OXFORD LECTURE SERIES IN MATHEMATICS AND ITS APPLICATIONS · 27

Introduction to the Mathematical Theory of Compressible Flow

Antonín Novotný
and Ivan Straškraba

Introduction to the Mathematical Theory of Compressible Flow

A. Novotný
Université du Sud Toulon–Var

I. Straškraba
Mathematical Institute of the Academy of Sciences of the Czech Republic

OXFORD
UNIVERSITY PRESS

OXFORD
UNIVERSITY PRESS

Great Clarendon Street, Oxford Ox2 6DP

Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship,
and education by publishing worldwide in

Oxford New York

Auckland Bangkok Buenos Aires Cape Town Chennai
Dar es Salaam Delhi Hong Kong Istanbul Karachi Kolkata
Kuala Lumpur Madrid Melbourne Mexico City Mumbai Nairobi
São Paulo Shanghai Taipei Tokyo Toronto

Oxford is a registered trade mark of Oxford University Press
in the UK and in certain other countries

Published in the United States
by Oxford University Press Inc., New York

First published 2004

British Library Cataloguing in Publication Data
Data available

Library of Congress Cataloging in Publication Data
Data available

ISBN 0-19-853084-6 (Hbk)

10 9 8 7 6 5 4 3 2 1

Typeset by the author in LaTeX
Printed in Great Britain
on acid-free paper by
Biddles Ltd. www.biddles.co.uk

Oxford Lecture Series in
Mathematics and its Applications 27

Series editors
John Ball Dominic Welsh

OXFORD LECTURE SERIES IN MATHEMATICS AND ITS APPLICATIONS

1. J. C. Baez (ed.): *Knots and quantum gravity*
2. I. Fonseca and W. Gangbo: *Degree theory in analysis and applications*
3. P.-L. Lions: *Mathematical topics in fluid mechanics, Vol. 1: Incompressible models*
4. J. E. Beasley (ed.): *Advances in linear and integer programming*
5. L. W. Beineke and R. J. Wilson (eds): *Graph connections: Relationships between graph theory and other areas of mathematics*
6. I. Anderson: *Combinatorial designs and tournaments*
7. G. David and S. W. Semmes: *Fractured fractals and broken dreams*
8. Oliver Pretzel: *Codes and algebraic curves*
9. M. Karpinski and W. Rytter: *Fast parallel algorithms for graph matching problems*
10. P.-L. Lions: *Mathematical topics in fluid mechanics, Vol. 2: Compressible models*
11. W. T. Tutte: *Graph theory as I have known it*
12. Andrea Braides and Anneliese Defranceschi: *Homogenization of multiple integrals*
13. Thierry Cazenave and Alain Haraux: *An introduction to semilinear evolution equations*
14. J. Y. Chemin: *Perfect incompressible fluids*
15. Giuseppe Buttazzo, Mariano Giaquinta and Stefan Hildebrandt: *One-dimensional variational problems: an introduction*
16. Alexander I. Bobenko and Ruedi Seiler: *Discrete integrable geometry and physics*
17. Doina Cioranescu and Patrizia Donato: *An introduction to homogenization*
18. E. J. Janse van Rensburg: *The statistical mechanics of interacting walks, polygons, animals and vesicles*
19. S. Kuksin: *Hamiltonian partial differential equations*
20. Alberto Bressan: *Hyperbolic systems of conservation laws: the one-dimensional Cauchy problem*
21. B. Perthame: *Kinetic formulation of conservation laws*
22. A. Braides: *Gamma-convergence for beginners*
23. Robert Leese and Stephen Hurley: *Methods and algorithms for radio channel assignment*
24. Charles Semple and Mike Steel: *Phylogenetics*
25. Luigi Ambrosio and Paolo Tilli: *Topics on analysis in metric spaces*
26. Eduard Feireisl: *Dynamics of viscous compressible fluids*
27. Antonín Novotný and Ivan Straškraba: *Introduction to mathematical theory of compressible flow*

For Stania, Filip
and Jakub

For Jana

PREFACE

The topic of this book is a mathematical description of Newtonian compressible fluids in the steady and unsteady regime. The history of attempts to describe rigorously the flow of a compressible fluid covered a long time beginning from observations of L. Euler in the middle of the 18th century and of C. Navier (Navier, 1827), H. Poisson (Poisson, 1829) and G. Stokes (Stokes, 1845) in the first half of the 19th century, and continue up to now. Despite the fact that the governing equations, called the Euler equations (in the inviscid case) and Navier–Stokes equations (in the viscous case), have been known for a very long time we are far from being satisfied with the completeness of their mathematical analysis (in both cases). Nevertheless the considerable effort of outstanding analysts cited throughout the book brought its fruits and a great number of nontrivial results for compressible fluids has been achieved. This book is an attempt to map the situation in the mathematical theory of compressible flow and present important and up-to-date results in a clear instructive form accessible to a wide audience despite the sophisticated techniques used to overcome modest a priori information derived directly from the equations.

