

# multivariable computer control a case study

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# MULTIVARIABLE COMPUTER CONTROL

A Case Study

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# MULTIVARIABLE COMPUTER CONTROL

## PREFACE

During the past decade there has been a tremendous growth in the number of analytical and design techniques available to the control engineer. Some of these are still in the initial state of development but others have been adapted and extended to the point where their tremendous potential can be exploited in industrial applications. Fortunately, this growth in available techniques has been accompanied, in fact preceded, by the availability of computing hardware that has the capability of implementing almost any control technique an engineer would like to apply to a real process. The rapid growth during the 1960's of real-time computer applications in industrial plants did not level out as many predicted, but due to the introduction of minicomputers and improved interface hardware, has actually expanded. The novelty of real-time computers has been largely overcome and applications of Direct Digital Control (DDC) and many management functions have become quite routine. There have been a few misguided computer applications, some of them highly publicized, that must be classified as failures. However, there have been many more applications that have been outstanding successes, as evidenced by repeated duplication of the original installations rather than by detailed public announcements. In fact the cumulative experience to date has led many knowledgeable people to believe that the future promises more reliable, more technically sophisticated, and more highly integrated process control and management systems.

At this point in time it would appear that the primary needs in the area of computer control are:

- (a) the availability of people with the technical training and vision to develop and implement the most promising of the presently available modern control techniques,
- (b) engineering evaluation of the currently available control techniques and the needs of industry,
- (c) experimental implementation and development of the most promising techniques.

Continuing research into new areas is also required and it was this need, plus considerations such as those listed above, that led to a series of computer-control projects in the Department of Chemical Engineering at the University of Alberta. The objective was to examine promising modern control techniques and with due consideration of the practical constraints of the personnel and equipment available, to evaluate the techniques by application to pilot plant units. This Case Study is concerned with the implementation of computer control on a pilot plant size, double effect evaporator and the subsequent evaluation of multivariable design, analysis and control techniques. The Case Study is compiled with the hope that a presentation of the results of applying several techniques, to a

single experimental unit, will enable the reader to see the relative advantages, disadvantages, potential and practicality of these methods.

It can be argued that an evaporator is not the most appropriate piece of equipment for evaluating techniques for industrial applications. However, its basic principles of operation are easily understood by anyone in the process industry, it is simple enough to be modelled from first principles yet complicated enough to require parameter estimation and process identification, and it includes an interacting set of flow, pressure, temperature and concentration variables. The evaporator itself is simpler and more highly instrumented than most industrial units. However, the experimental results should still give a good indication of how the new multivariable methods would work in actual applications. It is hoped that despite the shortcomings of this Case Study, the reader will develop insight into the methods presented and obtain a better basis from which to solve the problems of particular interest to him or her.

Since this Case Study consists of a collection of papers written over a period of several years, the presentation is not as smooth, continuous or complete as a textbook nor is the Case Study intended to be one. The introduction for each section should, however, help bridge the gaps and point out the interrelations between the different parts.

The reader will note from the co-authors for the individual articles that many different individuals have been involved in this work. We gratefully acknowledge the contributions made by our students as part of their thesis projects, by our colleagues, and by the staff in the department's Data Acquisition, Control and Simulation Centre. Without the financial support from the University, The National Research Council of Canada and from individual companies, this work would not have been possible. For any errors, omissions or shortcomings in this case study, we accept full responsibility.

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## Section 1: INTRODUCTION

### CONTENTS:

#### 1.1 Advanced Computer Control Improves Process Performance

D.G. Fisher and D.E. Seborg

### COMMENTS:

This section presents a review of some of the process dynamics and control projects undertaken in the Department of Chemical Engineering at the University of Alberta. There is no technical detail or theory that is not included in later sections of the Case Study but this section provides a good introduction and overview of the research activities. A description of the computer hardware and software in the Data Acquisition, Control and Simulation (DACS) Centre that was used in this work is available in the reference cited below.

### REFERENCE:

- [1] "Data Acquisition, Control and Simulation Centre", a booklet available from the Department of Chemical Engineering, University of Alberta, Edmonton, Canada (1974).

# Advanced Computer Control Improves Process Performance

D. G. FISHER and D. E. SEBORG, Univ. of Alberta

Multivariable and other modern control techniques can produce significantly better results than conventional methods and have tremendous potential for industrial application. This article presents specific results that have been obtained on a pilot plant evaporator, and reviews basic concepts and practical aspects of applying various techniques in advanced control.

