2

Printed in the United States of America

10 9 8 7 6 5 4 3

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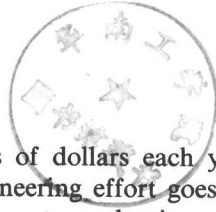
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**APPLIED LIFE  
DATA ANALYSIS**

**In Grateful Memory  
of my Mother and Father**

# Preface



Many major companies spend millions of dollars each year on product reliability. Much management and engineering effort goes into evaluating risks and liabilities, predicting warranty costs, evaluating replacement policies, assessing design changes, identifying causes of failure, and comparing alternate designs, vendors, materials, manufacturing methods, and the like. Major decisions are based on product life data, often from a few units. This book presents modern methods for extracting from life test and field data the information needed to make sound decisions. Such methods are successfully used in industry on a great variety of products by many who have modest statistical backgrounds.

This book is directed to engineers and industrial statisticians working on product life data. It will also aid workers in other fields where survival is studied, for example, in medicine, biology, actuarial science, economics, business, and criminology. Also, this book may supplement texts for many statistics and engineering courses, since it gives a wealth of practical examples with real data, emphasizes applied data analysis, employs computer programs, and systematically presents graphical methods, the method of maximum likelihood, censored data analysis, prediction methods, and linear estimation.

Life data generally contain running times on unfailed units, which require special statistical methods. In the past, these rapidly developing methods were associated with aerospace applications, but they are more widely used for consumer and industrial products. This book presents many applications to diverse products ranging from simple dielectrics and small appliances to locomotives and nuclear reactors.

This book draws from my experience teaching courses on life data analysis throughout the General Electric Company and at Rensselaer Polytechnic Institute and Union College. These courses have been popular with practicing engineers and graduate students in engineering, statistics, and operations research.



This book is organized to serve practitioners. The simplest and most widely useful material appears first. The book starts with basic models and simple graphical analyses of data, and it progresses through advanced analytic methods. All preliminary material for a topic is stated, and each topic is self-contained for easy reference, although this results in some repetition. Thus this book serves as a reference as well as a textbook. Derivations are generally omitted unless they help one understand the material. Such derivations appear in advanced sections for those who seek a fundamental understanding and wish to develop new statistical models and data analyses.

Readers of this book need a previous course in statistics and, for some advanced material, facility in calculus or matrix algebra. While many methods employ new and advanced statistical theory, the book emphasizes how to apply them. Certain methods (particularly those in Chapters 8 and 12), while important, are difficult to use unless one has special computer programs, which are now available.

There is much literature on life data analysis. So I have selected topics useful in my consulting. However, I briefly survey other topics in the final chapter.

Chapter 1 describes life data analysis, provides background material, and gives an overview of the book in detail. Chapter 2 presents basic concepts and statistical distributions for product life. Chapters 3 and 4 present graphical methods for estimating a life distribution from complete and censored life data. Chapter 5 explains statistical models and analyses for data on competing failure modes and on series systems. Chapters 6, 7, and 8 provide analytic methods, mainly linear and maximum likelihood methods, for estimating life distributions from complete and censored data. Chapter 9 provides methods for analyzing inspection data (quantal-response and interval data). Chapters 10, 11, and 12 provide methods for comparing samples (hypothesis tests) and for pooling estimates from a number of samples. Chapter 13 surveys other topics.

The real data in all examples come mostly from my consulting for the General Electric Company and other companies. Many of these real data sets are messy. Proprietary data were protected by vaguely naming a product and by multiplying the data by a factor. So engineers are advised not to use examples as typical of any product.

For help on this book I am overwhelmed with a great feeling of gratitude to many. Dr. Gerald J. Hahn, my co-worker, above all others, encouraged me, helped me to obtain support from General Electric, generously contributed much personal time reading the manuscript, and offered many useful suggestions. Gerry is the godfather of this book. I am much indebted for

support from management at General Electric Corporate Research and Development—Dr. Art Bueche, Mr. Stu Miller, Mr. Virg Lucke, Dr. Dick Shuey, Mr. E. Lloyd Rivest, Dr. Dave Oliver, Dr. Hal Chestnut, and Mr. Bill Chu. Professor Al Thimm, encouraged by Professor Josef Schmee, both of Union College, kindly provided me with an office, where I worked on this book, and a class that I taught from my manuscript during a leave from GE. Professor John Wilkinson of Rensselaer Polytechnic Institute gave me the original opportunity to teach courses and develop preliminary material for this book.

Colleagues have generously given much time reading the manuscript and offering their suggestions. I am particularly grateful to Paul Feder, Gerry Hahn, Joseph Kuzawinski, Bill Meeker, John McCool, Ron Regal, Josef Schmee, Bob Miller, Bill MacFarland, Leo Aroian, Jim King, Bill Tucker, and Carolyn Morgan.

Many clients generously let me use their data. They also inspired methods (such as hazard plotting) that I developed for their problems. Many students contributed suggestions. There are too many to name, unfortunately.

The illustrations are mostly the superb work of Mr. Dave Miller. The manuscript benefited much from the skillful technical typing of Jean Badalucco, Ceil Crandall, Edith White, and Ruth Dodd.

WAYNE NELSON

*Schenectady, New York*  
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# 1

## Overview And Background

### 1. INTRODUCTION

This chapter presents (1) an overview of this book's contents and (2) background information for the rest of the book. To read this chapter and the rest of the book, one needs a basic statistics course. Although addressed mostly to engineering applications, this book applies to many other fields. A key characteristic of life data analysis distinguishes it from other areas in statistics: namely, data are usually censored or incomplete in some way. Like other areas, it is concerned with estimates and confidence limits for population parameters and with predictions and prediction limits for future samples. The following paragraphs describe applications and the history of life data analysis.

**Applications.** This book presents methods for analysis of product life data and gives many engineering applications. In this book, examples of applications to products include diesel engine fans, transformers, locomotive controls, generator field windings, material strength, generator retaining rings, locomotive reliability, electrical insulating fluids and oils, the strength of electrical connections, Class-B and Class-H motor insulations, appliance cords, fuses, turbine disks, shave dies, alarm clocks, batteries, toasters, capacitors, cryogenic cables, motor coils, engine cylinders and pistons, power lines, large motor warranty costs, turbine wheels, and distribution transformers.