Coefficient Plane Models for Control System Analysis and Design

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1. Coefficient Plane Models for Control System Analysis and Design

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Preface

There has been a revolution in the art of control systems design due to the ready availability of digital computational facilities on a scale that could not have reasonably been envisaged a decade ago. In theory, the software now exists to solve most design problems "in absentia". However, in practice, this presupposes a great deal of intuition on the part of the system designer insofar as the devices within the system must be realistically modelled, and the structure of the system must be understood. The operational performance specifications must be properly interpreted as a reliable performance index capable of selecting an engineering optimum design, alternative forms of compensation still need to be evaluated and costed, and the advantages of feedback control properly exploited. Finally, products of any special-to-type software need to be evaluated against accepted norms.

This monograph is motivated by the need to provide practising engineers with the understanding of feedback systems necessary to make effective use of the new methodology and to give them added confidence in their design and development activities. It should prove of equal interest to mechanical, electrical, aerospace, maritime, hydraulic, and pneumatic systems specialists.

The monograph builds on the knowledge that most practising systems engineers have a nodding acquaintance with transfer function concepts. All University research groups working on dynamic analysis problems use a suite of classical control programmes to cross-check the feasibility of solutions obtained by modern control theory. It is implied, for example, that displaying system pole-zero arrays can provide an understanding of system behaviour over and above the information contained in a quadratic performance index. This philosophy dates back to concepts of damping ratio and natural frequency associated with a second order system. If the remainder of the poles of a high order system are "far-away" in the s-plane, the engineer can still visualise the dynamic behaviour of the system to an acceptable degree of approximation.

Concepts based on second order transfer functions are valuable items in the system engineer's tool kit. They do have fundamental limitations, and leave many practical questions unanswered. These include:-

(a) when does the use of second order transfer function approximations mislead the engineer into making false predictions?

(b) under what engineering circumstances should a high order system be designed so that dynamic performance is dominated by a few poles and zeros?

(c) what are the rules for approximating high order systems by low order models, and how accurate is the approximation?

(d) what is the optimum system pole-zero array to choose for a low order model, and what factors influence the definition of optimum?

(e) having arrived at a desired system transfer function meeting the performance specifications, how is the compensation to be chosen?

It is the purpose of the monograph to answer these questions using coefficient plane concepts which relate to third order models. Reducing a complex system to a coefficient plane model permits us to visualise cause and effect from two dimensional plots. This is because the third parameter, the normalisation frequency, can be interpreted as a time scaling factor similar in effect to the undamped natural pulsatance of a second order system. So the monograph concentrates on the presentation of visual aids such as percentage overshoot and bandwidth contours.

A number of system simplification methods found particularly useful by the author and his colleagues are included. These are polynomial truncation, Bode plot modelling, and s plane modelling via the time delay theorem. The "why" of good control system design is also explored in terms of reduced sensitivity to plant variation, and optimisation of performance in the presence of plant input saturation. The final chapter exploits the use of coefficient plane models in a CACSD suite of programmes in which the performance of a high order system is optimally matched to deterministic specifications.

The author has enjoyed working in the area of coefficient plane modelling with numerous colleagues. Particular thanks are due to Professor George J. Thaler (U.S. Naval Postgraduate School, Monterey), Lt. Cdr. Mike J. Ashworth (Royal Naval Engineering College, Manadon), and Mr. Peter Garnell (Royal Military College of Science, Shrivenham), who were kind enough to comment on the manuscript in various stages of preparation.

Contents

CHAPTER 1

The Time Delay Theorem Allied to Low Order Modelling

1.1. INTRODUCTION

There is much interest at this point in time in the derivation of low order models to represent complex plant, processes, and systems. For example, a recent survey of available techniques lists 40 references, but more importantly, 38 different authors are involved, and some 28 research establishments, universities, and industrial concerns are represented (110), thus indicating a broad base for this work. Consequently, we may expect to find a wide variety of reasons for using low order models, of which the following are thought to be the most important;

(i) To Simplify Understanding of the System, whether the problem is analysis, synthesis, or identification. An interpretation of such expressions as time constant, damping ratio, and natural frequency is second nature to the control engineer, particularly since standard curves describing performance of simple systems have been available for many years (57). It is therefore natural to attempt to reduce complex systems to such simple terms, even if considerable skill is required to obtain a realistic model.

(ii) To Reduce Computational Requirements, which means better use of support facilities as well as the computer itself, in addition to widening the scope of problems which can be handled on a given size of machine. For example, if it is necessary to evaluate the transient response sensitivity to parameter variations using classical sensitivity theory, then an n^{th} order system in general requires a simulation of order 2(n+1) (105). In this instance provision of a low order model adequately describing the dynamic response completely alters the scale of the computational problem.

(iii) To Reduce the Likelihood of Data Preparation Errors, simply on a law of averages basis, because less parameters are involved, and the range of numbers encountered reduced.

(iv) To Make Best Use of Scanty Experimental Data, by estimating a few parameters with confidence, rather than estimating more parameters with less confidence. The low order model is then a more reliable predictor of system performance (93).

(v) To Reduce Hardware Requirements in situations where an on-line system model is required; for example in monitoring the integrity of avionic equipment by comparing system and model outputs, or for use in self-adaptive systems. The same argument also applies to the design of reduced order state observers (38), and to the design of compensation networks (5).

(vi) To Generalise Results established on a particular system to comparable systems, using low order models which basically differ in time scale only. This is particularly useful in tolerancing performance for system checkout (21).

(vii) To Provide Guidelines for On-Line Interactive Modelling by selecting the most useful normalised test features, such as individual frequencies for signal injection (111).

(viii) To Improve the Methodology of Computer Aided Control System Design, by relating a suitable low