**THE PRIMARY CHALLENGE** facing people in the control field today is to formulate their problems more definitely, clarify their objectives and glean from the wealth of available alternatives the practical approaches that can be applied to their specific problems. This challenge led to a number of continuing projects in the Department of Chemical Engineering at the University of Alberta, to investigate and evaluate some of the most promising techniques of modern control theory by applying them to computer-controlled pilot plant units.

This article deals specifically with the application of several techniques to a pilot plant evaporator. The objective is not an all-encompassing review of available techniques but rather a review and discussion of results obtained in specific projects. The discussion does not include rigorous definitions or

derivations but instead concentrates on concepts, comparisons and practical aspects of the various techniques. Details, derivations, related work, and additional data are available in the references.

## Modeling multivariable systems

People in industry frequently dismiss modern control techniques because they think a suitable mathematical model cannot be developed. It is true that a model of the process is needed at some stage in most control system design techniques. However, powerful methods are available to derive suitable models through empirical testing or theoretical analysis or a combination of the two. Some control systems can even be designed to learn or adapt, so that they will compensate for unknown or changing factors after they are connected to the process.

The pertinent questions today are "What type of model is required? How complicated? How exact?" The general answer is that development of a suitable model is simply part of the control system design procedure. In practical applications, this might require several iterations rather than a single definitive step.

A schematic diagram of the pilot plant evaporator used for most of the investigations reported here is illustrated in Figure 1. It has a complex feed system which permits operation of the equipment in a cyclic fashion, as well as the introduction of load changes and disturbances in the feed conditions. Controlled flows of concentrated triethylene glycol solution and water are temperature-controlled by means of steam heaters, and then mixed in the proportions necessary to produce a feed stream having the desired concentration and flow rate. This blending equipment is not included in the model, however, because it has a relatively fast dynamic response and does not interact with the state variables of the main evaporator.

The first effect is a short-tube vertical calandria-type unit with natural circulation. The 9-in. diameter unit has an operating holdup of 2 to 4 gallons, and its 32 stainless steel tubes, 3/4-in. o.d. by 18 in. long, provide approximately 10 square feet of heat transfer surface altogether.

The second stage is a long-tube vertical effect set up for either natural or forced circulation. It has a heat

transfer area of 5 square feet and is made up of three 6-ft long, 1-in. o.d. tubes. Capacity of the circulating system is about 3 gallons.

The evaporator is fully instrumented with commercial electronic controllers. Solution concentrations are measured by in-line refractometers similar to those described by Stackhouse (Ref. 1).

Experimental studies on the evaporator have clearly shown that different models are desirable for different applications. For example, a steady state model yielded the gains necessary to obtain significant improvement with static feedforward control (Ref. 2). Simple transfer function models, relating specific pairs of input/output variables, have proven adequate for dynamic feedforward compensation (Ref. 2,3). Third-order state-space models have worked as the basis for the design of optimal feedback systems (Ref. 4,5). A fifth-order linear model appears to be reasonable for most multivariable control techniques (Ref. 5,6). A nonlinear dynamic model proved best for off-line simulation purposes (Ref. 5,7). For optimal servo control, parameter estimation (curve fitting) techniques proved necessary to modify some of the theoretically derived constants in a fifth-order model and obtain better agreement with experimental evaporator data (Ref. 7,8,9,10). In other words, a single model is seldom satisfactory for all stages in the design procedure or for use in all parts of the final control system.

The pilot plant evaporator discussed above is obviously simpler than a typical industrial unit as shown in Figure 2. Most industrial installations will also differ from one another, because of various possible combinations of forward, backward and parallel flow systems for both feed and heating medium; and because of design variations among the several evaporator effects. The question therefore arises, "Is there a rational general approach to modeling such

systems, or must each one be approached on an individual basis?"

A generalized approach to the modeling of evaporators has been presented by Newell and Fisher (Ref. 11). Using this four-step procedure to develop a model of the pilot plant evaporator of Figure 1 led to a system of 10 nonlinear, first-order ordinary differential equations. The equations were linearized by considering only perturbations around the steady state operating point and putting them into the standard state-space form:

$$\dot{\mathbf{x}}(t) = \mathbf{A} \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t) \quad (1)$$

$$\mathbf{y}(t) = \mathbf{C} \mathbf{x}(t)$$

where:

$\mathbf{u}$ ,  $\mathbf{y}$  and  $\mathbf{x}$  are the input, output and state vectors, respectively.

