Foundations of Boundary Layer Theory for Momentum, Heat, and Mass Transfer

JOSEPH
A.
SCHETZ

Joseph A. Schetz

Aerospace and Ocean Engineering Department
Virginia Polytechnic Institute and State University

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This book is dedicated to the memory of William H. Webb, builder of clipper ships and founder and endower of the Webb Institute of Naval Architecture, whose generosity made it possible for this grandson of uneducated Albanian, German, and Slovak immigrants to obtain an excellent technical education completely free.

PREFACE

This volume was planned and written as a textbook for advanced undergraduate or beginning graduate students with three primary goals. First, it was intended to present understandable coverage at that academic level of the advances in turbulence modeling and the application of large digital computers to boundary layer problems, which have revolutionized the field over the last two decades. None of the existing texts serve that purpose. Some have held that modern numerical methods cannot be taught to engineering undergraduates so that they finish the course with any usable *tools*. That view is rejected here. Indeed, this writer strongly believes that the modern university student grew up in the *computer age*, and that he or she finds this type of material easier to grasp than such classical topics as Laplace transforms or Fourier series. Repeated experience in the classroom has proven this view to be correct. The second goal was to treat mass transfer in an integrated manner with momentum and heat transfer. The phenomena and methods of analysis are so similar that it seems inefficient and confusing to split convective mass transfer off as a separate subject as has often been the practice. Finally, a determined effort has been made throughout to relate viscous phenomena in general to the real world.

This book is written to be applicable for courses for mechanical, aerospace, chemical, civil, and ocean engineering students. The treatment presumes that the student has had at least one undergraduate course in fluid mechanics. Tables following this preface suggest coverage for a one-semester or two-quarter course for different majors at both the advanced undergraduate and beginning graduate levels.

To achieve the goals set for coverage and length, it was necessary to omit any discussions of unsteady flows or truly three-dimensional cases. Those topics are, however, usually not discussed in any detail in courses at the intended level. Further, to leave room in the book and time in the classroom for thorough treatments of numerical methods and turbulent flows, much of the older material on laminar flows was also omitted. Although some of that material is quite elegant and interesting, it really has little actual use to the practicing engineer.

There is one other somewhat unusual feature to the organization of the material. Integral methods are introduced very early, before the derivation of the differential equations of motion. The purpose here was to provide the student with some tools, so that simple problems can be worked early in the course. The author has found this to be a helpful motivating factor for the student.

A book is a personal thing to any author and it obviously reflects his or her individual background, experience, and current view of the subject. This writer has been fortunate to have had the opportunity to interact with some of the most prolific workers in the field: Robert M. Drake, Jr., George Mellor, Antonio Ferri, Paul Libby, and Edward R. Van Driest. To them, sincere thanks are due. Special thanks are due Roger Eichhorn, who taught me the value of a combined experimental and analytical approach to any new boundary layer problem. Finally, several people were kind enough to read early versions of the manuscript and provide constructive comments. My thanks to David Rooney, Heehwan Lee, George Wills, Dinshaw Contractor, Herman Krier, Felix Pierce, and George Inger.

Joseph A. Schetz
Blacksburg, Va.
July 1982

Upper Division Mechanical or Chemical Engineering	Upper Division Aerospace Engineering	Upper Division Civil or Ocean Engineering
Chap. 1	Chap. 1	Chap. 1
Chap. 2	Chap. 2	Chap. 2 (Skip Secs. 2-4, 2-5, 2-6, and 2-7)
Chap. 3	Chap. 3	Chap. 3 (Skip Secs. 3-4 and 3-5)
Chap. 4	Chap. 4	Chap. 4 (Skip Secs. 4-4 and 4-5)
Skip Chap. 5	Chap. 5 (Skip Secs. 5-4, 5-6, and 5-8-1)	Skip Chap. 5
Chap. 6 (Skip Secs. 6-2, 6-3, and 6-5-6)	Chap. 6 (Skip Secs. 6-2 and 6-3)	Chap. 6 (Skip Secs. 6-2, 6-3, and 6-5-6)
Chap. 7 (Skip Secs. 7-10, 7-11, 7-12, 7-13, and 7-14)	Chap. 7 (Skip Secs. 7-10, 7-11, 7-12, 7-13, and 7-14)	Chap. 7 (Skip material on injection and Secs. 7-10, 7-11, 7-12, 7-13, and 7-14)
Chap. 8 (Skip Secs. 8-4, 8-5, 8-6, and 8-7)	Skip Chap. 8	Chap. 8 (Skip Secs. 8-2-2, 8-2-3, variable-density material, 8-3-2, 8-4, 8-5, 8-6, and 8-7)
Chap. 9 (Skip high-speed-flow material in Secs. 9-2-1 and 9-2-2, and Secs. 9-3-3, 9-3-4, and 9-3-5)	Chap. 9 (Skip Secs. 9-3-3, 9-3-4, and 9-3-5)	Skip Chap. 9
		Ocean Engineering *Add* outside material on free surface effects
		Civil Engineering *Add* outside material on open channel flows and sedimentation