We started with the realization of a rather different project including also a numerical treatment of the Euler and Navier–Stokes equations with M. Feistauer and J. Felcman around 1998. It soon appeared that the scope was too large for one monograph and so we accepted with great relief the proposal from Oxford University Press to split the book into two separate monographs, the first of which (Feistauer *et al.*, 2003) has already been published.

As already mentioned, the book covers Newtonian compressible fluids, more specifically Euler equations and Navier–Stokes equations in isentropic or barotropic regimes. We do not deal with heat conducting flows except for references to results for small data (that is under the assumption that appropriate norms of the given quantities are small enough). There is currently beeing published a research monograph by E. Feireisl (Feireisl, 2003*a*) devoted to this subject. There is a vast literature about different kinds of non-Newtonian fluids and we do not go into this business at all. So in this respect the present monograph covers only a part of the available mathematical results for compressible fluids.

We have adopted a textbook style. Even well-known basic theorems are recalled in the introductory chapter. This makes the book essentially selfcontained. There are sections called *heuristic approach*, where we describe the main ideas of proofs. These sections may be sufficient for an experienced reader to understand the subject without wasting time in numerous details. On the other hand, less experienced readers, nonspecialists and students will find in the book even standard technical details.

Let us briefly describe the contents. We start with the introductory Chapter 1, where different models for compressible fluids are derived and some fundamental mathematical results are surveyed. The results are given without proofs but detailed references are given there.

Chapter 2 surveys the theoretical aspects of the Euler system for inviscid compressible fluids with the necessary background from the theory of hyperbolic conservation laws. Representative local and global existence results are proved in detail, relying on recent publications, to give the present state of affairs. The generality here is modest, and this is mostly due to the lack of results for the Euler equations.

Chapter 3 is preparatory for the subsequent treatment of Navier–Stokes systems. Some specific mathematical tools for these equations, adjusted especially to steady equations, are developed here. The proofs, unlike Chapter 1, are mostly given and only in a few cases are they cited.

Chapter 4 is devoted to the theory of weak solutions for steady Navier–Stokes systems for compressible fluids with large data (that is without the restriction described above as the assumption of small data) in the barotropic regime. A complete and detailed proof of the existence of weak solutions is given and modifications for unbounded and exterior domains as well as for different boundary conditions are discussed thoroughly. A survey of known results on this issue is given in the bibliographic remarks.

Chapter 5 concerns strong solutions of steady Navier–Stokes equations. The existence of regular solutions is proved, paid for by the assumption of small data. Note that *unconditional* regularity of solutions both for steady and nonsteady equations is not known.

Chapter 6 again collects advanced mathematical tools, now adjusted to nonsteady problems. This includes properties of abstract functions in Bochner spaces, commutators and the study of the (renormalized) equation of continuity.

Chapter 7 is mainly devoted to the weak existence theory for nonsteady Navier–Stokes equations in the barotropic regime. Again a discussion of different regions and boundary conditions is included.

In Chapter 8, the global behavior of solutions in time is investigated and the related equilibrium problem is described.

In the final Chapter 9, the existence of strong solutions for nonsteady Navier–Stokes equations is studied and available existence and uniqueness results are reviewed.

Chapter 2 dealing with the Euler equations is essentially self-contained and can be read independently of the other chapters. The same is true for Chapter 4 which deals with weak solutions of steady barotropic Navier–Stokes equations (and which requires only Chapter 3 to be exhaustive) and for Chapter 7 dealing with weak solutions of nonsteady barotropic Navier–Stokes equations (which needs only Chapter 3 and Section 4.4 of Chapter 4 to form a complete treatment). Each of Chapters 5 (about strong solutions for steady equations), 8 (about large time behavior of weak solutions) and 9 (about strong solutions in the nonsteady regime) are essentially self-contained as well. Also, we attempt to treat all investigated systems in a uniform way taking into account their common nature.

We are grateful to E. Feireisl, G. P. Galdi, J. Heywood, P. Krejčí, V. Lovicar, J. Málek, S. Nazarov, J. Nečas, J. Neustupa, S. Novo, M. Padula, P. Penel, H. Petzeltová, K. Pileckas, M. Pokorný, M. Růžička, R. Salvi, A. Sequeira, A. Valli and A. Zlotnik, who are coauthors with at least one of us of several papers. We enjoyed working with them on more than one problem related to the subject.

In particular, we wish to express warm thanks to our friends who helped us to manage the task of the book project by reading the manuscript and with numerous valuable discussions; in alphabetical order: E. Feireisl, J. Heywood, S. Novo, M. Pokorný. Warm thanks also to S. Novotná who typeset, with care, part of the manuscript and compiled the bibliography.

Secondly, we would like to acknowledge three stays in Mathematisches Forschungsinstitut in Oberwolfach in the program Research in Pairs which helped us very much in the coordination of our work. We also thank the Grant Agency of the Czech Republic for financial support from projects nos. 201/02/0684, 201/02/0854, and the Czech and French Ministry of Education for support in the frame of project Barrande 99, no. 99004.

Our special thanks go, of course, to our families, without whose support we would not have been able to complete this extensive project.

Carqueiranne and Prague,
September 2003

A. N. & I. S.

CONTENTS