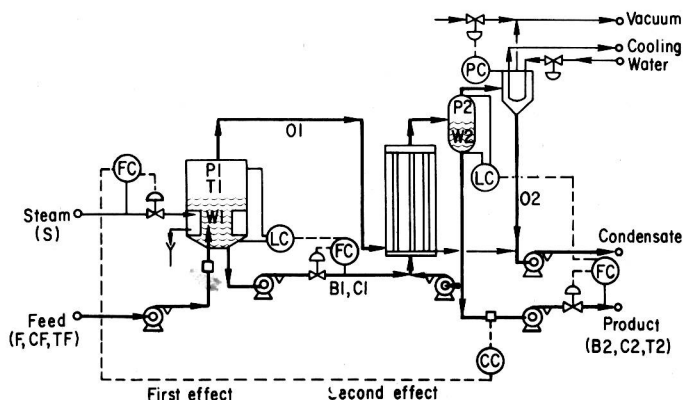
$\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are constant-coefficient matrices.

By neglecting factors such as the heat capacity of heat transfer surfaces, the state-space evaporator model can be reduced to lower order without significant loss in accuracy. The discussion and results that follow are based on a model with a fifth-order state vector and a sixth-order input vector made up of three control variables and three disturbance variables (see Nomenclature table).

### Simplicity vs complexity

Earlier investigators using the pilot plant evaporator did not adopt the approach outlined above, but instead derived their own models directly. These models ranged from first-order to fifth-order, and incorporated such physical assumptions as zero heat capacity in the heat transfer surfaces. Models obtained by reduction of the tenth-order model were in good agreement with those derived directly from physical assumptions (Ref. 5,6).

Figure 1. Schematic diagram of the double-effect, pilot plant evaporator at the University of Alberta. The control system shown is a single variable, multiloop configuration used as the base case. Over 50 process measurements and all final control elements are interfaced to an IBM 1800 digital computer. The evaporator is normally run under DDC or special multivariable control algorithms. Letter symbols in this and other figures are explained in the Nomenclature table.



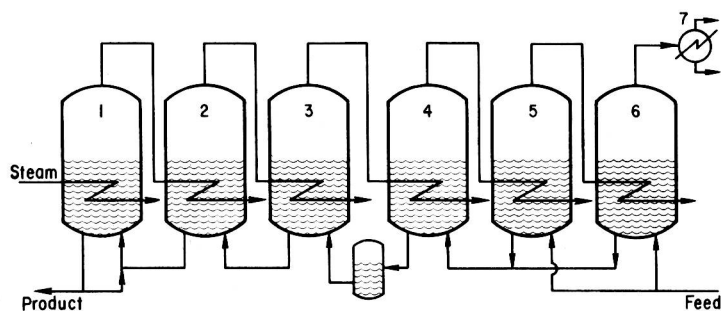


Figure 2. A schematic diagram of a typical six-effect industrial evaporator used to derive generalized material and energy balance equations.

Our experience with modeling has been that it is better to err on the side of complexity than to introduce assumptions prematurely or unnecessarily. Derivation of a rigorous model gives the engineer better insight and perspective and sometimes shows that the sum of certain factors is significant even though it would appear reasonable to neglect each when they are considered individually.

Most control engineers have access to a digital computer to assist with the design of control systems and frequently the final installation involves an on-line, real-time computer. Therefore, although complexity is not desirable for its own sake, it is no longer reasonable to reject techniques simply because they cannot be calculated by hand or implemented with conventional instruments.

### Process and parameter estimation

Techniques for process and parameter estimation are frequently required to modify the model so it will better approximate the actual performance of the process. This is a very broad and complex area that has recently been reviewed by Nieman *et al* (Ref. 12).

Quasi-linearization plus linear programming techniques were used by Nieman (Ref. 7,8) to adjust selected parameters of the theoretical nonlinear fifth-order state-space model so it would be in better agreement with the open-loop response data from the evaporator. Figure 3 compares the theoretical model response, the fitted model response and the actual process response for an equivalent disturbance. The change in product concentration here is much larger than occurs in most control studies and therefore tends to overemphasize the difference between the models. Although there is a significant error in the response of the theoretical model, it was adequate for most of the design methods discussed below.