First-Year Graduate Mechanical or Chemical Engineering	First-Year Graduate Aerospace Engineering	First-Year Graduate Civil or Ocean Engineering
Chap. 1 (Review only)	Chap. 1 (Review only)	Chap. 1 (Review only)
Appendix B	Appendix B	Appendix B
Skip Chap. 2 (*or* Review only)	Skip Chap. 2 (*or* Review only)	Skip Chap. 2 (*or* Review only)
Chap. 3 (Review only)	Chap. 3 (Review only)	Chap. 3 (Review only) (Skip Secs. 3-4 and 3-5)
Chap. 4	Chap. 4	Chap. 4
Skip Chap. 5	Chap. 5	Skip Chap. 5
Chap. 6 (Skip Sec. 6-5-6)	Chap. 6	Chap. 6 (Skip Sec. 6-5-6)
Chap. 7 (Review only Secs. 7-3-1, 7-3-2, and 7-7)	Chap. 7 (Review only Secs. 7-3-1, 7-3-2, and 7-7)	Chap. 7 (Skip material on injection, review only Secs. 7-3-1, 7-3-2, 7-7)
Chap. 8	Chap. 8	Chap. 8 (Skip Secs. 8-2-2 and 8-2-3 and variable-density material in Sec. 8-3-2)
Chap. 9 (Skip high-speed-flow material)	Chap. 9	Skip Chap. 9
		Civil Engineering *Add* advanced outside material on open channel flow and sedimentation
		Ocean Engineering *Add* advanced outside material on free surface effects

NOTATION

a	Speed of sound and amplification factor
A	Area or constant
$b_{1/2}$	Half-width
B_1, B_2, B'_1, B'_2	Constants
c	Average speed of molecules
c'_i	Fluctuating value of species concentration
c_i	Species concentration
C_i	Mean value of species concentration
C_1, C_2, etc.	Constants
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
c_r, c_i	Real and imaginary parts of the phase velocity
C_f	Skin friction coefficient
$\bar{C}_f$	Average skin friction coefficient
C_p	Pressure coefficient
C_D	Drag coefficient
D, d	Diameter
D_{ij}	Binary diffusion coefficient
D_T	Turbulent diffusion coefficient
e	Internal energy
$f(\cdot)$	Function of $(\cdot)$

f_i	Body force vector
g	Acceleration of gravity
h	Enthalpy
$\hbar$	Film coefficient
$\hbar_D$	Film coefficient for diffusion
$H(\Lambda)$	Shape factor
i	$\equiv \sqrt{-1}$
j	Index
J	Integrated momentum flux
k	Thermal conductivity and average roughness size
k_T	Turbulent thermal conductivity
k_1	Wave number of fluctuations
K_c, K_{cp}	Mass transfer parameters
K_1, K_2, etc.	Constants
K	Turbulent kinetic energy
$E_1(k_1)$	Kinetic energy of axial fluctuations at wave number k_1
ℓ	Turbulent scale length
ℓ_m	Mixing length
Le	Lewis number
Le_T	Turbulent Lewis number
m	Index along surface
$\dot{m}_i$	Diffusive mass flux of species i
M	Maximum value of m and Mach number
n	Index across layer
N	Maximum value of n
Nu	Nusselt number
Nu_{Diff}	Nusselt number for diffusion
p	Pressure
p_i	Partial pressure of species i
P	Mean pressure
P_C, P_T, P_V	Power law decay exponents
p'	Fluctuating pressure
Pr	Prandtl number
Pr_T	Turbulent Prandtl number
$\mathscr{P}$	Production of turbulent kinetic energy
q_i	Heat flux vector
q_T	Turbulent heat flux
q_w	Wall heat transfer rate