### Multivariable feedback control

The control objective of the pilot plant evaporator was defined as maintaining the output concentration,  $C_2$ , at a constant value in spite of disturbances in feed conditions. This can be accomplished using conven-

tional, cascaded, single-variable control loops as illustrated in Figure 1. That is,  $C_2$  is controlled by manipulating the inlet steam  $S$ ; and the holdup  $W_2$  is controlled by manipulating the outlet flow,  $B_2$ . Similarly  $W_1$  is controlled by  $B_1$ . The pairing of

### Nomenclature

Process variables	Steady state values
State vector, $x$ :	
W1 first effect holdup	14.6 kg
C1 first effect concentration	4.85%
H1 first effect enthalpy	335 kJ/kg
W2 second effect holdup	18.8 kg
C2 second effect concentration	9.64%
Control vector, $u$ :	
S steam flowrate	0.86 kg/min
B1 first effect bottoms	1.5 kg/min
B2 second effect bottoms	0.77 kg/min
Load vector, $d$ :	
F feed flowrate	2.27 kg/min
CF feed concentration	3.2 %
HF feed enthalpy	3.2 kJ/kg
Output vector, $y$ :	
equal to (W1, W2, C2)T	
Other variables:	
O1 overhead vapor from first effect	0.77 kg/min
O2 overhead vapor from second effect	0.73 kg/min
P1 pressure in first effect	68.9 kN/m <sup>2</sup>
P2 pressure in second effect	0.38 m Hg
TF temperature of feed	88 °C
T1 temperature in first effect	107 °C
T2 temperature in second effect	71 °C
Subscripts:	
FB feedback	
FF feedforward	
I integral	
m model	
SP setpoint	
ss steady state	
Superscripts:	
T matrix/vector transpose	
i counter for time intervals	
Abbreviations:	
CC Concentration controller	
LC Level controller	
FC Flow controller	
DDC Direct digital control	
P+I Proportional-plus-integral	

variables in this manner may be obvious, but it can also be derived from a sensitivity analysis (Ref. 5,13). Experimental studies showed that reasonable control could be obtained with this approach (Ref. 2,4); that feed flow was the most serious disturbance; that there were strong interactions between variables; that simple feedforward compensation would give significant improvement (Ref. 4,14); and that the process is non-linear in nature.

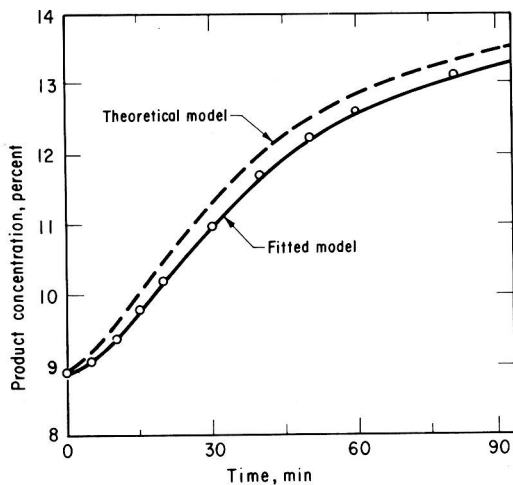


Figure 3. Comparison of a theoretical model with a fitted model and with experimental data is illustrated by this graph of product concentration versus time. Both models are fifth-order and nonlinear.

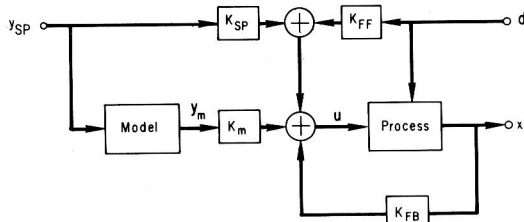


Figure 4. Here is a schematic representation of the multivariable feedback control law as defined by Equation (5). It includes proportional feedback, feedforward and setpoint control modes (See Nomenclature table).

Substitution of direct digital control algorithms for conventional industrial controllers showed that equivalent performance could be obtained with DDC, with due consideration of such additional factors as sampling time, filter constants and limits on input and output signals.

It is significant, relative to later discussion, that the fifth-order linear evaporator model is *not* suitable for determining values of the controller constants in the single-variable control system shown in Figure 1. The multiloop control configuration shown there was chosen by the user, based on experience and intuition. In multivariable control the situation is significantly different.