r	Radial coordinate and recovery factor
R	Pipe radius, gas constant, and radius of curvature
Ri	Richardson number
$r_0(x)$	Body radius
$r_{1/2}$	Half-radius
Re	Reynolds number
s	Transformed streamwise coordinate
Sc	Schmidt number
Sc_T	Turbulent Schmidt number
St	Stanton number
St_{Diff}	Stanton number for diffusion
$S(\Lambda)$	Shear parameter
t	Time
T	Static temperature
$\mathbf{T}_{x,y,z}$	Surface force vector
T_b	Bulk temperature
T^*	Reference temperature
T_t	Total (stagnation) temperature
T_0	Time period
$\bar{T}$	Mean temperature
T'	Fluctuating temperature
T_*	Heat transfer temperature
T^+	$\equiv (T_w - \bar{T})/T_*$
u	Streamwise velocity
u_{ave}	Average velocity
U	Mean velocity
$\tilde{U}$	Mass-weighted mean velocity
u'	Fluctuating velocity
u_*	Friction velocity
u^+	$\equiv U/u_*$
v	Transverse or radial velocity
v_w	Transverse velocity at the wall
V	Mean transverse velocity and general velocity
v_0^+	Dimensionless transverse velocity at the wall
v'	Fluctuating transverse velocity
x	Streamwise coordinate
X_i	Mole fraction of species i
y	Transverse coordinate

Y	Transformed transverse coordinate
$y^+ \equiv yu_*/\nu$	Transverse coordinate for the law of the wall
W_i	Molecular weight
$W(y/\delta)$	Wake function
Z	$\equiv k^m l^n$

Greek

α	Wave number and amplification factor
α_T	$\equiv k_T/\rho c_p$
β	Pressure gradient parameter and wave number
ψ	Planar stream function
Ψ	Axisymmetric stream function
$\hat{\psi}$	Disturbance stream function
ε	Dissipation of turbulent energy
$\varepsilon_{n,m}$	Truncation error
ε_{xy}	Strain
ρ	Density
λ	Pohlhausen pressure gradient parameter, pipe resistance coefficient, and second viscosity coefficient
λ^*	Mean free path between molecules
Λ	Thwaites–Walz pressure gradient parameter
Λ_t	Smith and Spalding parameter
τ, τ_{xy}	Shear
τ_T	Turbulent shear
Ω	Intermittency
μ	Laminar viscosity
μ_T	Turbulent viscosity
κ	Constant in the law of the wall
κ_T	Constant in the Temperature law of the wall
ν	Laminar kinematic viscosity
ν_T	Turbulent kinematic viscosity
ϕ	Amplitude function
δ	Boundary layer thickness
δ_t	Conduction thickness
δ_T	Thermal boundary layer thickness
δ_c	Concentration boundary layer thickness
δ^*, Δ^*	Displacement thickness
δ_k^*	Kinematic displacement thickness

θ	Momentum thickness
Θ, θ_r	Excess temperature
Δ	Clauser integral boundary layer thickness
ζ	$\equiv \delta_T/\delta$
$\phi_{1,2}$	Deformation angle
Π	Wake parameter
ξ	Dummy variable
η	Similarity variable
$\bar{\eta}$	Transformed transverse coordinate
ω	Dimensionless frequency and viscosity law exponent
$\sigma_{K,z,\tau}$	"Prandtl" numbers for K, Z, τ
Γ	See Eq. (9-8)
γ	Ratio of specific heats

Subscripts

c	Values on the centerline
e	Values at the edge of the boundary layer
j	Initial values in a jet
t	Stagnation values
w	Wall values
∞	Conditions in the approach flow

CONTENTS