The approach taken to develop a multivariable control system was the discrete version of the "linear-quadratic-Gaussian" control problem originally developed by Kalman (Ref. 15), and recently the subject of an extended review (Ref. 16). The first step is to derive a suitable linear, discrete, state-space model of the process in the following form:

$$\mathbf{x}[(n+1)T] = \Phi(T)\mathbf{x}(nT) + \Delta(T)\mathbf{u}(nT) \quad (2)$$

$$\mathbf{y}(nT) = \mathbf{C}\mathbf{x}(nT) \quad n = 0, 1, 2, \dots$$

The fundamental matrix  $\Phi(T)$  and the coefficient matrix  $\Delta(T)$  can be derived directly from the coefficient matrices in Equation (1) and the two models are equivalent at the sampling points if  $\mathbf{u}$  is constant over the sampling interval.

The control objective is then defined as a summed quadratic function of the state and control variables which can be written in simplified form as:

$$J = \sum_{i=1}^N [\mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i + (\mathbf{u}_{i-1})^T \mathbf{R} (\mathbf{u}_{i-1})] \quad (3)$$

where:

$i$  identifies the time interval.

$\mathbf{e}$  is the process error (actual minus desired value).

$N$  is a large integer ( $N \gg 0$ )

$\mathbf{Q}$  and  $\mathbf{R}$  are weighting matrices of constants specified by the user.

The optimization problem of finding the control output  $\mathbf{u}$  that will minimize the performance index  $J$  can be solved using the techniques of dynamic programming, and results in a feedback control law of the form:

$$\mathbf{u}(nT) = \mathbf{K}_{FB}\mathbf{x}(nT) \quad (4)$$

where  $\mathbf{K}_{FB}$  is a matrix of proportional control constants. For a given model this is essentially a synthesis procedure in which the control engineer specifies the matrices  $\mathbf{Q}$  and  $\mathbf{R}$  in Equation (3) and the optimization procedure generates both the form of the control law and the control constants (Ref. 5,6).

It is, however, extremely difficult to modify the problem to accommodate constraints on the state and control variables. Note that if  $\mathbf{K}_{FB}$  is diagonal, then

Equation (4) represents a multiloop proportional control system—that is, a set of independent, single-variable control loops.

The multivariable procedure has been extended by the authors and coworkers to include integral feedback control, feedforward control (Ref. 17) and provision for model following during setpoint changes (Ref. 6). The complete control law, illustrated in Figure 4, is given by:

$$u(nT) = K_{FB}x(nT) + K_m y_m(nT) + K_{SP}y_{SP}(nT) + K_{FF}d(nT) \quad (5)$$

Note that this one method generates the multivariable equivalent of conventional single-variable control techniques. However, more importantly, when applied to the evaporator the multivariable control law proved to be practical and robust, and gave significantly better control than the multiloop approach.

Figure 5 shows a direct comparison of the effect of a 20 percent increase in feed flow rate on the product concentration C2. It is obvious that multivariable control is significantly better than conventional methods. The economic advantage of the tighter control would depend on the particular application.

It is interesting that the multivariable feedback controller was designed using the same fifth-order model that was *not* suitable for the design of single-variable systems. Even though some of the elements of  $K_{FB}$  are an order of magnitude larger than the proportional control constants used experimentally with the multiloop system, the theoretically designed control law gave excellent control when applied experimentally (Ref. 6).

### Optimal servo control

Although the multivariable feedback control laws discussed in the previous section could handle setpoint changes, they were designed primarily to regulate a process about a set of constant operating conditions. A different approach is advantageous when it is desired to make a grade change on a production unit or to change the process from one set of operating conditions to another in an "optimum" manner.

Nieman and Fisher have shown (Ref. 9,10) that for a discrete, linear, time-invariant system such as defined by Equation (2), it is possible to formulate the optimization problem so it can be solved by a standard linear programming package. The problem must also have a linear performance criteria such as minimum time to implement the change or minimum sum-of-the-absolute error between actual and desired values. This technique also readily accommodates constraints on the state and control variables, and on their rates of change. Implementation consists of outputting the precalculated values of  $[u(nT), n = 1, 2, 3, \dots, N]$  at the appropriate control interval. On-line calculations are not required and in simple cases the implementation can be done manually.

When applied in an open-loop configuration, this technique is sensitive to modeling errors and unanticipated disturbances. Real-time calculation or adjustment of the bang-bang switching times can significantly reduce this problem. However, for practical industrial applications it is recommended that the optimal open-loop control policy  $u^*$ , for a specified change, should be added to the existing feedback control scheme. This is shown in Figure 6 by the dotted box. The calculated optimal trajectory  $y^*$  is introduced as the setpoints of the feedback controllers.

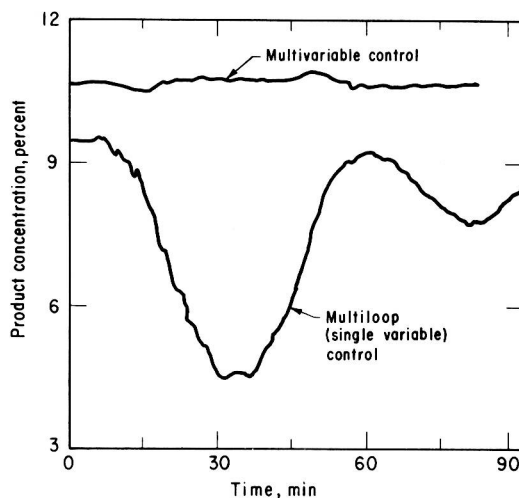


Figure 5. Comparison of experimental response data from the evaporator, under multivariable control and under multiloop proportional-plus-integral feedback control.

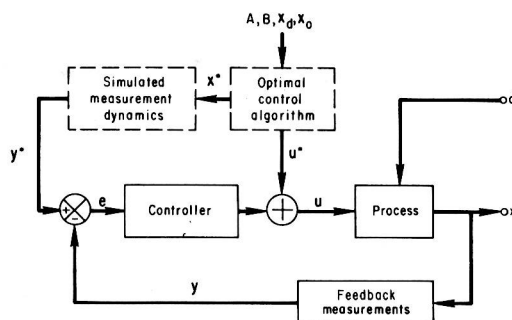


Figure 6. Schematic diagram illustrating how an optimal open-loop servo control policy can be added to an existing multiloop or multivariable feedback control system.

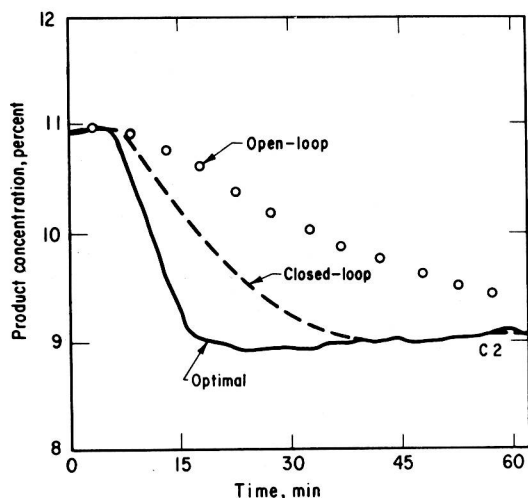


Figure 7. Comparison of open-loop, closed-loop and optimal response data for experimental runs of the evaporator. The open-loop response is to a step change in inlet steam flow; closed-loop response to a step change in the DDC setpoint; and optimal response is based on bang-bang switching of the steam flow.

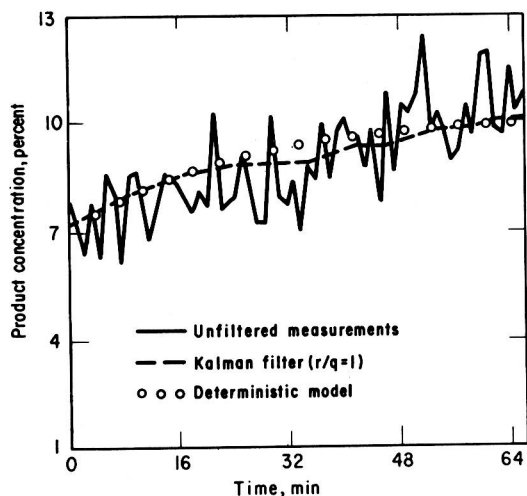


Figure 8. Simulated data illustrates the performance of a discrete Kalman filter designed for the evaporator. The solid line is the filter input, the dashed line is the filter output, and the circles depict noise-free response of the model.

If the calculated and actual process responses are identical, the transient proceeds as in the optimal open-loop case. However, if modeling errors or unexpected disturbances are present, then the feedback system, although suboptimal, tends to correct for these errors (Ref. 10).

The degree of improvement that can be achieved in the response of the evaporator product concentration is illustrated in Figure 7. Here, the optimal response time of the evaporator is about half of that obtained using conventional DDC control. Techniques that can produce this degree of improvement in pilot plant applications justify careful consideration by industry.

### Optimal filtering and estimation

Two problems that commonly arise in industrial applications are noise and unmeasured variables. Optimal multivariable feedback controllers, in particular, have relatively high gains which make them sensitive to measurement noise and which require that estimates of *all* the state variables of the process be available.

If a state-space model of the process is available and the characteristics of the noise are known, it is possible to design an optimal filter that will minimize the error between the actual and measured values of the process variables. A Kalman filter (Ref. 18) was implemented on the evaporator and at each sampling interval an estimate of the state vector  $\hat{x}$  was calculated from:

$$\hat{x}(n+1)T] = \bar{x}(nT) + K[y(nT) - C\bar{x}(nT)] \quad (6)$$

$K$  is a matrix of constants

$\bar{x}$  is the value of the state variables calculated from the process model.

In the ideal case the matrix  $K$  is calculated off-line from the process model and measured noise characteristics. But in applications to the pilot plant evaporator, the assumed noise characteristics were treated as design parameters and adjusted to modify the filter response for different process conditions.

Figure 8 shows a comparison of measured, filtered and model output values of the product concentration from a simulated evaporator run, starting with an initial error of 30 percent in C2 and returning to setpoint. In the ideal case, the filtered values would coincide with the calculated values.

This filter has been evaluated experimentally in a series of open-loop and closed-loop runs using different process conditions, disturbances and filters (Ref. 18). The conclusions reached are as follows:

- The filter is robust and practical and gives true values of the process state variables.
- If tightly tuned to reject measurement noise, the filter becomes sensitive to unmeasured disturbances. If the disturbances are measured, its performance is excellent.
- The performance of the filter is relatively insensitive to errors in the model parameters. (For example,

changes of  $\pm 25$  percent in the holdups in both evaporator effects produce negligible changes in the filter outputs).

- The filter can be extended or augmented to provide estimates of unmeasured state variables and/or constant biases in the noise signals. (It can also be extended to provide continuously updated estimates of process parameters that change with time.)

The Kalman optimal multivariable filter, as defined by Equation (6), was found to be much better in removing noise and introduced less attenuation and phase shift than single-variable exponential filters (Ref. 19).

### Other control approaches

Adaptive control techniques are particularly attractive for industrial application because the controller parameters are adjusted on-line to compensate for uncertainties in the process model and for changing process conditions. A multivariable, model-reference adaptive control system has recently been successfully applied to the pilot plant evaporator (Ref. 20).

A generalized computer program developed over the last few years makes use of network diagram techniques (as in CPM and PERT methods) to define interactions and sequencing in discrete control activities (Ref. 21,22). This program has been successfully used to automate startup of the evaporator.

The control projects summarized in this article demonstrate that modern control techniques such as optimal, multivariable feedback control, servo control and filtering produce significantly better results than conventional methods when applied to a pilot plant evaporator.

Computer applications must grow beyond simple emulation of conventional control instruments and/or automation of existing procedures, and reach out to encompass new approaches and new techniques.

### References

1. Stackhouse, G. E., "A New Refractometer for Continuous Black Liquor Analysis," ISA/CPPA International Symposium on Pulp and Paper Process Control, Vancouver, 1969, pp. 187-194.
2. Fehr, M., "Computer Control of an Evaporator," M.Sc. thesis, University of Alberta, Dept. of Chemical & Petroleum Engineering, 1969.
3. Wilson, A. H., "A Feedforward Control System for a Double Effect Evaporator," M.Sc. thesis, University of Alberta, Dept. of Chem. & Pet. Eng., 1966.
4. Jacobson, B. A. and Fisher, D. G., "Computer Control of a Pilot Plant Evaporator," *The Chemical Engineer*, (in press), England.
5. Newell, R. B., "Multivariable Computer Control of an Evaporator," Ph.D. thesis, University of Alberta, Dept. of Chem. & Pet. Eng., 1971.
6. Newell, R. B. and Fisher, D. G., "Experimental Evaluation of Optimal Multivariable Regulatory Controllers with Model-Following Capabilities," *Automatica*, Vol. 8, No. 3, 1972, pp. 247-262.
7. Nieman, R. E., "Application of Quasilinearization and Linear Programming to Control and Estimation Problems," Ph. D. thesis, University of Alberta, Dept. of Chem. & Pet. Eng., 1971.
8. Nieman, R. E. and Fisher, D. G., "Parameter Estimation Using Quasilinearization and Linear Programming," *Canadian Journal of Chemical Engineering*, Vol. 50, No. 6, 1972, pp. 802-806.
9. Nieman, R. E. and Fisher, D. G., "Experimental Evaluation of Time-Optimal, Open-Loop Control," *Transactions of the Institution of Chemical Engineers*, Vol. 51, No. 2, 1973, pp. 132-140.
10. Nieman, R. E. and Fisher, D. G., "Experimental Evaluation of Optimal Multivariable Servo Control in Conjunction with Conventional Regulatory Control," *Chemical Engineering Communications* (in press).
11. Newell, R. B. and Fisher, D. G., "Model Development, Reduction and Experimental Evaluation for a Double Effect Evaporator," *Industrial Engineering Chemical Process Design & Development*, Vol. 11, No. 2, 1972, pp. 213-221.
12. Nieman, R. E., Fisher, D. G. and Seborg, D. E., "A Review of Process Identification and Parameter Estimation Techniques," *International Journal of Control*, Vol. 13, No. 2, 1971, pp. 209-264.
13. Ritter, R. A. and Andre, H., "Evaporator Control System Design," *Canadian Journal of Chemical Engineering*, Vol. 48, No. 6, 1970, p. 696.
14. Nisenfeld, A. E. and Hoyle, D. L., "Dynamic Feedforward Control of Multi-effect Evaporators," *Instrumentation Technology*, Vol. 17, No. 2, 1970, pp. 49-54.
15. Kalman, R. E., "A New Approach to Linear Filtering and Prediction Theory," *Transactions ASME, Journal of Basic Engineering*, Vol. 83, 1960, p. 35; Vol. 83, 1961, p. 95.
16. *IEEE Transactions on Automatic Control*, Vol. AC-16, No. 6, 1971.
17. Newell, R. B., Fisher, D. G. and Seborg, D. E., "Computer Control Using Optimal, Multivariable Feedforward Feedback Algorithms," *AIChE Journal*, Vol. 18, No. 5, 1972, pp. 976-984.
18. Hamilton, J. C., Seborg, D. E. and Fisher, D. G., "An Experimental Evaluation of Kalman Filtering," *AIChE Journal* (in press).
19. Hamilton, J. C., "An Experimental Investigation of State Estimation in Multivariable Control Systems," M.Sc. thesis, University of Alberta, Dept. of Chem. & Pet. Eng., 1972.
20. Oliver, W. K., Seborg, D. E. and Fisher, D. G., "Model Reference Adaptive Control Based on Liapunov's Direct Method," *Chemical Engineering Communications* (in press).
21. Hayward, S. W., "Alberta Discrete Activity Monitor System (ADAMS)," M.Sc. thesis, University of Alberta, Dept. of Chem. & Pet. Eng., 1972.
22. Hayward, S. W. and Disher, D. G., "Alberta Discrete Activity Monitor Program (ADAMS)," 22nd Canadian Chemical Engineering Conference, Toronto, September 1972.

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## Section 2: MODELLING, SIMULATION AND DESIGN

### CONTENTS:

- 2.1 Model Development, Reduction, and Experimental Evaluation for an Evaporator  
R.B. Newell and D.G. Fisher
- 2.2 Description and Applications of a Computer Program for Control Systems Design  
D.G. Fisher, R.G. Wilson and W. Agostinis
- 2.3 Model Reduction For Discrete-time Dynamic Systems  
R.G. Wilson, D.G. Fisher and D.E. Seborg
- 2.4 Hybrid Simulation of a Computer Controlled Evaporator  
W.K. Oliver, D.E. Seborg and D.G. Fisher

### COMMENTS:

The first step in the application of most modern control techniques is to develop a suitable mathematical model of the process. In some cases this can be done by deriving the appropriate equations from first principles and calculating the parameters from physical property data and/or design correlations. In other cases the parameters must be determined by using actual plant data. When it is impractical to derive the equations analytically, then "process identification techniques" can be applied in experimental tests on the actual process.

Analog, digital or hybrid simulation of all or part of the system of interest is frequently helpful in order to give the control engineer a "feel" for system performance and to compare alternative design strategies. The procedure followed for each major control application is different but normally involves a sometimes iterative sequence of modelling, design, simulation and evaluation stages.

The first paper describes the development, reduction and evaluation of dynamics models of the pilot plant evaporator. Several different models are compared and it is concluded that the decision about which model is "best" depends on the particular way it is to be used in the final application.

Simulation on a digital computer is one way of comparing alternative models and paper 2.2 describes the GEneral, Multipurpose, Simulation and Control PackagE (GEMSCOPE) developed and used at